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Original Article

Investigating Instability of Regression Parameters and Structural Breaks in Nigerian Economic Data from 1984 to 2019

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Abstract *- Instability of regression coefficients can be an indication of structural break. This research investigated the existence of structural breaks and regression parameter instability using the Quandt Likelihood Ratio Test and the CUSUM Test. The Nigerian Real Gross Domestic Product was regressed on health and agricultural expenditure from 1984 to 2019. Five regression models were employed in this study (Linear, Logarithm, Inverse, Power and Exponential). The results revealed that the Quandt Likelihood Ratio (QLR) and CUSUM test identified instability of the regression parameters and structural break at different points for each of the models considered while the Harvey-Collier test was able to show non-linearity in all the models, hence confirming existence of structural breaks. It was observed that both tests were seen to identify close structural breaks using the inverse model. The cumulated sum of scaled residuals shows that the inverse model and the exponential model's scaled residual exhibited similar distribution patterns while linear, logarithm and power multiple regression models showed the same scaled residuals distribution behavior. Some of the structural breaks were seen 1998 which was the year the military regime ended, 1988, 2007, 2010 and 2015 which were the periods of change of various civilian regimes. In conclusion the study was able to show that the economic data considered requires to be spilt in other to avoid erroneous prediction of Nigeria Gross Domestic Product (GDP) based on agricultural and health expenditures.*

Keywords *- Cusum test, Harvey-Collier test, Quandt likelihood ratio test, Regression parameter instability and Structural break.*

1. Introduction

The Quandt (1960) likelihood ratio (QLR) test is an extention of the Chow test proposed by Chow (1960), where a *F*-test statistic is gotten for all likely breakpoints within a given range. This given range is usually dependent on the degrees of freedom required for the estimation of the parameters of a regression model. The QLR statistic is estimated as the test statistic that has the largest value across the range of all potential break points. The null hypothesis of no structural change would be rejected if the absolute value of the test statistic is relatively large. The appropriate asymtotic p-values for QLR statistic was provided by Andrews (1993) and Andrews & Ploberger (1994). The QLR test is normally applied as a sup*F*-test and it provides good power against the alternative of a breakpoint. It is worthy of note that in large samples with multiple discrete breaks or instances where the break comes in the form of a slow evolution of the regression function, the QLR statistic also rejects the null hypothesis with high probability. This lets us know that QLR statistic not only pinpointss single discrete breaks but can also pinpoint instability in the regression coefficients. Due to this, if the QLR statistic rejects the null hypothesis, it could be that there is a single discrete break or there are multiple discrete breaks or there is instability of regression coefficients. The null hypothesis states that the model parameters at constant at all points while the alternative states that there is a difference somewhere. The stability of linear models was investigated by Mustafa et al (2014) based on the structural changes of small number of observations. They discovered that the point where structural change occurs is a key point and if the point of the break is not known then recursive least square and recursive residual test will be the option.

CUSUM test was proposed by Page (1958) for mean change detection while Brown et al (1975) proposed using it for test of parameter stability. It checks the stability of coefficients β in a multiple linear regression model. It is best detailed as a test for instability of the variance of post-regression residuals. The CUSUM test is based on the recursive least square estimation of the model. It is hinged on the instinct that if there are changes from one period to the next then the one-step-ahead forecast will not be correct and the forecast error will be greater than zero. Hence the greater the CUSUM test statistic, the more the forecast error, which increases the statistical evidence in favor of parameter instability. The inference is based on a sequence of sums, or sums of squares, of recursive residuals; that is the standardized one-step-ahead forecast errors which is computed iteratively from nested subsamples of the data. Under the null hypothesis of coefficient constancy, values of the sequence outside an expected range suggest structural change in the model over time, though Paul and Trenkler (2008) proposed a flexible technique for detecting violation of constant coefficients in linear regression assumption as an alternative to CUSUM- and MOSUM-techniques.

The Harvey Collier test is used to test for linearity in model. It achieves this by performing a t-test using the degrees of freedom of the parameters on the recursive residuals. Recursive residuals are said to be independently and identically distributed and are obtained simply from linear transformations of the ordinary residuals. It is assumed that if the mean of the recursive residuals differs significantly from zero then it means that the true relationship in the model is not linear. The null hypothesis states that there is linearity.

