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Original Article

An EPQ Model for Deteriorating Items having a Fixed Expiry Date with Price and Credit Period Probability Demand under Trade Credit

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Abstract *- An EPQ version for deteriorating gadgets with promoting fee and credit score length touchy call for is advanced. This fashions address merchandise that have a hard and fast expiry date and then the product cannot be sold. Permissible put off is permitted to pay clients dues. The shortages are allowed and partly backlogged. In this version, we've got taken into consideration variable manufacturing price. Moreover, the call for for the gadgets is thought as probabilistic. Finally, the version is illustrated with numerical example. Sensitivity evaluation additionally studied with the exalternate of parameters.*

Keywords *- Deteriorating gadgets, constant expiry date, probabilistic call for, permissible put off length, credit score length etc.*

1. Introduction

In any production or enterprise operation it's miles very critical to keep an excellent stock for the clean and green functioning of the machine. Every object inclusive of beauty merchandise, shampoos, toothpaste and medicinal drug etc. which has a sure expiry date. So, the deterioration earlier than the expiry date of a product performs an crucial position with inside the stock modeling. Some current opinions on stock control machine are offered with the aid of using Chen et al. (2006), Patra (2010), Prasad and Mukherjee (2016), Palanivel and Uthayakumar (2017), Saha and Chakrabarti (2017b) and Saha and Chakrabarti (2017).

Again, there's additionally an crucial difficulty related to the inventory of bodily items, that is deterioration. Most bodily items go to pot over time, so the management and upkeep of manufacturing inventories of deteriorating gadgets have obtained a good deal interest of numerous researchers with inside the current years. It is widely recognized that sure merchandise inclusive of vegetables, medicinal drug, gasoline, and radioactive chemical substances get deteriorated or spoiled for the duration of their ordinary garage length. As a result, even as figuring out the premiere stock coverage of that sort of merchandise, the loss because of deterioration cannot be ignored.

Researchers taken into consideration the deterioration of the goods even as growing their version, amongst them the paintings executed with the aid of using Hou (2006), He et al. (2010), Min et al., (2012), Sicilia et al. (2014), Ghiami and Williams (2015) are really well worth mentioning. In any manufacturing stock machine, it's miles very crucial to decide an most suitable manufacturing price, due to the fact if the producer produces a massive quantum of items, it is able to bring about a loss because of deterioration of the goods, retaining value of the extra gadgets and massive funding withinside the manufacturing. Also, the goods might also additionally get obsoleted and therefore remained unsold. On the opposite hand, an inadequate quantity of inventory might also additionally bring about a shortage. Researchers have advanced stock fashions taking numerous forms of manufacturing price.

Su and Lin (2001) advanced a manufacturing stock version thinking about manufacturing price as call for and stock degree structured. Samanta& Ajanta (2004) mentioned deterministic stock version of deteriorating gadgets with quotes of manufacturing and shortages. Bhowmick et al. (2011) labored with deterministic stock version of deteriorating gadgets with quotes of productions, shortages and variable manufacturing cycle. Manna et al. (2016) derived an monetary order amount version with ramp kind call for price, regular deterioration price and unit manufacturing value.

Hsu et al .(2007) advanced an premiere ordering selection for deteriorating gadgets with expiration date and unsure lead time. Wu et al.(2014) explored premiere credit score length and lot length with the aid of using thinking about not on time charge time structured call for below default hazard for deteriorating gadgets with expiration dates. Tayal et al.(2014) offered a multi object stock version to optimize the unit time earnings of stock control for the goods having an expiration date and then the product cannot be sold. Singh and kumar(2018) advanced an stock version for premiere ordering and replenishment regulations with a hard and fast expiry date and credit score length touchy call for below alternate credit score. Chang et al. (2019) advanced an monetary manufacturing amount version for unique forms of charge structures with product expiration dates for perishable gadgets. However, maximum of the perishable gadgets go to pot non-without delay over time. Khan et al. (2019a, 2019b) advanced a version thinking about the most life of the goods below fee-touchy call for.

