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Original Article

A Novel Mathematical Model for Curbing the Spread of Covid-19 in Nigeria

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Abstract *- This paper considered a novel transmission and control model for curbing the spread of Covid-19 disease in Nigeria. The model dynamics was studied in the presence of vaccines on the Nigeria population. The analytical properties of the model which include local and global stabilities of the Covid-19-free and endemic equilibria were discussed. The basic reproductive number was obtained in order to ascertain the states of the eradication of Covid-19 from Nigeria. The parameters in the model were estimated based on the real data of COVID-19 infected cases, number of vaccinated and deaths recorded in Nigeria. The sensitivity of the model parameters was carried out and the results proved that high values of the parameters associated with the rate of vaccination and precautionary measures will drastically reduce the spread rate of Covid-19 in Nigeria. The trace and determinant of matrix method was applied to determine the local stability of Covid-19-free equilibrium. The Covid-19-free equilibrium was found to be globally stable when the reproductive number is strictly less than one and the endemic equilibrium is globally stable when the reproductive number is more than one. The numerical simulation and analysis of the model on various sets of parameters showed that there is a strong significant effect on the infectives if a minimum of fifty percent of the inhabitants is vaccinated against Covid - 19 in Nigeria.*

Keywords *- Covid-19 free equilibrium, Inclusion of vaccines, Infected symptomatics, Precautionary measures, Reproductive number.*

1. Introduction

Parasites, viruses and bacteria are the main causes of infectious diseases. These diseases are termed infectious because they can be transmitted from the host by various means. These include direct contact with the infected persons or surfaces [25], contact with some medium of transmission (e.g airborne pathogens, etc., [24]), the factor of biodiversity [30], or some climatic factors [43, 44]. A specific case of the above mentioned is the coronavirus disease, which was named Covid-19 following the discovery made by the Chinese Center for Disease Control (CCDC) on December 19, 2019, having 44 patients presenting with symptoms of “Severe Acute Respiratory Syndrome Coronavirus 2” (SARS-Cov 2) [46]. Since the first discovery of Covid-19, there had been records of the rapid spread of the disease across the globe. The pandemic had resulted in a declining economy [10, 47] and an increase in the death rates in most countries [51].

Nigeria in particular had her first case on February 27, 2020, which was imported by an Italian who traveled across some states in Nigeria. Reports from the Nigeria Center for Disease Control (NCDC) have shown that Nigeria has experienced five major waves since the first outbreak, with more severe and high number of cases in the second and fourth waves. So far, there are 263,694 confirmed cases of Covid-19 infected people with 3,148 recorded deaths. At the first instance, the Nigerian government in an attempt to eradicate the disease spread, had put measures in place. These measures include the use of facemask and hand sanitizers, social distancing, and closure of worship centers, educational institutions, business centers, and movement restrictions. However, despite these measures, there were noticeable hikes in the number of infected cases, which led to many more waves after the first one. These could be a result of non-adherence to the lock-down rules by the inhabitants of Nigeria [11], resulting from the lack of provision of incentives, palliatives, and inadequate health care systems and awareness programs [35], by the Nigerian government. Presently, the lock-down measures have so far been relaxed to a great extent; hence person to person contact rate is expected to be higher than during the lock-down period.

The discovery of a class of vaccines brought about a breakthrough in the world of health science, as this would help trigger the immune system of the susceptible population to recognize and fight the virus. The vaccine named Oxford AstraZeneca arrived in Nigeria on February 18, 2021, and was first administered on March 5, 2021. Although other types of the Covid 19

vaccines namely: Mordena: Spikevax, also known as mRNA-1273, Janssen, Spunik V, Pfizer-BioNTech and Sinopharm BIBP were approved for use in Nigeria, out of which a few were deployed. As of 21 August 2022, a total of 41, 103, 768 (19.9 percent of the total population) Nigerians have received the first dose; a total of 28,987,088 people (14.1 percent of the total population) are fully vaccinated. As of 2 September 2022, a total of 63, 346,714 vaccine doses have been administered. Owing to the above figures, it is clear that a good number of the Nigeria population belong to the exposed or infected-asymptomatic class. The question is this: will Covid-19 be eradicated from Nigeria if the vaccination exercise continues at this rate?

Mathematical models [3, 11, 22, 35, 38] with recent works in [9, 28] have been used to quantify the spread rate of the deadly Covid-19 in the literature in recent years. These models have analyzed mathematically the rate of spread of the disease under diverse conditions with the exemption of the effect of the vaccination on the spread of the disease. More so, since most nations' leaders have accepted the use of these vaccines, particularly in Nigeria, we are motivated to develop and analyze mathematically a new deterministic epidemiological model that can detect the effect of the inclusion of vaccines as part of the measures to curb the disease spread of Covid-19 in Nigeria. The aim of this research is to develop a novel epidemiological model with the inclusion of precautionary measures which include vaccines that can be used to curb the spread of Covid-19 in Nigeria. The remaining part of the paper is presented as follows. A thorough review of related works on the existing models of Covid-19 spread is given in the second section. Section three comprises the results of this work. These entail the formulation of the Covid-19 model, some basic properties of the model, the stability analysis of the model equilibria, parameter estimation, sensitivity analysis of the model numerical simulation and analysis and discussions. The summary of the work and conclusive remarks are given in the fourth section.

2. Review of Related Works on Models of COVID-19 Spread

Most mathematical models used in the literature for the spread of Covid-19 are nonlinear deterministic and probabilistic or stochastic models. First, we shall discuss a family of nonlinear deterministic models (which forms the larger class) and then review the works that are related to the stochastic models.

From time immemorial, the problem associated with epidemiological models has been solved by most Mathematicians [6, 34, 40]. The main aim of any well-posed mathematical epidemic model is to capture the rate, at which the disease can spread in a given space of time, the infected number of people in a given population, the rate at which the infected ones are removed from the population either by death or by some means of recovery and some possible means of curbing (control measures) the spread of the disease. The work of Daniel Bernoulli in 1766 has been known to be the first in the class of mathematical epidemic models [12, 15, 22]. The famous SIR model which is the Susceptible-Infectious-Removed model was birthed by Kermack and McKendrick in [31]. Their work showed the dynamics of the spread of the disease in a population as an infected person moves into a susceptible population. The SIR model has been extended in many ways. These include the Susceptible-Exposed-Infectious-Removed (SEIR) model [33], Susceptible-Infectious-Removed-Susceptible (SIRS) model in [17], and Susceptible-Infectious-Susceptible (SIS) model in [52].

Models showing the dynamics of the Covid-19 pandemic and evaluation of the effect of precautionary measures have been extensively treated by many researchers. At the early stage of the outbreak of Covid-19 in Nigeria, Edeki *et al.*, in [23], considered a five compartmental model for the spread dynamics in Nigeria and Spain. Their model and results suggest that a decrease in the contact rate of susceptible and infectious populations would cause a drastic decrease in the spread of the disease. The question as to how well can Nigerians' movement is restricted to reduce the contact rate brought about the work in [11]. The mathematical model in the work of Baba *et al.*, in [11] showed that the prolonged lock-down measures imposed on Nigerians could not be sustained due to inadequate facilities, incentives, and palliatives to keep the majority on hold. And this was found to have led to the second wave in the spread of the disease in Nigeria. According to Musa *et al.*, in [35], the low awareness rate of the spread of the disease in Nigeria has been discovered to be one of the causes of the hikes in the number of infected cases. Similar results of the works in [11, 23, 35] can be found in [27].