2. Related Literature

Economic time series are usually burdened with structural change, which may be dangerous to neglect. Economic relationships can be misinterpreted, estimates can be wrong, and policy suggestions might be misleading or worse. Early work on change point problems focused on identifying changes in mean and includes the work of Page (1954) and Hinkley (1970) who created the likelihood ratio and cumulative sum (CUSUM) test statistics, respectively. Structural break tests can be used to evaluate when and if there is a major change in our data. Bai et al. (1998) considered a single break case, where structural changes in co-integrated relationships in a system of equations were considered. Kejriwal and Perron (2008) obtained outcome results for multiple structural changes in a single co-integrating vector, following their work Esteve et al (2013) in their work considered the possibility that a linear co-integrated regression model with multiple structural changes would provide a better empirical description of the Spanish term structure of interest rates. Hansen (2000) stated that the Quandt-Likelihood Ratio (QLR) test is preferable for identifying structural change with unclear time. Zhou and Liu (2009) used a weighted CUSUM statistic for mean change detection in infinite variance AR(p) process while Qin et al.(2010) studied mean change detection in α-mixing processes using the CUSUM test. In their paper considered issues related to testing for multiple structural changes in co-integrated systems and the results show via simulations that their tests maintain the correct size in finite samples and are much more powerful than the commonly used LM tests, which suffer from important problems of non-monotonic power in the presence of serial correlation in the errors. Georgiev et al (2018), considered the works of Andrew (1993) and Nyblom (1989) where tests for structural change based on the SupF and Cramer–von-Mises type statistics were applied and the predictors display strong persistence. Paye and Timmermann (2006) in their paper applied a different already known structural change tests designed to dictate abrupt (deterministic) changes in a model's parameters in other to study the structural stability of PRs for stock returns in relation to structural breaks in the coefficients of state variables for a data-set of monthly stock returns for ten OECD countries. Bai and Perron (1998) considered theoretical issues related to the limiting distribution of estimators and test statistics in the linear model with multiple structural changes. Other related literatures include; Garcia and Perron (1996), Liu, Wu and Zidek (1997), Lumsdaine and Papell (1997), and Morimune and Nakagawa (1997).

3. Materials and Methods

This work considered Quandt Likelihood Ratio test and CUSUM test in identifying structural breaks and regression coefficient instability and also applied the Harvey Collier test for linearity. The tests were examined by applying them on five different models namely; Linear, Inverse, logarithm, power and exponential models. The analysis was done using economic data collected from National Bureau of Statistics (NBS) 2019 Bulletin which consists of Real Gross Domestic Product (RGDP) (as a proxy for economic growth) as the dependent variable and the independent variables are the expenditures on Health and the agricultural sector of Nigeria from 1984 to 2019.

3.1. Quandt Likelihood Ratio Test

The Quandt Likelihood Ratio (QLR) test is usually applied when there is a believe that the regression relationship has a sudden break at an unknown time point in the series say τ. Hence changing the constant model parameter say β with variance σ_β^2 to another constant parameter say δ with varianceσ_δ^2, then the likelihood ratio statistic is computed, for each observation *z*, where *z* runs from $z = p + 1$ to $z = T - p - 1$, given T to be the number of observations in the series and p is the number of parameters in the model, z=1,…,T. The Quandt likelihood ratio statistic is thus;

$$\gamma_z = log_{10}\left(\frac{maximum\ of\ observations|H_0}{maximum\ of\ observations|H_1}\right) \quad (1)$$

3.2. Model Specification

The models for this study are assumed to have the form:

Linear: $$RGDP_t = \beta_0 + \beta_1 Heath_t + \beta_2 Agric_t + \varepsilon_t \quad (2)$$

Logarithm: $$\ln(RGDP_t) = \beta_0 + \beta_1 \ln(Heath_t) + \beta_2 \ln(Agric_t) + \varepsilon_t \quad (3)$$

Inverse: $$RGDP_t = \beta_0 + \beta_1 \frac{1}{Heath_t} + \beta_2 \frac{1}{Agric_t} + \varepsilon_t \quad (4)$$

Power: $$RGDP_t = \beta_0 (Heath_t^{\beta_1})(Agric_t^{\beta_2}) + \varepsilon_t \quad (5)$$

Exponential: $$RGDP_t = \beta_0 e^{Heath_t(\beta_1) + Agric_t(\beta_2)} + \varepsilon_t \quad (6)$$

where, the dependent variable represents the Real Gross Domestic Product (RGDP) while the independent variables are the expenditures on Health and the agricultural sector of Nigeria from 1984 – 2019.

