# Original Article Viscous Dispassion and Chemical Reaction Impact on Axisymmetric Flow through a Radially Extending Surface

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**Abstract** - This study presents a truthful mathematical evaluation of axisymmetric flow and heat transfer of an electrically conducting fluid over a radially extending surface. The flow trends occur due to stretching of surface all along radial path with an exponential velocity. The governing highly nonlinear partial differential equations are transmuted into ordinary differential equations by mentioning new similarity transformations. Mathematical analysis for flow performing is achieved using Shooting technique. The velocity, thermal and solutal profiles are shown for a variety of values of dimensionless natural parameters and reviewed in fact. Further Than skin friction coefficient, Nusselt number and Sherwood number are charted and investigated for several values of important parameters.

Keywords - Axisymmetric flow, Heat & Mass Transfer, Mhd, Chemical reaction, Viscous dissipation, Shooting method.

## **1. Introduction**

Over the last few years, the boundary-layer flow and heat transfer of viscous fluid overextending surface has been more intensive researched due to its useful engineering purposes. Such flows are much interesting in cooling of melt down sheets or electrical chips, the aerodynamics of extrusion of plastic sheet, the boundary layer alongside a liquid film in upgrading methods and a few more. In the industry for production of glass fibre, getting plastic sheet, cooling, and dry up of papers, evaporating of polymers etc. Those procedures really vary on the mass transfer at the extending surface. Fairbanks and Wike [1] studied chemical reaction effect in an isothermal laminar flow over a flat plate. Andersson et al. [2] are reviewed diffusion of a chemical reactive species from a stretching sheet. Ahmed et al. [3] have researched axisymmetric flow and heat transfer in a power law fluid. A. Shahzad et al. [4] have debated on heat transfer analysis of viscous fluid over nonlinear radially stretching sheet. Bhattacharyya et al. [5] have conferred reactive solute transfer over a stretching sheet with suction/ blowing. Najib et al. [6] talk about stagnation point flow and mass transfer with chemical reaction past a stretching/shrinking cylinder. Further, Abbas et al. [7] investigated the effect of chemical reaction in stagnation-point flow of a third-grade fluid employing a hybrid numerical method. Tripathy et al. [8] have examined a two-dimensional steady flow over a moving vertical plate through porous media. M. Khan and Azeem shahed [9] studied on On axisymmetric flow of Sisko fluid over a radially stretching sheet. The stagnation point of casson nano fluid flow over a radially stretching sheet was studied by G Narender et.al.[10]similarly the Meraj Mutafa et.al. [11]found numerical solutions for asymmetrical flow over radially stretching sheet by considering nano fluid.

The flow of micropolar fluid was first initiated by Eringin [12] Ferrofluid, liquid crystals, blood, suspensions etc. are examples of micropolar fluid. The stagnation point micropolar fluid flow was determined by Guram and smith [13]The micropolar fluid flow over axisymmetric sheets with heat source was investigated by Gorla and Takha [14]. Mishra et al. [15] analyzed the flow of heat and mass transfer on MHD micropolar fluid in presence of heat source. Dash et al. [16] explored a numerical approach to boundary layer flow over a stretching/shrinking sheet. B. Nayak et al. [17] reviewed Chemical reaction effect of an axisymmetric flow over radially stretched sheet. Bala Siddulu Malga et al. [18] studied, Finite Element Analysis of Fully Developed Free Convection Flow Heat and Mass Transfer of a MHD / Micropolar Fluid over a Vertical Channel. Bala Siddulu Malga et.al [19] considered Finite element study of Soret number effects on MHD flow of Jeffrey fluid through a vertical permeable moving plate. Partial Differential. P. Pramodet.al [20] researched Heat transfer effects on free convection of viscous dissipative fluid flow over an inclined plate with thermal radiation in the presence of induced magnetic field. Recently, Azeem Shahzad et al. [21] investigated Axisymmetric flow with heat transfer over exponentially stretching sheet: A computational approach. Encouraged by the above studies and prospective applications, it is of

significance in the present work is to study heat transfer and mass transfer study of an axisymmetric flow of a viscous fluid over a nonlinear radially extending sheet. The renovated ordinary differential equations explained mathematically by using Runge-Kutta forth order method complemented with shooting method. Lastly, present result is equated with that of prior available work Azeem Shahzad [21] and evaluated in a specific case which is in good accord.

### 2. Mathematical Formulation

Let us take into account a steady two-dimensional axisymmetric MHD flow of an electrically conducting, viscous fluid in the existence of viscous dispassion and chemical reaction over a extending surface coinciding the plate z=0. The flow of conducting fluid is due to the stretching of the sheet along the path with  $U(r) = c_0 r$ , where  $c_0$  is a dimensional constant. It is accepted that the surface concentration of the sheet is  $C_w$  with a fluid concentration  $C_\infty$ . The sheet is kept at constant temperature  $T_w$ , while  $T_\infty$  conveys the ambient fluid temperature. The flow geometry is shown in Figure 1.