An extension of the work of Edeki *et al.*, 2020 in [23] is that of Kifle and Obsu (2022) in [32], where a six compartmental model was considered. An addition of quarantine and seriously sick and hospitalized individuals were part of the work in [32]. The model's properties such as, the solutions' boundedness, disease free and endemic equilibria's existence, local and global stability of equilibrium points were carried out. Methods of Jacobian matrix [7] and Routh Hurtwitz [4, 5, 14, 20, 48] conditions were used to determine the local stability of the Covid-19-free equilibrium point. Their empirical results showed that spread of the disease can be minimized when contact rates of infected persons are reduced. Two classes of susceptible (aware and unaware) and hospitalized (severe and mild) individuals in the Nigeria Covid 19 spread scenario were considered in the work of Musa et. al., (2021) in [36]. Their model excluded birth and death rates as a result of the short time frame of the epidemic. The basic reproductive number (R_0), which is the number of persons that can be infected by one infected individual in a susceptible

population, was captured via the next generation matrix method [49]. The empirical results suggest that the Nigeria government should create more awareness programs and timely adherence to both mild and severe cases of Covid 19 should be put into consideration. An improved probabilistic model for predicting the spread of Covid 19 in some selected African countries (South Africa, Egypt, Algeria, Nigeria Senegal and Kenya) was presented by Zhao et.al., (2020) in [53]. Their prediction shows a rapid growth in the spread rate of the disease during the second phase of the epidemic in Algeria, Nigeria and Kenya.

According to information from the World Health Organization (WHO) [29], the virus that is responsible for the spread of Covid 19 can present itself in different forms (variants) constantly. Since the outbreak of the disease, different variants have been observed by virologist. These include the alpha, beta, delta and most recently the omicron variant. In view of this, some concerned mathematical researchers, Khan and Atangana in [18], have studied the asymptotic properties of the omicron model based on the conditions or values obtained for the basic reproduction number in the model. Extensions of the omicron model were carried out to obtain the second order differential equations for waves and a fractional stochastic version of the omicron variant model. Lagos state, which is highly populated in Nigeria has been found with very high number of Covid 19 infected cases [38]. The work of Okuonghae and Omame [39], proved that social distancing and face masking would be a solution for reducing the quick spread of the disease in a place like Lagos. An application of the methods used in [31] was used in the Pakistan's Covid 19 spread in [42].

In the class of stochastic models for the spread of Covid 19, a mixture of fractional epidemiological and stochastic models was achieved in [41]. The Euler and Euler-Murayama methods were used to achieve the numerical solutions in the Egypt Covid 19 spread of the new models. To capture the uncertainties in the spread of Covid 19 in Spain, Adak *et al.* (2021) in [2] addressed these uncertainties via an extended deterministic model. Optimal distribution of the covid 19 vaccines in the United States was achieved using a stochastic model in [19].

3. Main Results

In this section, the main results and discussion of the work was presented. These entail the formulation of the Covid19 model, some basic properties of the model (feasibility and positivity properties), the stability analysis of the model equilibria, parameter estimation, sensitivity analysis of the model, numerical simulation and analysis.

3.1. Model Formulation

The model was considered in an open population with vaccination as one of the control measures. The total population under consideration at time t denoted by $N(t)$, was slitted into the mutually exclusive compartments of Susceptible (S(t)), Vaccinated (V(t)), Exposed (E(t)), Asymptomatic-infectious ((I_A(t)), Symptomatic-infectious (I_S(t)), Quarantined (Q(t)), Treated (T(t)) and Recovered/Removed (R(t)) individuals.
Thus,

$$N(t) = S(t) + V(t) + E(t) + I_A(t) + I_S(t) + Q(t) + T(t) + R(t) \qquad (1)$$

Based on the above description, the diagrammatic flow chart for the $SVEI_AI_SQTR$ compartmental dynamics for the Covid-19 spread rate is shown in Figure 1 below.

In Fig. 1, newly recruited individuals enter directly into the $S(t)$ compartment at ∧ rate, by immunization, immigration or birth. $S(t)$ class also increases due to the wanes of vaccine immunity at the rate of η, then, through the loss of immunity after some period of time in the recovery compartment, the recovered returns to the $S(t)$ class at the rate α. Also, $S(t)$ compartment decreases due to μ, asymptomatic and symptomatic infection contact rate $\beta(1-\tau\omega)$. Due to immunity gained successively at the initial stage at the rate of θ and vaccinated proportion at the rate of κ the S(t) compartment reduces. The vaccination component increases due to the rate (κ) of vaccinating the susceptible and also reduces when the vaccine based immunity wanes at the rate of η and due to natural death that may occur during the process of vaccination at the rate of μ. The exposed human component escalate due to the increase of $S(t)$ who are exposed and infected with the covid at $\beta(1-\tau\omega)$. The component reduces as a result of μ rate and through the progression to the asymptomatic and symptomatic infectious compartment at the rate of $\sigma\rho$ and (1 - σ)ρ. The asymptomatic-infectious class increases due to progression of the exposed individuals at the rate of $\sigma\rho$.The component decreases due to the direct treatment at the rate of δ_A, natural mortality μ and death rate due to the case of being asymptomatic to the covid infection. The symptomatic-infectious increases due to the progression rate $(1-\sigma)\rho$. Then the compartment decreases (through the progression of) due to the intervention of quarantine process at the rate of ϕ_S, natural mortality rate μ, mortality due to the symptomatic-infectious at the rate of μ_S and also through the treatment rate δ_S. The quarantine class increases due to the symptomatic-infectious rate ϕ_S. The component reduced through the treatment given during quarantine process at the rate of ε, natural mortality rate of μ and death that occur from quarantine

process at the rate of ψ. The treatment class increases due to the direct treatment given to the asymptomatic-infectious at the rate of δ_A and symptomatic- infectious at the rate of δ_S respectively, then treatment given to the quarantine individuals at the rate of ε and it then reduces due to cure of infectious patient at γ rate and natural death during the process of treatment at the rate of μ and immunity loss after recovery at the rate of α.