4. Results and Discussion

4.1. Quandt Likelihood Ratio (QLR) statistic on the Different Models Subheadings

The application of Quandt Likelihood Ratio (QLR) test produced the results in table 4.2 below. The critical values of this test statistic are reported in Stock and Watson and follow the usual F-distribution (2003). The findings are thus;

Table 1. Quandt Likelihood Ratio (QLR) statistic on the Different Regression Models

Model	Quandt Likelihood Ratio Test	Structural Breaks (Year)
Linear	The maximum F(3,33) = 32.8804*	occurs at observation 2010
Logarithm	The maximum F(3,33) = 341.847*	occurs at observation 2006
Inverse	The maximum F(3,33) = 87.6122*	occurs at observation 1988
Power	The maximum F(3,33) = 42.6355*	occurs at observation 2004
Exponential	The maximum F(3,33) = 133.361*	occurs at observation 1998

Footnote: Significant at the 5 percent level (5% critical value = 6.02)

In table 1 above, QLR test statistic show that F-distribution values are greater than the critical F-distribution value of 6.02 listed underneath the table for the entire multiple regression model considered, hence showing significance, in other words the null hypothesis of constant parameters along the whole series is rejected. Observing the models reveals that the linear model identified a structural break in 2010. In 2010 there was a change in the presidency of Nigeria; Dr. Goodluck Ebele Jonathan was sworn in as the new president. Logarithm model identified a structural point in 2006; the inverse model identified a structural point in 1986. In 1986 the country adopted the structural adjustment program (SAP) where the country reformed its foreign exchange system, trade policies, and business and agricultural regulations thus causes major change in the economy. The power model identified a structural point in 2004, while exponential multiple regression model identified a structural break point in 1998, which is the year that military regime ended and civilian democratic Government took over. This results reveals the data should be spilt into two giving rise two regression models; for instance the linear model

4.2. CUSUM test statistic on the Different Regression Models

The CUSUM test for stability of parameters and identification of the structural break were done using the recursive residuals with the help of Gretl statistical software. The Harvey-Collier test runs a t-test on the recursive residuals (with parameter degrees of freedom). The test assumes that if the underlying connection is not linear but convex or concave, the mean of the recursive residuals should be significantly different from zero. This means that a significant result (rejecting the null) occurs when the fit is better with a range restriction (which is what happens if the model is nonlinear).

The Harvey-Collier test shows significance at 5% for all the models, which is an indication that relationship between health and agricultural expenditure on RGDP for each model is not linear. Hence it proves that the recursive residual means significantly deviates from zero thereby exhibiting instability in the parameters of all the models.

Also the CUSUM test statistic on all the Models shows structural break at different points; the linear model identified a structural break in 2013. Logarithm multiple regression model identified it in 2015, this was the year President Muhammadu Buhari was sworn into office. The inverse multiple regression model identified it in 1988 while the power multiple regression model identified it in 2007. In 2007 President Umaru Musa Yar'Adua was sworn in as the new president of Nigeria. While the exponential multiple regression model identified it in 2003.

Table 2. CUSUM test statistic on the Different Regression Models

Model	CUSUM Test	Harvey-Collier	Structural Breaks (Year)
Linear	mean of scaled residuals = 1340.41 sigma hat = 2424.92	Harvey-Collier t(35) = 3.31659 with p-value 0.002132**	occurs at observation 2013
Logarithm	mean of scaled residuals = 15575.4 sigma hat = 20226.8	Harvey-Collier t(35) = 4.62022 with p-value 5.024e-005**	occurs at observation 2015
Inverse	mean of scaled residuals = -0.000115049 sigma hat = 0.000240772	Harvey-Collier t(35) = -2.86701 with p-value 0.006971**	occurs at observation 1986
Power	mean of scaled residuals = 0.115731 sigma hat = 0.109354	Harvey-Collier t(35) = 6.3499 with p-value 2.679e-007**	occurs at observation 2007
Exponential	mean of scaled residuals = -1.00329 sigma hat = 0.899328	Harvey-Collier t(35) = -6.69357 with p-value 9.532e-008**	occurs at observation 2003