Table 1. Contributions of some authors related to inventory model

Authors	EOQ model	EPQ model	Constant deterioration rate	Other deterioration rate(linear, weibull etc)	Constant demand	Other demand rate (price sensitive, credit sensitive etc)	With fixed expiry date	Items backlogged	Permissible delay in payments
Su CT, Lin CW		√	√			√			
Samanta GP, Roy A		√		√		√			
Chen et. al.	√			√		√			
Hou KL	√		√			√		√	√
Hsu et. al.	√		√		√		√		
He et al.		√	√			√			
Patra SK	√		√		√			√	√
Bhowmick et al.	√			√		√		√	
Min et al.		√		√		√			√
Sicilia et al.	√		√			√		√	
Tayal et al.	√		√			√	√	√	
Wu et al.	√			√		√	√		
Ghiami Y, Williams T		√	√		√				√
Prasad K, Mukherjee	√			√		√		√	
Manna et al.	√		√			√			
Palanivel M, Uthayakumar		√		√		√		√	
Saha S, Chakrabarti T	√		√			√			
Saha S, Chakrabarti T	√		√			√		√	
Singh D., Kumar N	√			√		√	√		
Chang et al.	√			√		√	√		√
Khan et al.	√			√		√		√	√
Khan et al.	√			√		√	√	√	√

We develop an inventory model for the retailer under the following scenario:

1. The supplier provides a permissible delay period to the retailer.
2. The demand is a probabilistic function of selling price and the length of credit period.
3. Shortages are allowed and partially backlogged.
4. The inventory system deals with the deterioration and fixed expiry date.

2. Assumption

Following assumptions are adopted.

- The demand D(s,n) of the product is function of selling price and credit period. For this D(s,n)=a-bs+cn, a,b and c are non-negative parameters.
- Holding cost h(t)=$h_1 + h_2 t$, where $h_1, h_2 \geq 0$.
- Shortages are allowed and partially backlogged with a constant rate η $(0 \leq \boldsymbol{\eta} < \mathbf{1})$.
- **A** permissible delay period of time n is allowed by the supplier to the retailer to pay all his dues. After a fixed time interest will be charged on his dues.
- The production rate, P(t) is a multivariable function of stock level and demand and given by the following : $P(t) = \begin{cases} \alpha + \beta D(t) - \gamma I(t), & I(t) \geq 0 \\ \alpha + \beta D(t) & , I(t) < 0 \end{cases}$, where I(t) is inventory level at any time t, $\alpha(> 0), \beta(0 \leq \beta < 1)$ are initial and stock dependent consumption rate parameters and $0 \leq \gamma < 1$.
- Deteriorating items having a fixed expiry date and no repair or replacement of the deteriorating items, for this $\theta(t) = \frac{1}{1+v-t}$, where $0 \leq t \leq v$, so deterioration rate tends to 1 as time tends to expiry date of the product.

3. Notations

- A, b, c : demand parameters
- h_1, h_2 :holding cost parameters
- η : backlogging parameter
- p : purchasing price of the product per unit
- s : selling price of product per unit
- k : shortages cost per unit per unit time
- l : lost sales cost per unit per unit time
- P : production cost of the product
- Q_1 : initial stock level
- Q_2 : maximum backorder quantity
- Q : order quantity per cycle
- v : expiry date of product
- T : cycle time
- Z : the total retailer profit per unit time for a complete cycle

4. Mathematical Modeling

Here, the retailer receives the stock Q_1 at initially. During the time interval $[o, v]$, inventory reduces due to demand and deterioration, due to expiration of the product stock becomes zero at $t = v$ and therefore shortages occur. During the time interval $[v, T]$, shortages are accumulated due to demand, until it reaches to maximum allowable shortage level Q_2 which is partially backlogged at $t = T$. Therefore, the inventory level at any instant of time t is described by the following differential equations:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(s,n), \qquad 0 \leq t \leq v \tag{1}$$

$$\frac{dI(t)}{dt} = -\eta\, D(s,n), \qquad v \leq t \leq T \tag{2}$$

With the boundary condition I(v)=0,

The solution of (1) and (2) are given by

$$I(t) = (a - bs + cn)\{(1 + v - t)\log(1 + v - t)\}, \quad 0 \le t \le v \tag{3}$$

