

# A Study On Bisets

K. Karpagam<sup>1</sup> And G. Ramesh <sup>2</sup>

<sup>1</sup> Research Scholar, Department of Mathematics,  
Government Arts College(Autonomous), Kumbakonam, Tamil Nadu, India. Affiliated to  
Bharathidasan University, Tiruchirappalli-620 024.

<sup>2</sup> Associate Professor, Department of Mathematics,  
Government Arts College(Autonomous), Kumbakonam, Tamil Nadu, India. Affiliated to  
Bharathidasan University, Tiruchirappalli-620 024.

## Abstract

The concept of biset is introduced, equality, cardinality, complement, subsets of a biset, union, intersection, symmetric difference of two bisets are defined. We also investigated the concepts of functions on a biset and discussed some of their properties.

**AMS Subject Classification:** 03C55, 03E15, 03E20

**Keywords:** Bisets,  $\mathcal{B}$ -union,  $\mathcal{B}$ -intersection,  $\mathcal{B}$ -open and  $\mathcal{B}$ -closed sets.

## 1 Introduction

The modern study of set theory was initiated by the German mathematician Richard Dedekind and Georg Cantor in 1870s. In particular, Georg Cantor is commonly considered as the founder of set theory. The non-formalized systems investigated during this early stage go under the name of naïve set theory. In mathematics, a set is a collection of elements [7][14][18]. The elements that make up a set can be any kind of mathematical objects: numbers, symbols, points in space, lines, other geometrical shapes, variables, or even other sets [5] The set with no element is the empty set; a set with a single element is a singleton. A set may have a finite number of elements or be an infinite set. Two sets are equal if and only if they have precisely the same elements [16].

Sets are ubiquitous in modern mathematics. Indeed, set theory, more specifically Zermelo-Fraenkel set theory, has been the standard way to provide rigorous foundations for all branches of mathematics since the first half of the 20th century [5].

The concept of a set emerged in mathematics at the end of the 19th century [8]. The foremost property of a set is that it can have elements, also called members; this property is called the extensionality of sets [5].

Once applications to analysis began to be found, however, attitude began to change, and by the 1890s Cantor's ideas and results were gaining acceptance. By 1900, set theory was recognized as a distinct branch of mathematics. Gottlob Frege, discovered, on his own, the fundamental ideas that have made possible the whole modern development of logic and thereby invented an entire discipline.

In this paper, we define biset, equal bisets, cardinality, complement, subsets of a biset, union, intersection and symmetric difference of two bisets, functions on

a biset, open and closed biset(that is,  $\mathcal{B}$ -open and  $\mathcal{B}$ -closed). Then , we study its related properties and obtained some relations between set and biset.

## 2 Preliminaries

In this section we present the preliminary definitions with examples.

**Definition 2.1** *A biset is simply a pair of collection of distinct objects. The number of objects contained within a biset may be finite, countably infinite or uncountably infinite.*

*It is written as  $A_B = A_1 \cup A_2$ , where  $A_1$  and  $A_2$  are ordinary sets with  $A_1 \neq A_2$ . That is, a biset  $A_B$  is defined as a pair of two sets  $A_1$  and  $A_2$  where ' $\cup$ ' is just for the notational convenience(symbol)only.*

**Example 2.2** *Let  $A_B = A_1 \cup A_2 = \{a, b, c\} \cup \{1, 2\}$ , here  $A_1 = \{a, b, c\}$  and  $A_2 = \{1, 2\}$ .*

*Let  $A_B = A_1 \cup A_2 = \{2, 3, 4\} \cup \{p, q, r\}$ , here  $A_1 = \{2, 3, 4\}$  and  $A_2 = \{p, q, r\}$ .*

**Result 2.3** *If  $A_B = A_1 \cup A_2$  is a biset, then the sets  $A_1$  and  $A_2$  are called components of the biset.*

*If  $A_B = A_1 \cup A_2$  with  $A_1 = A_2$  then  $A_B$  is not a biset, except for the empty set.*

*We consider the components  $A_1, A_2$  are of different characteristics.*

**Remark 2.4** *A biset is said to be finite if both components are finite.*

*A biset is said to be infinite if any one or both the components are infinite.*

*A biset is said to be countable if both components are countable.*

*A biset is said to be uncountable if any one or both components are uncountable.*