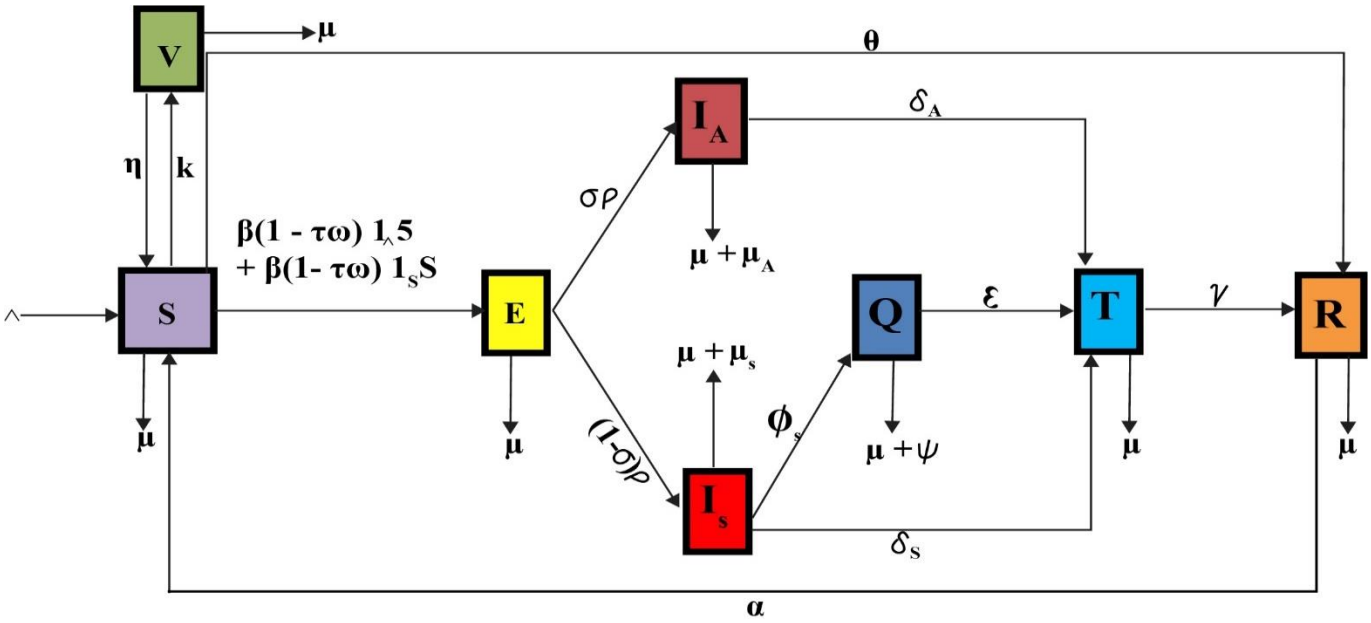


Fig. 1 Schematic flow chart on the SVEIQRT compartmental dynamics for covid-19 spread rate.

Table 1. State variables of the model

Parameters	Description
$S(t)$	Number of susceptible at time t
$V(t)$	Vaccination at time t
$E(t)$	Number of exposed at time t, who have been infected but not yet infectious
$I_A(t)$	Number of asymptomatic-infectious at time t, who are capable of transmitting the disease without symptoms
$I_S(t)$	Number of symptomatic-infectious at time t, who are capable of transmitting the disease with the symptoms
Q(t)	Number of quarantine at time t
T(t)	Number of treated at time t
R(t)	Number of recovered/removed individuals at time t
$N(t)$	Total number of individuals at time t

Table 2. Parameter values of the model

Symbol	Description
Λ	Recruitment of individuals
β	Contact rate
μ	Natural mortality rate
δA	Asymptomatic self-treatment rate
δS	Symptomatic self-treatment rate
γ	Recovery rate
θ	Progression rate of individuals who are not exposed to the infection
μ_A	The death rate due to the asymptomatic infection
μ_S	The death rate due to the symptomatic infection

ϕ_S	Progression rate from symptomatic infectious class to quarantine class
ψ	The death rate due to quarantine
α	The rate at which a recovered individual becomes susceptible
ε	Treatment rate of the quarantined individuals
$1-\sigma$	Fraction of exposed individuals who are symptomatic to the infection at the end of incubation
ρ	Rate of progression from exposed compartment to infectious compartment (if $\frac{1}{\rho}$ is incubation period)
τ	efficacy of precautionary measures
ω	The proportion of individuals who comply with the precautionary measures
k	The proportion of susceptible to be vaccinated
η	The rate at which vaccine-based immunity wanes

The variables and parameters in Table 1 and 2 respectively in the model, satisfies equation (1). Suppose that each state variables and parameters of the model are non-negative for $t \geq 0$, hence, the model was analyzed in a suitable region. The following are the differential equations of the flow chart:

$$\left.\begin{aligned}
\dot{S} &= \Lambda - (\mu + \kappa + \theta)\mathrm{S} - (\beta(1-\tau\omega)I_A + \beta(1-\tau\omega)I_S)\mathrm{S} + \eta\mathrm{V} + \alpha\mathrm{R} \\
\dot{V} &= \kappa S - (\mu + \eta)V \\
\dot{E} &= (\beta(1-\tau\omega)I_A + \beta(1-\tau\omega)I_S)S - (\mu + \sigma\rho)E - (1-\sigma)E \\
\dot{I}_A &= \sigma\rho\mathrm{E} - (\delta_A + \mu + \mu_A)I_A \\
\dot{I}_S &= (1-\sigma)\rho\mathrm{E} - (\mu + \mu_S + \phi_S + \delta_S)I_S \\
\dot{Q} &= \Phi_S I_S - (\mu + \psi + \epsilon)\mathrm{Q} \\
\dot{T} &= \delta_A I_A + \delta_S I_S + \epsilon\mathrm{Q} - (\mu + \gamma)\mathrm{T} \\
\dot{R} &= \theta\mathrm{S} + \gamma\mathrm{T} - (\mu + \alpha)\mathrm{R}
\end{aligned}\right\} \quad (2)$$

with
$S(0) = S_0$, $V(0) = V_0$, $E(0) = E_0$, $I_A(0) = I_{A0}$, $I_S(0) = I_{S0}$, $Q(0) = Q_0$, $T(0) = T_0$, $R(0) = R_0$

where,

$$N(t) = S + V + E + I_A + I_S + Q + T + R$$

Hence, the dynamics of the population $N(t)$ can be described as:

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dV}{dt} + \frac{dE}{dt} + \frac{dI_A}{dt} + \frac{dI_S}{dt} + \frac{dQ}{dt} + \frac{dT}{dt} + \frac{dR}{dt} \quad (3)$$

Substituting (1) into (3) gives:

$$\frac{dN}{dt} = \Lambda - \mu(S + V + E + I_A + I_S + Q + T + R) - \mu_A - \mu_S - \psi Q \quad (4)$$

Finally, equation (4) leads to the simplified form of the population dynamics:

$$\frac{dN}{dt} = \Lambda - \mu N - \mu_A I_A - \mu_S I_S - \psi Q \quad (5)$$

Hence, (5) depicts the population changes known as population dynamics

3.2. Feasibility and Positivity of the Model

3.2.1. Feasibility of the model

The model is said to be feasible if the solution of (1) lies in a well-defined region and possesses some biological meaning.
The next theorem state the feasible of the model.