In viewpoint of the above statements, the governing steady two-dimensional boundary layer calculations for momentum and heat transmission of a viscous electrically performing fluid following Azeem Shahzad [21] are:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = v\left(\frac{\partial^2 u}{\partial z^2}\right) - \frac{\sigma}{\rho}B^2(r)u$$
(2)

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \alpha \left(\frac{\partial^2 T}{\partial z^2}\right) + \frac{v}{c_p} \left(\frac{\partial u}{\partial z}\right)^2 \tag{3}$$

$$u\frac{\partial c}{\partial r} + w\frac{\partial c}{\partial z} = D\left(\frac{\partial^2 c}{\partial z^2}\right) - \kappa_1(C - C_\infty)$$
(4)

wherever u and w be the velocity elements along the axial and transverse direction, correspondingly. v is the kinematic viscosity,  $\rho$  is the fluid density, T the temperature of fluid,  $\alpha = \frac{k}{\rho c_p}$  is the thermal diffusivity of fluid,  $c_p$  reveals the specific heat.C is the concentration of the fluid, D is the diffusion coefficient,  $C_{\infty}$  is the ambient concentration and  $B(r) = B_0 r$  is the varying magnetic field,  $\sigma$  the electrical conductivity. The consequent boundary is for the velocity elements and the concentration are:

$$u = U(r) = C_0 r, w = 0, C = C_w = C_\infty + C_0 r^n \text{ at } z = 0, T = T_w,$$
$$u \to 0, C \to C_\infty T \to T_\infty \text{ as } z \to \infty$$
(5)

To nondimensionalize the equations (2)-(4), we introduced the following new similarity transformations:

$$u = U(r)f'(\eta), w = -\sqrt{C_0 v} f(\eta), \theta(\eta) = \left(\frac{T - T_{\infty}}{T_w - T_{\infty}}\right), \ \phi(\eta) = \left(\frac{C - C_{\infty}}{C_w - C_{\infty}}\right) and \ \eta = \sqrt{\frac{C_0}{v}} z$$
(6)

where  $C_w$  is the varying surface concentration,  $C_0$  is a constant, n is power-law exponent indicates the change of quantity of solute in r -direction.

We consider the steady axisymmetric MHD flow of an electrically conducting incompressible viscous flow engaged over a extending sheet. The geometry of the problem is demonstrated in Figure 1. The flow of conductive fluids is affected by the sheet stretching in the  $U(r) = C_0 r$  direction, where  $C_0$  is a dimensional constant and the surface is stretching along z-direction.

Moreover, it is assumed that  $T_w$ , and  $C_w$  conveys the extending sheet temperature, solute concentration, respectively. At the infinity, the T $\infty$ , C $\infty$  and are the ambient temperature, ambientsolute concentration, respectively. In the current study,  $B(\mathbf{r}) = B_0 \mathbf{r}$  recaps the strength of magnetic field, where  $B_0$  is a uniform magnetic field strength. The magnetic field

acts along normal to the sheet in the positive Z-direction. When the magneticfield is differing in the radial direction, it also differs in the vertical direction, since the divergence should be zero. Under the boundary layer approximation, the equations of mass conservation, linear momentum conservation, energy conservation can be obtained. Staying in view Eq. (6), Eq. (1) is the same satisfied while the momentum Eq. (2) temperature Eq. (3) and concentration Eq. (4) along with boundary conditions in Eqs. (5) reduces to the following

$$f'''(\eta) + f(\eta)f''(\eta) - (f'(\eta))^2 - Mf'(\eta) = 0$$
<sup>(7)</sup>

$$\theta''(\eta) + Prf(\eta)\theta'(\eta) + PrEc(f''(\eta))^2 = 0$$
(8)

$$\phi''(\eta) - nSc\phi(\eta)f'(\eta) + Scf(\eta)\phi'(\eta) - \gamma Sc\phi(\eta) = 0$$
(9)

Where  $Pr = \frac{v}{\alpha}$  Prandtl number,  $Sc = \frac{v}{D}$  is the Schmidt number,  $\gamma = \frac{k_1}{c_0}$  is the chemical reaction parameter and  $M = \frac{\sigma B_0^2}{\rho c_0}$  is the magnetic parameter  $Ec = \frac{U^2}{(T_w - T_\infty)c_p}$  is the Eckert number.