**Definition 2.5** *The biset membership symbol  $\in$  is used to say that an object or a pair is a member of the biset  $A_B$ . It has a partner symbol  $\notin$  which is used to say an object is not in the components.*

**Example 2.6** *Let  $A_B = A_1 \cup A_2$ , let  $x_B = x_1 \cup x_2$  or  $(x_1, x_2) \in A_B$  which implies  $x_1 \in A_1$  and  $x_2 \in A_2$ . Let  $A_B = \{1, 2, 3\} \cup \{a, b, c\}$  where  $A_1 = \{1, 2, 3\}$  and  $A_2 = \{a, b, c\}$ , then 1,2,3 are elements of  $A_1$  and a,b,c are elements of  $A_2$ .*

*That is,  $A_B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c)\}$ .*

**Definition 2.7** *Two biset  $A_B$  and  $C_B$  are said to be equal (briefly  $\mathcal{B}$ -equal) if they have exactly the same components. If  $A_B = A_1 \cup A_2$  and  $B_B = B_1 \cup B_2$  be two bisets we say that  $A_B$  and  $B_B$  are equal and written as  $A_B = B_B$  if and only if  $A_1$  and  $B_1$  are identical and  $A_2$  and  $B_2$  are identical.*

*That is,  $A_1 = B_1$  and  $A_2 = B_2$ . Otherwise they are not equal.*

**Example 2.8** *Let  $A_B = \{x/x \text{ is a prime number, } x < 7\} \cup \{y/y \text{ is a vowel of the alphabet}\}$  and  $B_B = \{2, 3, 5\} \cup \{a, e, i\}$ , then  $A_B = B_B$ .*

*Let  $A_B = \{x/x \text{ is not a prime number, } x < 7\} \cup \{y/y \text{ is a vowel of the alphabet}\}$  and  $B_B = \{1, 2, 4\} \cup \{b, c, d, m\}$  then  $A_B \neq B_B$ .*

**Definition 2.9** Let  $A_B = A_1 \cup A_2$ , let  $|A_1| = \alpha_1$  and  $|A_2| = \alpha_2$  then the cardinality  $A_B$  denoted by  $|A_B| = \alpha_1\alpha_2$  notationally it is written as  $(\alpha_1, \alpha_2)$ .

The cardinality of biset maybe  $(0,0)$  then, it is called empty biset. In other words, the empty biset is defined as  $\emptyset_B = \emptyset$  where  $\emptyset$  is a empty set.

**Definition 2.10** The union of two bisets  $A_B$  and  $B_B$  is the collection of pair of all objects they are in either one of the components. It is written as

$C_B = C_1 \cup C_2 = A_B \cup B_B$ , where  $C_1 = A_1 \cup B_1$  and  $C_2 = A_2 \cup B_2$ . Using set builder form,

$$C_B = A_B \cup B_B = \{x_B = (x_1, x_2) : x_1 \in A_1 \cup B_1 \text{ and } x_2 \in A_2 \cup B_2\}$$

where  $A_B = A_1 \cup A_2$  and  $B_B = B_1 \cup B_2$ . It is also called as  $\mathcal{B}$ -union.

**Definition 2.11** The intersection of two bisets  $A_B$  and  $B_B$  is the collection of pair of all objects they are in both the components. It is written as

$$D_B = D_1 \cap D_2 = A_B \cap B_B, \text{ where } D_1 = A_1 \cap B_1 \text{ and } D_2 = A_2 \cap B_2. \text{ That is,}$$

$$D_B = A_B \cap B_B = \{x_B = (x_1, x_2) : x_1 \in A_1 \cap B_1 \text{ and } x_2 \in A_2 \cap B_2\}$$

where  $A_B = A_1 \cup A_2$  and  $B_B = B_1 \cup B_2$ . It is also called as  $\mathcal{B}$ -intersection.

**Example 2.12** For the biset  $A_B = \{1, 2, 3, 4, 5\} \cup \{a, b, c, d\}$  where  $A_1 = \{1, 2, 3, 4, 5\}$  and  $A_2 = \{a, b, c, d\}$ , then we have  $|A_B| = (5, 4)$  with  $|A_1| = 5$  and  $|A_2| = 4$ .