Theorem 1: Given that the condition in equation (5) holds. Every solutions of the model in equation (1) with initial conditions in $\mathbb{R}_+^8$ tends to and is contained a compact set (Γ) as $t \to \infty$. The feasible solution which is a positively invariant set of the model is given by:

$$\Gamma = \left\{(S, V, E, I_A, I_S, Q, T, R) \in \mathbb{R}_+^8 : N(t) \leq \frac{\Lambda}{\mu}\right\}.$$

Proof: In what follows we give a proof to ascertain Theorem 1. From (5) where changes of *N* leads to changes of all variables in the population (i.e *N* = *S, V, E, I_A, I_S, Q, T, R*) we have

$$\frac{dN}{dt} = \Lambda - \mu N - \mu_A - \mu_S - \psi Q \tag{6}$$

In the absence of infection ($\mu_A = \mu_S = \psi$), (6) eases to

$$\frac{dN}{dt} \leq \Lambda - \mu N \tag{7}$$

Solving inequality in (7) following [13] gives

$$N(t) \leq N(0)e^{-\mu t} + \frac{\Lambda}{\mu}(1 - e^{-\mu t}) \tag{8}$$

Hence, as $t \to \infty$ *N*(*t*), approaches $0 \leq N \leq \frac{\Lambda}{\mu} \Longrightarrow N(t) \leq \frac{\Lambda}{\mu}$, then paths of the model (1) are closed. Hence, the feasible solution set for the equation (1) is given by $\Gamma = \left\{(S, V, E, I_A, I_S, Q, T, R) \in \mathbb{R}_+^8 : N(t) \leq \frac{\Lambda}{\mu}\right\}$

In this case, whenever $> \frac{\Lambda}{\mu}$, then $\frac{dN}{dt} < 0$, this implies that the population N(t) decreases asymptotically to its carrying capacity. Therefore, solutions of $S, V, E, I_A, I_S, Q, T, R$ along the initial conditions in R^8_+ is bounded in Γ for $t > 0$, and is a positively invariant set under the flow induced by the model (1). Therefore, equation (1) is meaningful biologically and well-posed in Γ.

3.2.2. Positivity Solution of the State Variables

This explains the solutions of state variables in (1) being non-negativity. That is, for the condition of positivity, we shall establish that the solutions (1) with positive initial conditions, remain positive for all time *t* > 0.

Theorem 2: Let the initial value of (1) be {(*S*(0), *V* (0), *E*(0), I_A(0) ,I_S(0), *Q*(0), *T*(0), *R*(0)) ≥ 0}∈ Γ. Then, the solution set $\{S(t), V(t), E(t), I_A(t), I_S, Q(t), T(t), R(t)\}$ in (1) is positive for all *t* > 0.

Proof. Consider the first equation in (1),

$$\frac{dN}{dt} = \Lambda - \mu + \kappa + \theta)S - \beta(1 - \tau\omega)[I_A + I_S]S + \eta V + \alpha R \geq -(\mu + \kappa + \theta)S,$$

for $\beta(1 - \tau\omega) \in [0,1)$ and $\beta(1 - \tau\omega) \leq \frac{\wedge}{[I_A + I_S]S}$

$$\frac{dS}{dt} = -(\mu + \kappa + \theta)S. \tag{9}$$

Integrating inequality in (9) and by the method of separation of variables, we have:

$$S(t) \geq Ae^{-(\mu+\kappa+\theta)t}. \tag{10}$$

where A is a constant

Setting t = 0 and putting on the initial condition, the result yield

$$S(t) \geq S(0)e - (\mu + \kappa + \theta)t \geq 0, since\ (\mu + \kappa + \theta) > 0. \tag{11}$$

Hence, *S* is positive for *t* > 0. The above steps are also applicable to the second to last term of (1) which establishes that the variables $S(t), V(t), E(t), I_A(t), I_S, Q(t), T(t)$, and $R(t)$ are positive for all *t* > 0.

3.3. Stability Analysis of the Model Equilibria

Here, the equilibria states are determined and the stabilities of these states are analyzed.

3.3.1. Existence of Equilibria Points

Let $\mathbb{E}(S, V, E, I_A, I_S, Q, T, R) \in \Gamma$ represent the equilibrium point of (1). The equilibrium points are generated by imploring the condition:

$$\frac{dS}{dt} = \frac{dV}{dt} = \frac{dE}{dt} = \frac{dI_A}{dt} = \frac{dI_S}{dt} = \frac{dQ}{dt} = \frac{dT}{dt} = \frac{dR}{dt} = 0$$

This implies,

$$\wedge - (\mu + \kappa + \theta)S - (\beta(1 - \tau\omega)I_A + \beta(1 - \tau\omega)I_S)S + \eta V + \alpha R = 0$$

$$\kappa S - (\mu + \eta)V = 0$$

$$(\beta(1 - \tau\omega)I_A + \beta(1 - \tau\omega)I_S)S - (\mu + \sigma\rho)E - (1 - \sigma)\rho E = 0 \quad (12)$$

$$\sigma\rho E - (\delta_A + \mu + \mu_A)I_A = 0$$
$$(1 - \sigma)\rho E - (\mu + \mu_S + \phi_S + \delta_S)I_S = 0$$
$$\phi_S I_S - (\mu + \psi + \epsilon)Q = 0$$
$$\delta I_A + \delta_S I_S + \epsilon Q - (\mu + \gamma)T = 0$$
$$\theta S + \gamma T - (\mu + \alpha)R = 0$$

Let Γ^0 and Γ^* represent the boundary and the interior of Γ in $\mathbb{R}^8$ respectively. Then, by solving equation (12), it has two equilibria: the Covid-19-free equilibrium and endemic equilibrium

3.3.2. Covid-19 - Free Equilibrium (CFE) Point

The CFE points of the model will be generated when ${I_A}^0 = {I_S}^0 = 0$.

Let $\mathbb{E}^0 = (S^0 + V^0 + E^0 + {I_A}^0 + {I_S}^0 + Q^0 + T^0 + R^0)$ be the CFE. From equation (1), we have

$$\begin{aligned}
&\wedge - (\mu + \kappa + \theta)\mathrm{S}^0 - \left(\beta(1 - \tau\omega){I_A}^0 + \beta(1 - \tau\omega){I_S}^0\right)\mathrm{S}^0 + \eta\mathrm{V}^0 + \alpha R^0 = 0\\
&\kappa\mathrm{S}^0 - (\mu + \eta)\mathrm{V}^0 = 0\\
&\left(\beta(1 - \tau\omega){I_A}^0 + \beta(1 - \tau\omega){I_S}^0\right)S - (\mu + \sigma\rho)\mathrm{E}^0 - (1 - \sigma)\rho\mathrm{E}^0 = 0\\
&\sigma\rho\mathrm{E}^0 - (\delta_A + \mu + \mu_A){I_A}^0 = 0\\
&(1 - \sigma)\rho\mathrm{E}^0 - (\mu + \mu_S + \phi_S + \delta_S){I_S}^0 = 0\\
&\phi_S I_S - (\mu + \psi + \epsilon)Q^0 = 0\\
&\delta{I_A}^0 + \delta_S{I_S}^0 + \epsilon Q^0 - (\mu + \gamma)T^0 = 0\\
&\theta\mathrm{S}^0 + \gamma T^0 - (\mu + \alpha)R^0 = 0
\end{aligned} \quad (13)$$

Solving the system of equation (13) simultaneously to obtain the covid-19-free equilibrium state

$$\mathbb{E}^0 = (S^0 + V^0 + E^0 + {I_A}^0 + {I_S}^0 + Q^0 + T^0 + R^0) = \left(\frac{\mu(\mu+\eta)(\mu+\alpha)}{\mu[(\alpha+\mu)(\mu+\kappa+\eta)+\theta(\mu+\eta)]}, \frac{\Lambda\kappa(\mu+\alpha)}{\mu[(\alpha+\mu)(\mu+\kappa+\eta)+\theta(\mu+\eta)]}, 0,0,0,0, \frac{\Lambda\theta(\mu+\eta)}{\mu[(\alpha+\mu)(\mu+\kappa+\eta)+\theta(\mu+\eta)]}\right) \quad (14)$$

This represents the state in which there is no infection in the nation.