The required physical boundary conditions of the above governing Equations are as follows

$$f(\eta) = 0, f'(\eta) = 1, \theta(\eta) = 1, \varphi(\eta) = 1 \text{ as } \eta = 0$$
$$f(\eta) \to 0, \theta(\eta) \to 0, \phi(\eta) \to 0 \text{ as } \eta \to \infty$$
(10)

#### 3. Method of solution

Solution to the problem of the system of Eqs. (7)–(9) all together with boundary conditions (10) is calculated by using an effective numerical technique known as shooting method. Here we transform the nonlinear equations into system of seven first order ordinary differential equations by considering the variables i.e.

$$f = y_1, f' = y_2, f'' = y_3, \qquad f''' = y_2^2 + My_2 - y_1y_3,$$
  

$$\theta = y_4, \qquad \theta' = y_5, \qquad \theta'' = -Pry_1y_5 - PrE_cy_3^2,$$
  

$$\phi = y_6, \phi' = y_7, \ \phi'' = nScy_6y_2 - Scy_7y_1 + \gamma Scy_4 \qquad (11)$$

together with boundary conditions

$$y_1(0) = 0, y_2(0) = 1, y_2(\infty) = 0,$$
  
 $y_4(0) = 1, y_4(\infty) = 0, y_6(0) = 1, y_6(\infty) = 0.(12)$ 

The essential mechanism of flow, heat and mass transfer are coefficient of skin-friction  $C_f$ , local nusselt number  $N_u$ , and Sherwood number  $Sh_r$  defined as:

$$C_{f} = \frac{2\tau_{\omega}|_{z=0}}{\rho U^{2}}, \quad N_{u} = \frac{rq_{\omega}|_{z=0}}{k(T_{w} - T_{\infty})}, \quad Sh_{r} = \frac{rq_{m}|_{z=0}}{D(C_{w} - C_{\infty})}$$
(13)

Where  $\tau_{\omega}$ ,  $q_{\omega}$ , and  $q_m$  are the wall shear stress, wall heat flux and wall mass flux separately having the following expressions:

$$\tau_{\omega} = \mu \frac{\partial U}{\partial z}\Big|_{z=0}, q_{\omega} = -k \frac{\partial T}{\partial z}\Big|_{z=0}, q_m = -D \frac{\partial C}{\partial z}\Big|_{z=0}$$
(14)

Using Eqs. (6), (11) and (12) the drag, heat and mass transfer get the following form:

$$\rho r \frac{\sqrt{c_0 v}}{2\mu} C_f = f''(0), \quad \frac{1}{r} \sqrt{\frac{v}{c_0}} N_u = -\theta'(0), \\ \frac{1}{r} \sqrt{\frac{v}{c_0}} Sh_r = \phi'(0)$$
(15)

All the mathematical calculations are completed with the support of computer-based software "MATLAB". To work out the transformed differential equations mathematically we follow highly effective mathematical shooting method with fourth order Runge-Kutta system. In this technique it is most vital to choose the suitable finite values of  $\eta \rightarrow \infty$ . To select  $\eta_{\infty}$ , some initial guess values are taken and solve the problem with some set of parameters to obtain f''(0),  $\theta'(0)$  and  $\varphi'(0)$ . The explanationmethod is reiterated with a large new value of  $\eta_{\infty}$  up to two consecutive values of f''(0),  $\theta'(0)$  and  $\varphi'(0)$  differ only after desired accuracy signifying the limit of the boundary along  $\eta$ . The last value of  $\eta_{\infty}$  is chosen as applicable value of limit  $\eta \rightarrow \infty$  for that set of parameters. The resultant differential equations can be integrated by fourth order Runge-Kutta scheme. The one above technique is reiterated until we get the results up to the preferred grade of accurateness10<sup>-6</sup>.

## 4. Results and Discussion

We analysed the impacts of numerous values of parameters engaged in the equations viz. magnetic parameter-M, Prandtl number-Pr, Schmidt number Sc, Chemical reaction rate parameter- $\gamma$ , and the power-law exponent-n. In this analysis for mathematical calculation we used, n = 1 and $\gamma$  = 0.3. Ec=0.8, Sc=0.5, Pr=0.7. These values are kept as common in the entire survey except for varied values as exhibited in Figures 2–13. The confirmation of the result is obtained in a certain case and found to be in excellent deal with the result of B. Nayak [11]and Azeem Shahzad [15].

Figures 2(a)-2(c) represent the influence of the velocity, temperature, and concentration profiles with magnetic parameter M. It is noticed from these figures that the velocity distribution decline due to increment in the magnetic field. It is clearly seen in Figure 2(a) increase in the value of M slow down the momentum and hence a decline in radial velocity. Physically, the magnetic parameter produced Lorentz force which slows down the motion of the fluid. In figure 2(b) &(2c) temperature, and concentration profiles are observed increases near to the boundary wall. The escalation in the values of magnetic triggers increment in thermal and solute boundary thickness.