Let  $A_B = \{1, 2, 3, 4, 5\} \cup \{a, b, c, d\}$  and  $B_B = \{1, 3, 5, 6\} \cup \{a, c, e\}$  where  $A_B = A_1 \cup A_2$  and  $B_B = B_1 \cup B_2$ .

$$\begin{aligned} \text{Then } C_B &= A_B \cup B_B \\ &= (A_1 \cup A_2) \cup (B_1 \cup B_2) \\ &= (A_1 \cup B_1) \cup (A_2 \cup B_2) \\ C_B &= \{1, 2, 3, 4, 5, 6\} \cup \{a, b, c, d, e\}. \end{aligned}$$

$$\begin{aligned} \text{Then } D_B &= A_B \cap B_B \\ &= (A_1 \cup A_2) \cap (B_1 \cup B_2) \\ &= (A_1 \cap B_1) \cup (A_2 \cap B_2) \\ D_B &= \{1, 3, 5\} \cup \{a, c\}. \end{aligned}$$

**Definition 2.13** If  $A_B$  and  $B_B$  are bisets and  $A_B \cap B_B = \emptyset$ , that is,  $A_1 \cap B_1 = \emptyset$  and  $A_2 \cap B_2 = \emptyset$ , then we say that  $A_B$  and  $B_B$  are disjoint bisets.

**Example 2.14** If  $A_B = \{1, 2\} \cup \{a, b, c\}$  and  $D_B = \{3, 4\} \cup \{d, e, f\}$  Then,  $A_B \cap B_B = \emptyset$ .

**Definition 2.15** The universal biset is the set of pair of all possible objects. The universal bisets is commonly written as  $U_B = U_1 \cup U_2$  where  $U_1$  and  $U_2$  are universal sets of the first and second components.

**Definition 2.16** The complement of a biset  $A_B$  is the collection of pair of all objects in the universal bisets that are not in  $A_B$ . That is,  $A_B^c = A_1^c \cup A_2^c$ . It is also called as  $\mathcal{B}$ -complement.

In set builder form,

$$\begin{aligned} A_B^c &= \{x_B : x_1 \notin A_1 \text{ and } x_2 \notin A_2\} \text{ or more compactly as} \\ A_B^c &= \{x_B : (x_B \notin A_B)\} \end{aligned}$$

**Definition 2.17** The symmetric difference of two bisets  $A_B$  and  $B_B$  is the biset of pair of objects that are in one and only one of the bisets. The symmetric difference is written as  $A_B \Delta B_B = D_B$ .

In curly brace notation

$$D_B = A_B \Delta B_B = \{(A_B - B_B) \cup (B_B - A_B)\}$$

**Example 2.18** Suppose that  $U_B = \{1, 2, 3, 4, 5\} \cup \{a, b, c, d\}$ , if  $A_B = \{1, 2, 3\} \cup \{a, b, d\}$  and  $B_B = \{1, 3, 5\} \cup \{a, c, d\}$ ,

$$\begin{aligned} \text{Then } A_B - B_B &= (A_1 \cup A_2) - (B_1 \cup B_2) \\ &= (A_1 - B_1) \cup (A_2 - B_2) \end{aligned}$$

$$A_B - B_B = \{2\} \cup \{b\}.$$

Let  $A_B = \{1, 2, 3\} \cup \{a, b, d\}$  and  $B_B = \{1, 3, 5\} \cup \{a, c, d\}$

$$\begin{aligned} \text{Then } B_B - A_B &= (B_1 \cup B_2) - (A_1 \cup A_2) \\ &= (B_1 - A_1) \cup (B_2 - A_2) \end{aligned}$$

$$B_B - A_B = \{5\} \cup \{c\}$$

$$D_B = A_B \Delta B_B = \{(A_B - B_B) \cup (B_B - A_B)\}$$

$$D_B = \{[(2) \cup (b)] \cup [(5) \cup (c)]\}$$

$$D_B = \{(2, 5) \cup (b, c)\}$$

**Definition 2.19** For two bisets  $A_B = A_1 \cup A_2$  and  $B_B = B_1 \cup B_2$ , we say that  $A_B$  is a subset of a biset  $B_B$  if each element of  $A_1$  is also an element of  $B_1$  and each element of  $A_2$  is also an element of  $B_2$ . In formal notation we write  $A_B \subseteq B_B$  where  $A_1 \subseteq B_1$  and  $A_2 \subseteq B_2$ .