3.3.3. Basic Reproductive Number

R_0 is the average number of secondary infections caused by a single infectious individual during their lifetime. The value of R_0 obtained suggest whether a disease has died out or is persistent in a community. Next, the effective reproduction number R_0will be derived using the next generation approach method as described in [49] and [21]. According to Diekmann *et al.*, in [21], $\mathcal{F}\nu^{-1}$ is known the next generation matrix and we set $R_0 = \rho\,(\mathcal{F}\nu^{-1})$ where, ρ denotes the spectral radius of a matrix $\mathcal{F}\nu^{-1}$. For the R_0, differential equations associated with the *E*, I_A, I_S, *Q*, *T*, compartments given below was used. The functions $(\mathcal{F}_i)$ for the new infection rate and (ν_i) for the transfer rate into and out of the infected and Recovered compartments as indicated in Figure 1 was computed.

Rearranging the equation (1) in this order; *E*, I_A, I_S, *Q*, *T* infected classes and followed by uninfected classes; *S*, *V*, *R* also from the two populations, results in

$$\left.\begin{array}{c}
\dot{E} = (\beta(1-\tau\omega)I_A + \beta(1-\tau\omega)I_S)S - (\mu+\sigma\rho)E - (1-\sigma)E \\
\dot{I}_A = \sigma\rho\mathrm{E} - (\delta_A + \mu + \mu_A)I_A \\
\dot{I}_S = (1-\sigma)\rho\mathrm{E} - (\mu + \mu_S + \phi_S + \delta_S)I_S \\
\dot{Q} = \Phi_S I_S - (\mu + \psi + \epsilon)\mathrm{Q} \\
\dot{T} = \delta_A I_A + \delta_S I_S + \epsilon\mathrm{Q} - (\mu+\gamma)\mathrm{T} \\
\dot{S} = \wedge - (\mu + \kappa + \theta)\mathrm{S} - (\beta(1-\tau\omega)I_A + \beta(1-\tau\Gamma)I_S)\mathrm{S} + \eta\mathrm{V} + \alpha\mathrm{R} \\
\dot{V} = \kappa\mathrm{S} - (\mu+\eta)\mathrm{V} \\
\dot{R} = \theta\mathrm{S} + \gamma\mathrm{T} - (\mu+\alpha)\mathrm{R}
\end{array}\right\} \quad (15)$$

The rate of appearance of a new infection in compartments E has been derived using the method of next generation matrix from the system (15) the matrices $\mathcal{F}\nu$ are as follows:

$$\mathcal{F}_i = \begin{bmatrix} \beta(1-\tau\omega)I_A + \beta(1-\tau\omega)I_S)S \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

The Jacobian matrix of $\mathcal{F}$ at the CFE point is obtained below:

$$\mathcal{F} = \begin{bmatrix} 0 & \beta(1-\tau\omega)\mathrm{S} & \beta(1-\tau\omega)\mathrm{S} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (17)$$

The transfer of individuals out of the compartments of the system (3.15) by all other means is

$$\mathcal{V}_i = \begin{bmatrix} (\mu + \sigma\rho)\mathrm{E} + (1-\sigma)\rho\mathrm{E} \\ -\sigma\rho\mathrm{E} + (\delta_A + \mu + \mu_A)I_A \\ -(1-\sigma)\rho\mathrm{E} + (\mu + \mu_S + \Phi_S + \delta_S)I_S \\ -Q_S I_S + (\mu + \psi + \epsilon)\mathrm{Q} \\ -\delta_A I_A - \delta_S I_S - \epsilon\mathrm{Q} - (\mu+\gamma)\mathrm{T} \end{bmatrix} \quad (18)$$

Obtaining the partial derivatives of (18) with respect to E, I_A, I_S, Q, T we obtain

$$\mathcal{V} = \begin{bmatrix} \mu+\rho & 0 & 0 & 0 & 0 \\ -\sigma\rho & \delta_A + \mu + \mu_A & 0 & 0 & 0 \\ -(1-\sigma)\rho & 0 & \mu + \mu_S + \Phi_S + \delta_S & 0 & 0 \\ 0 & 0 & -\Phi_S & \mu + \psi + \epsilon & 0 \\ 0 & -\delta_A & -\delta_S & -\epsilon & \mu+\gamma \end{bmatrix} \quad (19)$$

The next step is to compute the inverse of ν and this gives

$$\mathcal{V}^{-1} = \begin{bmatrix} \frac{1}{(\mu+\rho)} & 0 & 0 & 0 & 0 \\ \frac{\sigma\rho}{(\mu+\rho)(\delta_A+\mu+\mu_A)} & \frac{1}{(\delta_A+\mu+\mu_A)} & 0 & 0 & 0 \\ -\frac{(-1+\sigma)\rho}{(\mu+\rho)(\mu+\mu_S+\Phi_S+\delta_S)} & 0 & \frac{1}{\mu+\mu_S+\Phi_S+\delta_S} & 0 & 0 \\ -\frac{\Phi_S(-1+\sigma)\rho}{(\mu+\rho)(\mu+\mu_S+\Phi_S+\delta_S)(\mu+\psi+\epsilon)} & 0 & \frac{\Phi_S}{(\mu+\mu_S+\Phi_S+\delta_S)(\mu+\psi+\epsilon)} & \frac{1}{(\mu+\psi+\epsilon)} & 0 \\ \frac{\rho\vartheta}{(\mu+\rho)(\delta_A+\mu+\mu_A)(\mu+\mu_S+\Phi_S+\delta_S)(\mu+\psi+\epsilon)(\mu+\gamma)} & \frac{\delta_A}{(\delta_A+\mu+\mu_A)(\mu+\gamma)} & \frac{\delta_S\mu+\delta_S\psi+\delta_S\epsilon+\phi_S\epsilon}{(\mu+\mu_S+\Phi_S+\delta_S)(\mu+\psi+\epsilon)(\mu+\gamma)} & \frac{\epsilon}{(\mu+\psi+\epsilon)(\mu+\gamma)} & \frac{1}{(\mu+\gamma)} \end{bmatrix} \quad (20)$$

where,

$\vartheta = \phi_s\epsilon\delta_A + \phi_s\epsilon\mu_A - \delta_S\sigma\mu^2 + \delta_S\mu_A\epsilon + \delta_S\mu\psi + \delta_S\delta_A\mu + \phi_s\epsilon\mu + \delta_S\delta_A\epsilon + \delta_A\sigma\mu^2 + \delta_S\delta_A\psi + \delta_S\mu_A + \delta_S\mu\epsilon + \delta_S\mu^2 + \delta_A\sigma\mu\psi + \delta_A\sigma\mu\epsilon + \delta_A\sigma\mu_S\mu + \delta_A\sigma\mu_S\psi + \delta_A\sigma\mu_S\epsilon + \delta_A\sigma\phi_S\mu + \delta_A\sigma\phi_S\psi - \delta_S\sigma\mu\psi - \delta_S\sigma\mu\epsilon - \delta_S\sigma\mu_A\mu - \delta_S\sigma\mu_A\psi - \delta_S\sigma\mu_A\epsilon - \phi_S\epsilon\sigma\mu - \phi_S\epsilon\sigma\mu_A$

Solving $\mathcal{F}\nu^{-1}$, the basic reproductive number for the populations is obtained as

$$R_0 = \frac{\beta\Lambda\,\rho(1-\tau\omega)\,(\mu+\eta)\,(\mu+\alpha)((\delta_A+\mu+\mu_A)+\sigma\,(\mu_S+\Phi_S+\delta_S-\delta_A-\mu_A))}{\mu[(\alpha+\mu)(\mu+\kappa+\eta)+\theta(\mu+\eta)](\mu+\mu_S+\phi_S+\delta_S)(\delta_A+\mu+\mu_A)(\mu+\rho)} \tag{21}$$

Remark 3.1: Epidemiologically,

- if $R_0 < 1$, Covid-19 spread rate will diminish and go into extinction.
- if $R_0 = 1$,Covid-19 spread rate remain unchanged.
- if $R_0 > 1$ the spread rate of the Covid-19 will increase and keep spreading.