$$I(t) = -\eta(a - bs + cn)(t - v), \qquad v \le t \le T \tag{4}$$

From (3) and (4) we can get initial inventory level

$$Q_1 = I(0) = (a - bs + cn)\{(1 + v)\log(1 + v)\} \tag{5}$$

And the maximum backorder quantity

$$Q_2 = -I(t) = \eta(a - bs + cn)(T - v) \tag{6}$$

So, order quantity per cycle is

$$Q = Q_1 + Q_2 = (a - bs + cn)\{(1 + v)\log(1 + v) + \eta(T - v)\} \tag{7}$$

Setup cost : the setup cost per production run is

$$\text{OC=O} \tag{8}$$

Production cost : the production cost is calculated as

$$\text{PC}=p\left\{\int_0^v P(t)dt + \int_v^T P(t)dt\right\}$$

$$\text{PC}=p\left\{\{\alpha + \beta(a - bs + cn)\}T - \frac{1}{4}\gamma(a - bs + cn)\right\} \tag{9}$$

Holding cost: cost associated with the holding of stock is

$$\text{HC}=\int_0^v (h_1 + h_2 t)I(t)dt = (a - bs + cn)\left\{\begin{array}{c} h_1\left\{-\frac{3}{2}v - \frac{5}{4}v^2 + \frac{3}{2}(1 + v)^2\log(1 + v)\right\} \\ +h_2\left\{\frac{1}{6}v + \frac{5}{12}v^2 + \frac{7}{36}v^3 - \frac{1}{6}(1 + v)^3\log(1 + v)\right\} \end{array}\right\} \tag{10}$$

Deterioration cost: cost associated with deterioration is calculated as

$$\text{DC=d}\left\{Q_1 - \int_0^v D(s,n)dt\right\} = d(a - bs + cn)\{(1 + v)\log(1 + v) - v\} \tag{11}$$

Shortage cost: the shortage cost per cycle is

$$\text{SC}=k\int_v^T\{-I(t)\}dt = \frac{k}{2}\eta(a - bs)(T - v)^2 \tag{12}$$

Lost sales cost: during shortage not all customers waiting for the next lot of item, they make their purchase from any other place. So, the lost sales is calculated as

$$\text{LSC}=l\int_v^T(1 - \eta)D(s,n)dt = l(a - bs)(1 - \eta)(T - v) \tag{13}$$

Hence, the total cost per unit time of the production system will be

$$\text{T.C.}=\frac{1}{T}\left\{\begin{array}{c} O+p\left\{\{\alpha+\beta(a-bs+cn)\}T-\frac{1}{4}\gamma(a-bs+cn)\right\}+ \\ (a-bs+cn)\left\{\begin{array}{c} h_1\left\{-\frac{3}{2}v-\frac{5}{4}v^2+\frac{3}{2}(1+v)^2\log(1+v)\right\} \\ +h_2\left\{\frac{1}{6}v+\frac{5}{12}v^2+\frac{7}{36}v^3-\frac{1}{6}(1+v)^3\log(1+v)\right\} \end{array}\right\} \\ +d(a-bs+cn)\{(1+v)\log(1+v)-v\} \\ +\frac{k}{2}\eta(a-bs)(T-v)^2+l(a-bs)(1-\eta)(T-v) \end{array}\right\} \qquad (14)$$

We observe that the T.C. of this production inventory system is a function of v and T .

The total cost of the production system per unit time will minimize if

$\frac{\partial(T.C.)}{\partial v}=0$ and $\frac{\partial(T.C.)}{\partial T}=0$

Provided that $\frac{\partial^2(T\cdot C.)}{\partial v^2}\times\frac{\partial^2(T\cdot C.)}{\partial T^2}-(\frac{\partial^2(T\cdot C.)}{\partial v\partial T})>0$