If  $A_B \subseteq B_B$  then we also say  $B_B$  contains  $A_B$  which can be written as  $B_B \supseteq A_B$ . If  $A_B \subseteq B_B$  and  $A_B \neq B_B$ , then we write  $A_B \subset B_B$  and we say that  $A_B$  is a proper subset of the biset  $B_B$ .

**Example 2.20** If  $A_B = \{1, 2, 3\} \cup \{a, b\}$  then

$$C_B = \{1, 2\} \cup \{a, b\},$$

$$D_B = \{2, 3\} \cup \{b\},$$

$$E_B = \{\} \cup \{a\} \text{ are some different subsets of the biset } A_B.$$

**Definition 2.21** The set of all subsets of a biset  $A_B$  is called the powerset of the biset  $A_B$ . We denote it is  $\wp(A_B)$ .

$$\text{That is, } \wp(A_B) = \wp(A_1) \cup \wp(A_2)$$

**Example 2.22** Let  $A_B = \{1, 2, 3\} \cup \{a, b, c\}$ , then  $\wp(A_B)$

$$= \{\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\} \cup \{\{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

**Definition 2.23** If  $A_B$  and  $B_B$  are non-empty bisets, then a mapping on a biset  $f_B : A_B \rightarrow B_B$  is a rule which assigns to each member of  $A_B$  a unique member in  $B_B$ . We call  $A_B$  as the  $\mathcal{B}$ -domain and  $B_B$  as the  $\mathcal{B}$ -codomain of the mapping. If  $f_B$  is a function on a biset then we denote the domain of  $f_B$  by  $\mathcal{B}\text{-dom}(f_B)$  and the range of  $f_B$  by  $\mathcal{B}\text{-rng}(f_B)$ .

That is, if  $f_B : A_B \rightarrow B_B$  is mapping on a biset ( $\mathcal{B}$ -mapping), so that  $f_B(x_1, x_2) = (y_1, y_2) = (f_1(x_1), f_2(x_2))$  for all  $x_1 \in A_1, x_2 \in A_2$  and  $y_1 \in B_1, y_2 \in B_2$ .

**Definition 2.24** An ordered pair of two bisets is a collection of components with the ordered property that one element comes first and another element comes second. The biset containing only  $x_B$  and  $y_B$  (for  $x_B \neq y_B$ ) is written as  $(x_B, y_B)$ . The ordered pair of two bisets containing  $x_B$  and  $y_B$  with  $x_B$  first is written as  $(x_B, y_B)$ .

**Definition 2.25** A function on a biset  $f_B : A_B \rightarrow B_B$  is  $\mathcal{B}$ -one-one ( $\mathcal{B}$ -injective) if distinct elements in  $A_B$  have distinct images in  $B_B$  under  $f_B$ .

In other words,  $f_B$  is one-one, if  $(x_B, y_B) \in A_B$  and  $x_B \neq y_B$

That is,  $(x_1, x_2) \neq (y_1, y_2)$  implies that  $f_B(x_1, x_2) \neq f_B(y_1, y_2)$  or equivalently,  $f_B(x_1, x_2) = f_B(y_1, y_2)$  implies that  $(x_1, x_2) = (y_1, y_2)$ .

The mapping on a biset ( $\mathcal{B}$ -mapping)  $f_B$  is called  $\mathcal{B}$ -onto ( $\mathcal{B}$ -surjective) if the range of  $f_B$  is  $\mathcal{B}$ -equal to  $B_B$ . Thus if  $f_B$  is  $\mathcal{B}$ -onto, every element of  $B_B$  has a pre-image in  $A_B$ .

If  $f_B : A_B \rightarrow B_B$  is both one-one and onto then  $f_B$  is called a  $\mathcal{B}$ -bijection.

**Definition 2.26** Let  $X_B, Y_B$  and  $Z_B$  are bisets. The bicomposition of two functions on a biset  $f_B : X_B \rightarrow Y_B$  and  $g_B : Y_B \rightarrow Z_B$  are function on a biset  $h_B : X_B \rightarrow Z_B$  for which  $h_B(x_B) = g_B[f_B(x_B)]$  for all  $x_B \in X_B$ .