3.3.4. Local Stability Analysis of the CFE State

To achieve the local stability, the certain conditions that must be achieved for the CFE state to be stable and at the same time to be fully removed from the population are determined. Hence, the Jacobian matrix *J* of model (3.1) is determined by differentiating each equation in the system with respect to the state variables *S, V, E, IA, IS, Q, T, R*.

Theorem 3: The Covid-19-free equilibrium point is locally asymptotically stable if *R*0 < 1 for $tr(J^0(\mathbb{E}^0)) < 0$ and $detJ^0(\mathbb{E}^0) > 0$, and unstable if $R_0 > 1$ for $tr(J^0(\mathbb{E}^0)) > 0$ and $detJ^0(\mathbb{E}^0) < 0$

Proof. In the proof, a Jacobian matrix for the model system (1) evaluated at the CFE ($\mathbb{E}^0$) is built as follows:

$$\mathcal{J}_{E^0}{}^0 = \begin{bmatrix} \mathcal{H} & \eta & 0 & -\beta(1-\tau\omega)S^0 & -\beta(1-\tau\omega)S^0 & 0 & 0 & \alpha \\ -\kappa & -\mu-\eta & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathcal{G} & 0 & -\mu-\rho & \beta(1-\tau\omega)S^0 & \beta(1-\tau\omega)S^0 & 0 & 0 & 0 \\ 0 & 0 & \rho\sigma & -\delta_A-\mu-\mu_A & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\sigma)\rho & 0 & -\mu-\mu_S-\phi_S-\delta_S & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_S & -\mu-\psi-\epsilon & 0 & 0 \\ 0 & 0 & 0 & \delta_A & \delta_S & -\mu & -\gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu+\rho & -\gamma & -\mu-\alpha \end{bmatrix} \tag{22}$$

where, $\mathcal{H} = \mu - \kappa - \theta - \beta\,(1-\tau\,\omega)\,(I_A^0 + I_S^0)$ and $\mathcal{G} = \beta\,(1-\tau\,\omega)\,(I_A^0 + I_S^0)$.

Therefore, at CFE (*E*0), *IA*0 = *IS*0 = 0 and $S^0 = \frac{\Lambda(\mu+\eta)(\mu+\alpha)}{\mu[(\alpha+\mu)(\mu+\kappa+\eta)+\theta(\mu+\eta)]}$, the Jacobian matrix J^0 in (22) reduced to

$$\mathcal{J}_{E^0}{}^0 = \begin{bmatrix} -a & \eta & 0 & -b & -b & 0 & 0 & \alpha \\ -\kappa & -c & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -d & b & b & 0 & 0 & 0 \\ 0 & 0 & \rho\sigma & -e & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\sigma)\rho & 0 & -f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_S & -g & 0 & 0 \\ 0 & 0 & 0 & \delta_A & \delta_S & -\epsilon & -h & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma & -X \end{bmatrix} \tag{23}$$

where,

$a = (\mu+\kappa+\theta)$, $b\ \frac{\beta\,(1-\tau\,\omega)\,(\mu+\eta)(\mu+\alpha)\Lambda}{\mu[(\alpha+\mu)(\mu+\kappa+\eta)+\theta(\mu+\eta)]}$; $c = -(\mu+\eta)$; $d = (\mu+\rho)$; $e = (\delta_A+\mu+\mu_A)$; $f = (\mu+\mu_S+\phi_S+\delta_S)$; $g = (\mu+\psi+\varepsilon)$; $h = (\mu+\gamma)$; $X = (\mu+\alpha)$

Next, the stability of the CFE state is determined. This means the conditions that determines the total eradication of Covid-19 from the Nigeria population. The method of evaluating the trace and determinant of the system of equations without explicitly calculating the eigenvalues is employed here.

The matrix $J^0(\mathbb{E}^0)$ in (23) of dimension (11) is stable if its trace is negative i.e $tr\left(J^0(\mathbb{E}^0)\right) < 0$ and its determinant is positive i.e $detJ^0(\mathbb{E}^0) \geq 0$. The trace of matrix $J^0(\mathbb{E}^0)$ is obtained as:

$$trJ^0(\mathbb{E}^0) = -a-c-d-e-f-g-h-X$$

$$= -(a+c+d+e+f+g+h+X)$$

$$= -(8\mu + k + \theta + \eta + \rho + \delta_A + \mu_A + \mu_s + \phi_s + \delta_s + \psi + \epsilon + \gamma + \alpha)$$
$$trJ^0(\mathbb{E}^0) < 0 \quad (24)$$

Also, the determinant of matrix $J^0(\mathbb{E}^0)$is generated as

$detJ^0(\mathbb{E}^0) = gh(-\kappa\eta X + acX - \theta ca)(def - \sigma\rho bf - \rho eb + \rho eb\sigma)$
$= gh(acX - \kappa\eta X - \theta ca)(\rho eb\sigma + \text{def} - \sigma\rho bf - \rho eb)$
Hence

$$detJ^0(\mathbb{E}^0) > 0 \quad (25)$$

Iff $acX > \kappa\eta X + \theta c\alpha$, and $e > \frac{\rho b(\sigma f + e)}{(\rho b\sigma + df)}$
After simplification of equation (25), we obtain the following:
$detJ^0(\mathbb{E}^0) = \mu(\mu + \psi + \varepsilon)(\mu + \gamma)(\mu + \alpha)[(\mu + \eta)(\alpha + k + \mu + \theta)](\mu + \rho)(\delta A + \mu + \mu A)(\mu + \mu s + \phi S + \delta S)(1 - R0)$ (26)
Hence, $detJ^0(\mathbb{E}^0) > 0$ if $R_0 < 1$. Thus biologically, Covid -19 infection will be eradicating from the population.