Numerical Example

The following input data are used to illustrate the model

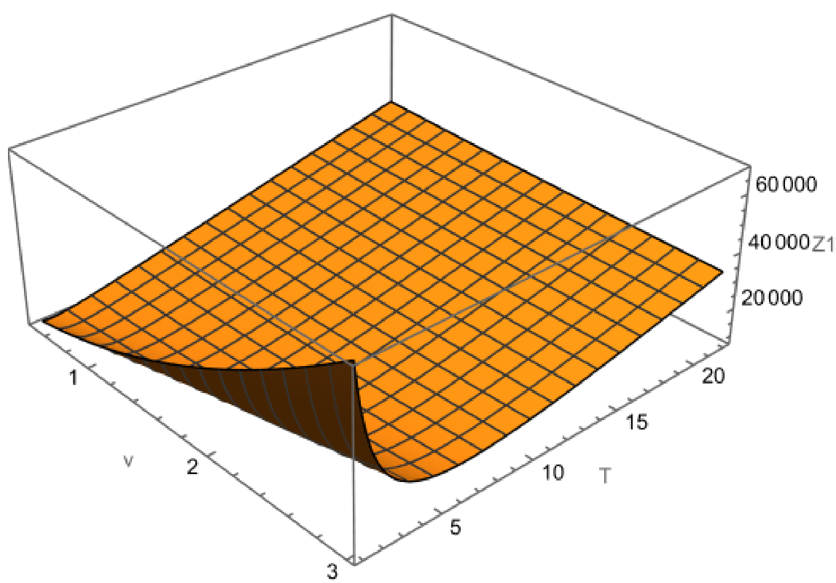
$$\alpha=5,\beta=.2,\gamma=.1,\quad a=300,b=1.5,c=175,P=60,s=150,d=20,n=2.8,k=15,$$

$$l=12,O=200,h_1=1,h_2=.5,\eta=.95$$

Corresponding to these input values, the optimal values of v=0.68 and T=3.04

Substitute these in equation (7) and (14), we get $Q=1475\ and\ T.C.=9847.$

Graphical presentation of the convexity of TC



5. Sensitivity Analysis

Based on the above-mentioned example, a sensitivity analysis is carried out by varying the values of one parameter whereas the other parameters are kept unchanged at the same time. In first table we study the changes in a, b and c.

- Decrease (increase) in a decreases (increases) the value of T.C., v and Q but increases (decreases) the value of T.
- Decrease (increase) in b increases (decreases) the value of T.C. , Q, T and v.
- Decrease (increase) in c decreases (increases) the value of T.C. and Q but increases (decreases) the value of T and v.

In second table changes in parameters α, β and γ are studied.

- Increases in α,β increase the value of T.C. while Q unchanged.
- Increases in γ decreases the value of T.C., Q and T while v increases.

Parameters	% change in parameters	T.C.	Q	T	V
A	-20	7449	1385	4	0.5
	-10	8648	1425	3.52	0.59
	10	11485	1579	3.26	0.908
	20	12760	1649	3.22	1.006
B	-20	12147	1629	3.31	0.95
	-10	11125	1543	3.21	0.88
	10	9135	1378	3.07	0.78
	20	8282	1322	2.91	0.71
C	-20	8821	1220	3.18	0.86
	-10	9334	1280	3.01	0.82
	10	9647	1356	2.80	0.62
	20	12126	1423	2.64	0.5

Parameters	% change in parameters	T.C.	Q	T	V
α	-20	9787	1475	3.04	0.68
	-10	9817	1475	3.04	0.68
	10	9877	1475	3.04	0.68
	20	9907	1475	3.04	0.68
β	-20	8491	1475	3.04	0.68
	-10	9169	1475	3.04	0.68
	10	10525	1475	3.04	0.68
	20	11203	1475	3.04	0.68
γ	-20	9921	1485	3.06	0.683
	-10	9884	1479	3.05	0.684
	10	9809	1468	3.03	0.685
	20	9772	1456	3.01	0.687

6. Conclusion

In this model, we have considered variable production rate. Moreover, the demand for the items is assumed as probabilistic. We develop an inventory model for the retailer under The supplier provides a permissible delay period to the retailer. The demand is a probabilistic function of selling price and the length of credit period, shortages are allowed and partially backlogged. The inventory system deals with the deterioration and fixed expiry date. By the numerical analysis proposed model is verified. This model can further be improved by taking some more realistic situation, such as inflation induced demand rate, different replenishment rate, and ramp type demand rate etc.

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