That is,  $f_B(x_1, x_2) = f_B(y_1, y_2)$  and  $g_B(x_1, x_2) = (z_1, z_2)$

Then  $h_B(x_1, x_2) = g_B(y_1, y_2) = (z_1, z_2)$ . We write  $g_B \circ f_B = h_B$  for the composition of  $g_B$  with  $f_B$ .

**Definition 2.27** Let  $f_B : A_B \rightarrow B_B$  be a bijection on a biset  $A_B$ . Then for each  $b_B \in B_B$ , there exists a unique element  $a_B \in A_B$  such that  $f_B(a_1, a_2) = (b_1, b_2)$ .

Define  $f_B^{-1} : A_B \rightarrow B_B$  by  $f_B^{-1}(b_1, b_2) = (a_1, a_2)$ ,  $f_B^{-1}$  is called the  $\mathcal{B}$ -inverse of the  $\mathcal{B}$ -function  $f_B$ .

**Definition 2.28** Let  $A_B$  be any biset. The functions on a biset  $i_{A_B} : A_B \rightarrow A_B$  defined by  $i_{A_B}(x_B) = x_B$  for all  $x_B \in A_B$  is called the identity function on the biset  $A_B$ .

That is,  $i_{A_B}(x_1, x_2) = (x_1, x_2)$  where  $x_1 \in A_1$  and  $x_2 \in A_2$ . Thus  $i_{A_B}$  leaves every element of  $A_B$  fixed.

**Definition 2.29** If  $A_B$  and  $B_B$  are bisets then the set of all ordered pair of two bisets with the first element from  $A_B$  and the second from  $B_B$  is called the  $\mathcal{B}$ -cartesian product of  $A_B$  and  $B_B$ . It is denoted by  $A_B \times B_B$ . That is,

$$A_B \times B_B = (A_1 \times B_1) \cup (A_2 \times B_2) \\ = \{(a_1, b_1) \cup (a_2, b_2) : a_1 \in A_1, a_2 \in A_2 \text{ and } b_1 \in B_1, b_2 \in B_2\}.$$

**Definition 2.30** A biset  $N_B \subset R_B$  is said to be a  $\mathcal{B}$ -neighbourhood of a point  $p_B \in R_B$  if there exists a  $\mathcal{B}$ -open interval  $]a_1, b_1[ \cup ]a_2, b_2[$  containing  $p_B$  and contained in  $N_B$ . If  $N_B$  is a neighbourhood of  $p_B$ , then we say that  $p_B$  is a  $\mathcal{B}$ -interior point of  $N_B$ . That is,  $p_1$  is an interior point of  $N_1$  and  $p_2$  is an interior point of  $N_2$ .

**Definition 2.31** A biset  $G_B \subset R_B$  is said to be  $\mathcal{B}$ -open if it is a neighbourhood of each of its points. A biset  $G_B \subset R_B$  is said to be  $\mathcal{B}$ -open if for each  $p_B \in R_B$ , there exists  $\epsilon_1$  and  $\epsilon_2 > 0$  such that,  $]p_1 - \epsilon_1, p_1 + \epsilon_1[ \subset G_1$  and  $]p_2 - \epsilon_2, p_2 + \epsilon_2[ \subset G_2$ .

**Definition 2.32** A biset  $F_B \subset R_B$  is said to be  $\mathcal{B}$ -closed if its complement (that is,  $R_B \sim F_B$ ) is  $\mathcal{B}$ -open.

**Definition 2.33** A point  $p_B \in R_B$  is said to be a limit point (or an accumulation  $\mathcal{B}$ -point) of a biset  $S_B \subset R_B$ , that is,  $S_1 \subset R_1$  and  $S_2 \subset R_2$ . If for each  $(\epsilon_1, \epsilon_2) > 0$  then  $\mathcal{B}$ -open interval  $]p_1 - \epsilon, p_1 + \epsilon[ \cup ]p_2 - \epsilon, p_2 + \epsilon[$  contains a point of  $S_B$  other than  $p_B$ .

**Proposition 2.34** Two bisets are equal ( $\mathcal{B}$ -equal) if and only if each is contained in another biset

that is,  $(A_B = B_B) \Leftrightarrow (A_B \subseteq B_B) \wedge (B_B \subseteq A_B)$ .