3.3.5. Endemic Equilibrium (EE)

Here, the EE of equation (1) is obtained. Endemic equilibrium state is the state where the disease cannot be completely removed but continue in the entire population. That is, $(S^*, V^*, E^*, I_A{}^*, I_S{}^*, Q^*, T^*, R^*) \neq (0,0,0,0,0,0,0,0)$

To obtain the endemic equilibrium state, one need to solve (1) by equating them to zero. The solution obtained after solving (1) simultaneously is represented in vector form below:

$$S^* = \frac{\vartheta_1(\mu + \rho)\vartheta_2}{\vartheta_5(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2)}$$

$$V^* = \frac{\kappa\vartheta_1(\mu + \rho)\vartheta_2}{(\mu + \eta)\vartheta_5(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2)}$$

$$E^* = \frac{(\mu + \gamma)\vartheta_6\vartheta_1\vartheta_2[\vartheta_5\Lambda(\mu + \alpha)(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2) - \vartheta_1\vartheta_2(\mu + \rho)(\mu\vartheta_3 + \alpha\vartheta_4)]}{\rho\vartheta_5(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2)[\vartheta_2(\mu + \rho)(\mu + \alpha)(\mu + \gamma)\vartheta_6\vartheta_1 - \rho\alpha\gamma(\sigma(\delta_A\vartheta_6\vartheta_1 - \vartheta_2\vartheta_7) + \vartheta_2\vartheta_7)]}$$

$$I_A{}^* = \frac{\sigma(\mu + \gamma)\vartheta_6\vartheta_1[\vartheta_5\Lambda(\mu + \alpha)(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2) - \vartheta_1\vartheta_2(\mu + \rho)(\mu\vartheta_3 + \alpha\vartheta_4)]}{\vartheta_5(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2)[\vartheta_2(\mu + \rho)(\mu + \alpha)(\mu + \gamma)\vartheta_6\vartheta_1 - \rho\alpha\gamma(\sigma(\delta_A\vartheta_6\vartheta_1 - \vartheta_2\vartheta_7) + \vartheta_2\vartheta_7)]}$$

$$I_S{}^* = \frac{(1 - \sigma)(\mu + \gamma)\vartheta_6\vartheta_2[\vartheta_5\Lambda(\mu + \alpha)(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2) - \vartheta_1\vartheta_2(\mu + \rho)(\mu\vartheta_3 + \alpha\vartheta_4)]}{\vartheta_5(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2)[\vartheta_2(\mu + \rho)(\mu + \alpha)(\mu + \gamma)\vartheta_6\vartheta_1 - \rho\alpha\gamma(\sigma(\delta_A\vartheta_6\vartheta_1 - \vartheta_2\vartheta_7) + \vartheta_2\vartheta_7)]}$$

$$Q^* = \frac{\phi_S(1 - \sigma)(\mu + \gamma)\vartheta_2[\vartheta_5\Lambda(\mu + \alpha)(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2) - \vartheta_1\vartheta_2(\mu + \rho)(\mu\vartheta_3 + \alpha\vartheta_4)]}{\vartheta_5(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2)[\vartheta_2(\mu + \rho)(\mu + \alpha)(\mu + \gamma)\vartheta_6\vartheta_1 - \rho\alpha\gamma(\sigma(\delta_A\vartheta_6\vartheta_1 - \vartheta_2\vartheta_7) + \vartheta_2\vartheta_7)]}$$

$$T^* = \frac{[\delta_A\vartheta_6\vartheta_1 - \vartheta_2\vartheta_7][\vartheta_5\Lambda(\mu + \alpha)(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2) - \vartheta_1\vartheta_2(\mu + \rho)(\mu\vartheta_3 + \alpha\vartheta_4)]}{\vartheta_5(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2)[\vartheta_2(\mu + \rho)(\mu + \alpha)(\mu + \gamma)\vartheta_6\vartheta_1 - \rho\alpha\gamma(\sigma(\delta_A\vartheta_6\vartheta_1 - \vartheta_2\vartheta_7) + \vartheta_2\vartheta_7)]}$$

$$R^* = \frac{\vartheta_1\vartheta_2\theta(\mu + \rho)[\vartheta_5\Lambda(\mu + \alpha)(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2) - \vartheta_1\vartheta_2(\mu + \rho)(\mu\vartheta_3 + \alpha\vartheta_4)] + \gamma\emptyset}{\vartheta_5(\mu + \alpha)(\sigma(\vartheta_1 - \vartheta_2) + \vartheta_2)[\vartheta_2(\mu + \rho)(\mu + \alpha)(\mu + \gamma)\vartheta_6\vartheta_1 - \rho\alpha\gamma(\sigma(\delta_A\vartheta_6\vartheta_1 - \vartheta_2\vartheta_7) + \vartheta_2\vartheta_7)]}$$

Where,
$\vartheta_1 = \mu + \mu_S + \phi_S + \delta_S$, $\vartheta_2 = \delta_A + \mu + \mu_A$, $\vartheta_3 = \mu + \kappa + \theta$, $\vartheta_4 = \mu + 2\kappa$, $\vartheta_5 = \beta\rho(1 - \tau\omega)$, $\vartheta_6 = (\mu + \psi + \varepsilon), \vartheta_7 = (\delta_S\mu + \phi_S\varepsilon + \delta_S\psi + \delta_S\varepsilon$

3.4. The Nigeria Covid-19 Data

The Nigeria Covid-19 total confirmed cases, recovered, number of death recorded and number of vaccinated are presented in Table 3 below.

Table 3. COVID-19 total confirmed cases, Recovered, Number of Death recorded and number of vaccinated in Nigeria

Months	Year	Total Cases	Death	Vaccinated (1 dose)	Vaccinated (complete dose)
March, 31	2021	162891	2056	718418	0
April,30	2021	165110	2063	1191563	0
May, 31	2021	166518	2099	1956598	73465
June, 30	2021	167618	2120	2241662	1155810
July, 31	2021	173908	2149	2994933	1437349
August, 31	2021	192431	2469	2994933	1437349
Sept., 30	2021	205765	2720	4790943	1943821
Oct., 30	2021	211961	2896	5693300	3079239
Nov., 30	2021	214218	2977	6329967	3516215
Dec.,31	2021	242341	3031	10310476	4482899
Jan.	2022	253181	3142	14834559	5446849
Feb.	2022	254660	3142	17735985	8091251
March, 31	2022	255415	3142	21049754	9565143
April,30	2022	255716	3143	25056663	14624418
May, 31	2022	256028	3143	27401621	16817979
June, 30	2022	257290	3144	28340766	20986761
July, 31	2022	257362	3146	36549506	24675659
August, 31	2022	257999	3147	41103768	28987008
Sept., 30	2022	264014	3148	49076780	36133196

3.5. Parameter Estimation

In this section, the parameters in equation (1) based on the real datasets of COVID-19 infected cases, number of vaccinated and death cases recorded in Nigeria were estimated. The COVID-19 monthly confirmed cases at the inception of the vaccination exercise which commenced on March 5, 2021, until the month of rounding up the research (September 31, 2022) in Nigeria was considered (see Table 3). The data were collected from NCDC online site [37] and also available online at [37]. Some of the model parameters were obtained from the literature while some were assumed based on the research focus. The average life expectancy (ALE) of Nigerians for the year 2021 is 55.12 ([26]) and hence, the natural death rate of individuals was calculated by taking the reciprocal of the ALE (in months), that is, $\mu = \frac{1}{(55.12 \times 12)}$.

The total population of Nigeria for the year 2021 is about 211,400,708, that is $N(0) = 211{,}400{,}708$, people. The recruitment rate of susceptible individuals ($\wedge$) was obtained from $\frac{\Lambda}{\mu} = N(0)$, and this resulted to 319,638. The incubation period $\left(\frac{1}{\sigma}\right)$ for COVID-19, which is the time from exposure to symptom development, is an average of five to seven days. Therefore, $\sigma = \frac{1}{7}$. The quarantine period $\left(\frac{1}{\phi_S}\right)$ for COVID-19 is 2 weeks. That is, $\phi_S = \frac{1}{14} days^{-1}$.