*Footnote: **Significant at the 5 percent level*

4.3. Cumulated Sum of Scaled Residuals on the Different Regression Models

Table 3. Cumulated sum of scaled residuals of all regression models and structural break point (or the year of occurrence)

YEAR	Linear	Logarithm	Inverse	Power	Exponential
1984	-0.001	0	0.155	-0.074	-0.011
1985	0.003	0	-1.263	0.434	0.041
1986	0.007	0.001	-2.701	0.937	0.086
1987	0.009	0.002	-4.837	1.543	0.102
1988	0.009	0.004	-7.8138	2.245	0.032
1989	0.024	0.007	-11.484	3.825	0.053
1990	0.035	0.012	-14.27	5.159	-0.006
1991	0.049	0.015	-15.804	6.221	-0.092
1992	0.046	0.015	-14.821	5.75	-0.113
1993	0.011	0.021	-14.476	5.288	-0.705
1994	-0.016	0.041	-14.636	5.451	-1.773
1995	-0.021	0.095	-15.182	6.443	-2.45
1996	-0.019	0.163	-15.806	7.642	-2.552
1997	-0.057	0.22	-16.288	8.352	-2.74
1998	-0.146	0.281	-16.348	8.299	-4.226
1999	-0.077	0.355	-16.417	8.837	-5.271
2000	0.006	0.444	-16.347	9.552	-6.08
2001	-0.204	0.533	-16.273	9.34	-6.98
2002	-0.971	0.752	-16.277	7.922	-10.344
2003	-0.978	1.013	-16.308	7.474	-11.826

2004	-0.046	1.404	-16.379	8.668	-12.884
2005	1.214	1.95	-16.464	10.439	-13.994
2006	2.621	2.7	-16.556	12.62	-15.068
2007	3.723	3.536	-16.639	14.733	-16.115
2008	4.618	4.532	-16.716	16.819	-17.18
2009	5.002	5.629	-16.785	18.673	-18.151
2010	6.96	7.063	-16.854	21.192	-19.211
2011	10.126	8.706	-16.915	23.866	-21.329
2012	12.997	10.551	-16.968	26.362	-23.748
2013	16.046	12.541	-17.013	28.769	-26.804
2014	17.777	14.64	-17.052	30.929	-29.304
2015	17.492	16.675	-17.085	32.527	-30.973
2016	16.744	18.779	-17.115	33.823	-32.423
2017	17.432	21.252	-17.145	35.256	-34.258
2018	18.974	24.213	-17.174	36.749	-36.987
2019	19.9	27.721	-17.202	38.099	-40.161

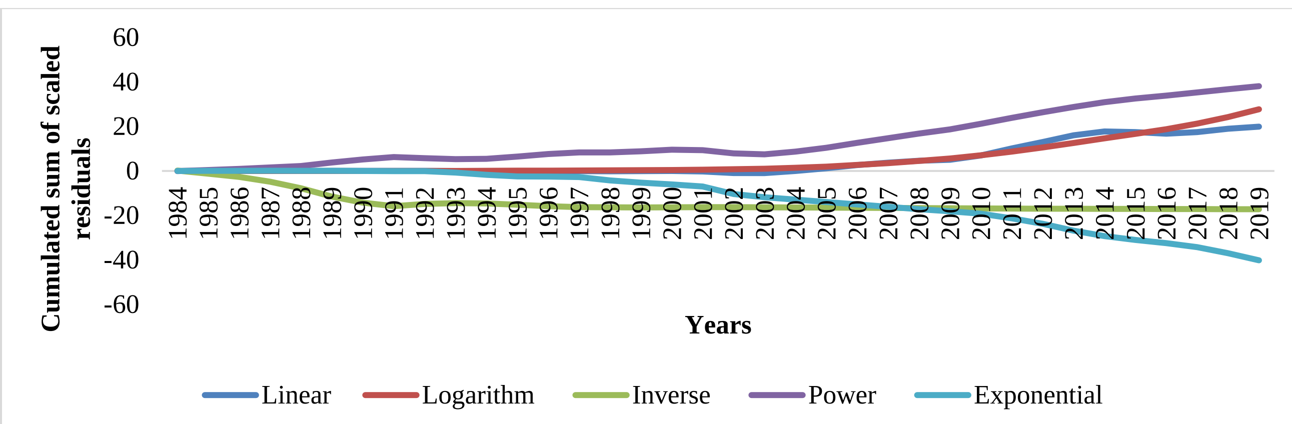

Fig. 1 Cumulated sum of scaled residuals plot

In figure 1 the cumulated sum of scaled residuals plot for the inverse and exponential multiple regression models have similarly distribution behavior, while linear, logarithm and power multiple regression models have the same scaled residuals distribution behavior.