**Proof:** Let  $A_B$  and  $B_B$  be two bisets. Assuming that  $A_B = B_B$ , that is,  $A_1 = B_1$  and  $A_2 = B_2$ . We know that every biset is a subset of a biset to itself, so  $A_B \subseteq A_B$ . That is  $A_1 \subseteq A_1$  and  $A_2 \subseteq A_2$ .

Since  $A_B = B_B$ , we may substitute into this expression on the left and obtain  $B_B \subseteq A_B$ . That is,  $B_1 \subseteq A_1$  and  $B_2 \subseteq A_2$ . Similarly, we may substitute on the right and obtain  $A_B \subseteq B_B$ .

That is,  $A_1 \subseteq B_1$  and  $A_2 \subseteq B_2$ . We have thus demonstrated that if  $A_B = B_B$  then  $A_B$  and  $B_B$  are both subsets of a biset each other.

Conversely, assume that  $A_B \subseteq B_B$  and  $B_B \subseteq A_B$ . Then by definition of subset of a biset, we have any element of  $A_B$  is an element of  $B_B$ . That is,  $A_1$  is an element of  $B_1$  and  $A_2$  is an element of  $B_2$ .

Similarly, any element of  $B_B$  is an element of  $A_B$ . This means that  $A_B$  and  $B_B$  have the same elements. Therefore,  $A_B = B_B$ .  $\square$

**Theorem 2.35** For the given an arbitrary family  $\mathcal{F}$  of open bisets in  $R_B$ . Then the union of two bisets ( $\mathcal{B}$ -union) is  $\mathcal{B}$ -open.

**Proof:** Let  $F_B$  be the union of two bisets of an arbitrary family  $\mathcal{F}$  of open bisets in  $R_B$ .

To prove that  $F_B$  is an open biset.

Consider any  $p_B \in F_B$ . since  $F_B$  is the  $\mathcal{B}$ -union of components of  $\mathcal{F}$ .

Therefore, there must exist an open biset  $H_B \in \mathcal{F}$  such that  $p_B \in H_B \subset F_B$ .

Since  $H_B$  is an open biset and  $p_B \in H_B$ , therefore, there must exist  $(\epsilon_1, \epsilon_2) > 0$  such that  $]p_1 - \epsilon, p_1 + \epsilon[ \cup ]p_2 - \epsilon, p_2 + \epsilon[ \subset H_B \subset F_B$ .

Again, since  $]p_1 - \epsilon, p_1 + \epsilon[ \cup ]p_2 - \epsilon, p_2 + \epsilon[$  is contained in  $F_B$ .

Therefore,  $F_B$  is a neighbourhood of  $p_B$ . Since  $p_B$  is any point of  $F_B$ , it follows that  $F_B$  is a neighbourhood of each of its points, and consequently,  $F_B$  is an open biset.  $\square$

**Theorem 2.36** Let  $G_B$  and  $H_B$  are two open bisets in  $R_B$ . Then intersection of two open bisets is  $\mathcal{B}$ -open

**Proof:** Let  $G_B \subset R_B$  and  $H_B \subset R_B$ . That is,  $G_1 \subset R_1$  and  $H_1 \subset R_1$ ,  $G_2 \subset R_2$  and  $H_2 \subset R_2$  be two open bisets.

If  $G_B \cap H_B = \emptyset_B$ , that is,  $G_1 \cap H_1 = \emptyset_1$  and  $G_2 \cap H_2 = \emptyset_2$ , then it is  $\mathcal{B}$ -open.

If  $G_B \cap H_B \neq \emptyset_B$ , let  $p_B$  be any point of  $G_B \cap H_B$ . Now,  $p_B \in G_B \cap H_B$  which implies that  $p_B \in G_B$  and  $p_B \in H_B$ .

That is,  $p_1 \in G_1 \cup G_2$  and  $p_1 \in H_1 \cup H_2$ ,  $p_2 \in G_1 \cup G_2$  and  $p_2 \in H_1 \cup H_2$ , which implies that  $G_B$  and  $H_B$  are  $\mathcal{B}$ -neighbourhoods of  $p_B$ .