Figure 3(a) analyses the effect of Prandtl number over dimensionless temperature distribution. We can observe that an increase in prandtl number reduced the thermal boundary layer. Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. In case of smaller values of Prandtl number, the thermal boundary layer thickness increases, further causing a reduction in heat transfer.

Figure 3(b) depicts the temperature distribution for different values of Eckert number (Ec), it can be noticed that an increase in the Eckert number Ec results the enhancement in temperature profiles because of the heat energy stored in the fluid due to frictional heating. This physical parameter expresses the relationship between the kinetic energy and internal energy.

Figure 4(a) explores A chemical reaction is a process leading to the chemical transformation of one compilation of chemicals into a new. The effects of chemical effect  $\gamma$  in the fluid. It is depicted from the graph that escalating values of chemical effect creates some reaction in the fluid flow and slow down the centrifugally extending velocity, temperature, and concentration profiles of the fluid. Raising the temperature profile closed to the wall also show that the boost up the value of chemical response that the effect is lessening the concentration profile away to the wall but closed to the wall it barely growing effect find-out. Hence destructive chemical reaction is not helpful to improve the concentration profile

Figure 4(b) explains the rate of mass transmission profile for various values of chemical reaction parameter. It is quite clear to say that here is a point of variation inside the boundary layer. It is also noticed that the rate of mass transmission falls considerably up to the variation area and later the reverse result is noted with the rise in chemical reaction. Therefore, it is determined that the double character on the profile is due to the addition of stronger species along with the magnetic parameter.

Figure 5(a) depicts the effect of power-law coefficient (stretching parameter) on the concentration and rate of mass transfer, correspondingly. On the wise examination of Figure 5(a) it is seen that the concentration profile slow down with an increase in power-law factor alongside the width of the concentration boundary layer. Further than, Figure 5(b) demonstrates that the rate of mass transfer diminishes with an upsurge in the value of stretching parameter-n up to the variation region and then reversal trend appears after that. We examined to reduce concentration on the boundary layer with the increase in the stretching parameter n.

Figure 6(a) is plotted to explain the impact of Schmidt number on the concentration profile. It is observed that with an improving value of Schmidt number Sc i.e., for heavier responsive species concentration profile falls. Hence, it is determined that the reactive species with positive reaction rate coefficient and species with low diffusivity i.e., for high values of Sc concentration profile decreases. Figure 6(b) is drawn to the influence of Schmidt number on the rate of mass transfer profile. It is observed that for n=0 the profile is nearly linear and further the profile drops for rising value of n up to certain region and then rate of mass transfer gets improved.

Figure 7 reveals the values of rate of mass transfer versus chemical reaction parameter for numerous values of power-law factor n. It is remarkable that the rate of mass transfer profile boosted with an increase in power-law factor considerably. Hence it is decided that increase in devastating chemical reaction rises the rate of mass transfer i.e., presence of stronger species is encouraging to increase in rate of mass transfer.

S.No	M	Pr	Ec	n	Sc	γ	<i>f</i> ''( <b>0</b> )	$\theta'(0)$	$\phi'(0)$
1	0.0	0.7	0.8	0.0	0.05	0.0	-1.0000	0.2118	0.1216
2	1.0						-1.4142	0.0205	0.1160
3	2.0						-1.7321	-0.1143	0.1133
4	3.0						-2.0000	-0.2206	0.1117
5		2.0						-0.7199	0.1117
6		4.0						-1.5114	0.1117
7		6.0						-2.3052	0.1117
8			1.0					-3.2617	0.1117
9			2.0					-8.0439	0.1117
10			3.0					-12.8262	0.1117
11				1.0					0.1341
12				2.0					0.1562
13				3.0					0.1781
14					0.1				0.2542
15					0.2				0.4001
16					0.5				0.7925
17						0.1			0.8590
18						0.3			0.9549
19						0.6			1.0635
20						1.0			1.1802

Table 1. value of  $f''(0), \theta'(0), \phi'(0)$  with respect to various parameters



Fig. 1 Physical configuration and coordinate system.



Fig. 2(b) Effect of Magnetic parameter-M on Temperature.



Fig. 2(c) Effect of Magnetic parameter-M on concentration.



Fig. 3(a) Effect of Prandtl number-Pr on Temperature.







Fig. 4(a) Effect of Chemical reaction parameter- $\gamma$  on mass distribution.



Fig. 4(b) Effect of Chemical reaction parameter- $\gamma$  on rate of mass distribution.



Fig. 5(a) Effect of power-law coefficient-n on mass distribution.



Fig. 5(a).Effect of power-law coefficient-n on rate of mass distribution.



Fig. 6(a) Effect of Smidt number-Sc on mass distribution.









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