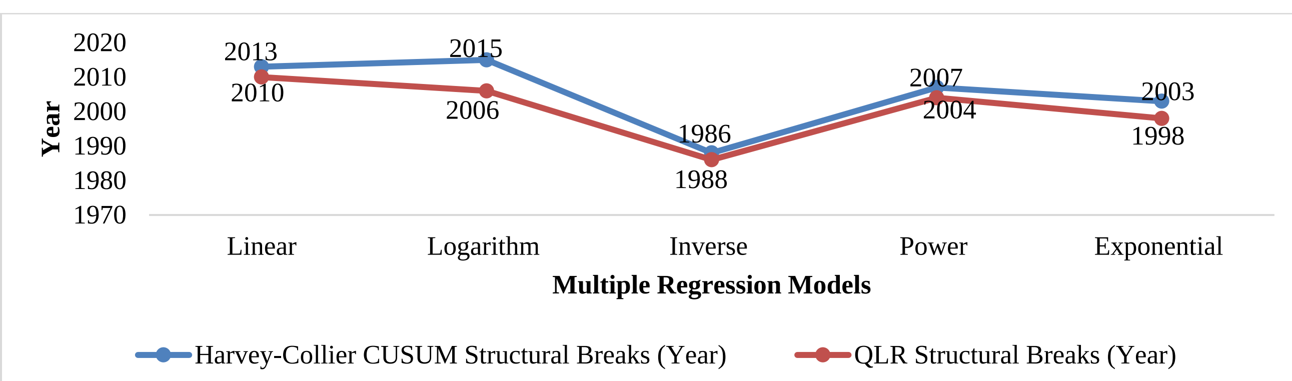

Fig. 2 Summary of Structural breaks point (CUSUM and QLR test Methods)

The results in the above figure 2 shows that the structural break points occurred at all the models considered (Linear, Logarithm, Inverse, Power and Exponential). Also note that CUSUM and QLR tests identified a close structural break point using the inverse model. The two test methods used in the study show that, there exist a structural break in these models; hence the data needs to be spilt to represents the structural breaks. For instance, the 1988 structural break, the Inverse model can be spilt and represented thus:

$$From\ 1984\ to\ 1988: \quad RGDP_t = \beta_0 + \beta_1 \frac{1}{Heath_t} + \beta_2 \frac{1}{Agric_t} + \varepsilon_t$$

$$from\ 1988\ to\ 2019: \quad RGDP_t = \delta_0 + \delta_1 \frac{1}{Heath_t} + \delta_2 \frac{1}{Agric_t} + \varepsilon_t$$

Where $\beta's$ and $\delta's$ and regression parameters for the new sets of spilt data. The first data will be from 1984 to 1988 and the second set of data will be from 1988-2019.

5. Conclusion

The research work investigated the existence of structural breaks and consistency of regression parameters of some multiple regression models namely; linear, logarithm, inverse, power and exponential models by applying the Quandt Likelihood Ratio Test and the CUSUM Test. Different models were considered because identification of structural break is subject to the particular specification of model that is under consideration. In the results both tests were able to identify the existence of structural breaks at different points in the data for each model. The Harvey-Collier test identified instability of the regression parameters in all the models. The cumulated sum of scaled residuals plot in fig 4.6 shows that the inverse model and the exponential model's scaled residual exhibited similar distribution patterns while linear, logarithm and power multiple regression models showed the same scaled residuals distribution behavior. Some of the structural breaks were seen to occur at the years were there were changes in government or adoption of new economic policies. Both tests were seen to identify close structural breaks using the inverse model. This study was able to show that this data requires to be spilt in other to obtain valid predictions.

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