Hence  $G_B \cap H_B$  is a  $\mathcal{B}$ -neighbourhood of  $p_B$ . That is,  $G_1 \cap H_1$  is a neighbourhood of  $p_1$  and  $G_2 \cap H_2$  is a neighbourhood of  $p_2$ .

Since  $p_B$  is any point of  $G_B \cap H_B$ , therefore, it follows that  $G_B \cap H_B$  is a  $\mathcal{B}$ -neighbourhood of each of its points, and consequently,  $G_B \cap H_B$  is an open biset.  $\square$

### 3 Conclusion

In this paper, we introduce the concept of biset, subsets of a biset,  $\mathcal{B}$ -union and  $\mathcal{B}$ -intersection of two bisets. Also, we studied the properties of  $\mathcal{B}$ -open and  $\mathcal{B}$ -closed concept in bispaces. In future, using this concept various types of spaces will be developed.

### References

- [1] Angelo Margaris, First order Mathematical Logic, 1990.
- [2] Dauben, Joseph W.(1979), Georg Cantor: His Mathematics and Philosophy of the Infinite. Boston: Harvard University Press, ISBN 0-691-02447-2.
- [3] Dubreil P., and Dubreil-Jacotin, M.L., Lectures on Modern Algebra, Oliver and Boyd., Edinburgh, 1967.
- [4] Greub, W.H., Linear Algebra, Fourth Edition, Springer-Verlag, 1974.
- [5] Halmos, Paul R. (1960), Naive set Theory, Princeton, N.J.: Van Nostrand ISBN 0-387-90092-6.
- [6] Herstein,IN., Topics in Algebra, John Wiley, 1975.
- [7] Jain P.K.; Khalil Ahmad; P. Ahuja (1995), Functional Analysis, New Age International(P)Limited, publishers, 1995.
- [8] Jose Ferreiro's (1 Nov. 2001)Labyrinth of Thought: A History of set theory and its Role in Modern Mathematics, Springer Science and Business Media, ISBN 978-3-7643-5749-8.
- [9] Jose Ferriro's (16 August 2007). Labyrinth of Thought: A History of set theory and its Role in Modern Mathematics ISBN 978-3-7643-8349-7.
- [10] Kostrikin, A.I., and Manin, Y.I., Linear Algebra and Geometry, Gordon and Breach Science Publishers, 1989.
- [11] Lay, D.C., Linear Algebra and its Applications, Addison Wesley, 2003.
- [12] Molodtsov, D.A: Soft set theory-first results. Comp. and Math. With App.37, 19-31(1999).
- [13] Padilla, R., Smarandache Notions Journal, 9 (1998)36-38.

- [14] Samuel Goldberg, Probability: An introduction, ISBN 978-0-486-65252-8, 1960.
- [15] Smarandache, Florentine, Collected papers II, University of Kishinev Press, Kishinev, 1997.
- [16] Stoll, Robert R (1974); Sets, Logic and Axiomatic Theories, W.H.Freeman and company.Pp, ISBN 978071604577.
- [17] Stoll, Robert R (1979); set theory and Logic. Mineola, N.Y.: Dover Publications, ISBN 0-486-63829-4.
- [18] Thomas H. Cormen; Charles E Leiserson; Ronald L Rivest; Clifford stein (2001), Introduction to Algorithms, MIT press.p.1070. ISBN 978-0-262-03293-3.
- [19] Vasantha Kandasamy, W.B., Bivector spaces, U.Sci. Phy.Sci., 11(1999)186-190.
- [20] Vasantha Kandasamy, W.B., Bialgebraic Structures and Smarandache Bialgebraic Structures, Amercian Research Press, Rehoboth, 2003.
- [21] Vasantha Kandasamy, W.B., Florentin smarandache, Iianthenral, K.,Introduction to Bimatrices, 2005
- [22] Vasantha Kandasamy, W.B.,Florentin Smarandache, Iianthenral, K., Introduction to Linear Bialgebra, sep.2005
- [23] Velleman, Daniel, How to prove It: A Structured Approach,Cambridge University Press,(2006) ISBN 0-521-67599-5.
- [24] Voyevodin, V.V., Linear Algebra, Mir Publishers, 1983.
- [25] Zadeh, L.A., Fuzzy Sets, Inform. and Control, 8, (1965), 338-353.
- [26] Zelinsky, D., A First Course in Linear Algebra, Academic Press, 1973.