To estimate the remaining parameters in the model, the initial conditions for the state variables are used. According to the NCDC report, the total confirmed cases of COVID-19 in Nigeria was 158,042 as at March 5, 2021, that is, $I_S(0) = 158{,}042$. As in other parts of the world, the administration of the vaccine kicked off on the 5th of March 2021 in Nigeria, with just one vaccinated individual. Also as at March 5, 2021, in Nigeria, there was limited COVID-19 test being carried out and hence, the possibility for the presence of asymptomatic ($I_A(0)$) individuals was found to be 40.50% of Symptomatic patients ($I_S(0)$). This resulted to the value of $I_A(0) = 64{,}007.01$. Thus, we estimate the initial conditions for the exposed, quarantined, treated and recovered individuals to account for the possible actual number of cases in Nigeria as at march 5, 2021. The values are: $V(0) = 1, E(0) = 125657, I_A(0) = 64007, I_S(0) = 158042, Q(0) = 9538, T(0) = 193172, R(0) = 96586$. Now, it follows that the initial susceptible population is given as: $S(0) = N(0) - (V(0) + E(0) + I_A(0) + I_S(0) + Q(0) + T(0) + R(0))$. Therefore, $S(0) = 211{,}400{,}708 - (1 + 125657 + 64007 + 158042 + 9538 + 193172 + 96586) = 210{,}753{,}705$. The summary of the estimated and calculated parameters in the model are presented in Table 4 above.

3.6. Sensitivity Analysis of the Model

The relative contributions of the parameters used in the model responsible for the transmission and prevalence of Covid-19 is discussed here. The parameters used in this section are shown in Table 4, chosen from the realistic ranges for illustrative purpose.

Definition 3.1: The normalized forward sensitivity index of a variable Q that depends, differentially, on a parameter m, is defined as: $Z_m^Q = \frac{\partial Q}{\partial m} \times \frac{m}{Q}$ where, $Z_m^{R_0}$ is the sensitivity index of R_0 with respect to parameter, m.

Table 4. Parameter values used in carrying out the sensitivity analysis of the model

Symbol	Baseline values	Reference
Λ	319638	Estimated
β	0.0005944	Abdul rahman et al 2020 [1]
μ	0.1512	Estimated
δA	0.8	Asumed
δS	0.85	Assumed
γ	0.05	Assumed
θ	[0-1.0]	Assumed
μ_A	0.0321	Estimated
μ_S	0.013	Salihu 2021[45]
ϕ_S	0.071	Estimated
ψ	0.06	Assumed
α	0.01	Assumed
ε	0.8	Abdul rahman et al 2020 [1]
σ	0.143	Estimated
ρ	0.00147	Estimated
τ	[0 - 1.0]	Assumed
ω	0.6	Zenebe et al 2021 [54]
κ	[0 - 1.0]	Assumed
η	0.05	Assumed

3.6.1. Sensitivity Indices of R_0

Given the explicit form of R_0; we now obtain an analytical expression for its sensitivity to each parameter applying the normalized forward sensitivity index by [16] and given as:

$$Z_m^{R_0} = \frac{\partial R_0}{\partial m} \times \frac{m}{R_0} \tag{28}$$

where $Z_m^{R_0}$ is the sensitivity index of R_0 with respect to parameter, m.

Sensitivity indices of R_0 corresponding to these parameters:
$\beta,\ \Lambda, \rho, \tau, \alpha, \sigma, \omega, \delta_A, \mu_A, \kappa, \theta, \eta, \delta_S, \mu_S, \mu, \phi_S$ was derived and computed. For instance, the sensitivity index of R_0 with respect to Λ and β are obtained as follows:

$$\left.\begin{aligned} Z_\Lambda^{R_0} &= \frac{\partial R_0}{\partial \Lambda} \times \frac{\Lambda}{R_0} = +1.0 \\ Z_\beta^{R_0} &= \frac{\partial R_0}{\partial \beta} \times \frac{\beta}{R_0} = +1.0 \end{aligned}\right\} \tag{29}$$

The sensitivity indices of R_0 with respect to the parameters Λ and β equals +1.0, and does not depend on any parameter value. The remaining indices are obtained by following the same method.
Using the parameter values in Table 4 for the sensitivity analysis, the results of the sensitivity analysis are presented in Table 5.

Table 5 shows the sensitivity indices of R_0 with respect to the sixteen parameters involved in the reproductive numbers. The highly sensitive parameters to the reproductive number include, μ followed by $\delta_S, \kappa, \tau, \omega, \theta, \delta_A, \phi s, \mu_S, \mu_A$, and while the lesser sensitive parameter is the contact rate β, and the recruitment rate Λ followed by $\rho, \eta\ \sigma, \alpha$.

From Table 5, the indices with positive signs show that the value of R_0, increases when the corresponding parameters increases and indices with negative signs indicates that, the value of R_0 decreases with increase in the corresponding parameters. This analysis is done to ascertain which parameters dominate the results of our analysis.

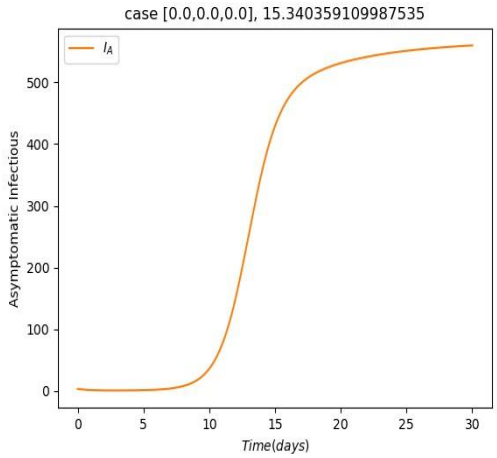
Therefore, it is clear from the Table 5 above that the R_0 will reduce with increases in the values of the relevant parameters, since the sensitivity indices of these parameters are negative.

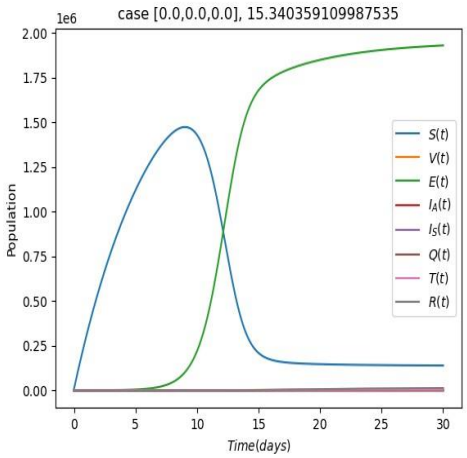
Table 5. Sensitivity indices of R_0 Parameters

Parameters	Sensitivity indices
	R_0
Λ	+1.000000000
β	+1.000000000
μ	-1.487199257
δ_A	-0.1349785823
δ_S	-0.640577200
θ	-0.2631578947
μ_A	-0.001687232279
μ_S	-0.03089842965
ϕ_S	-0.05350703670
α	+0.01644736844
σ	+0.02214037174
ρ	+0.9902951079
τ	-0.4814814814
ω	-0.4814814812
κ	-0.5263157896
η	+0.1315789474

3.7. Numerical Simulations and Analysis

Numerical simulations were achieved via the estimated parameters and some imported values from several literatures as shown in Table 1. The system in (1) was implemented with Maple-15 and Julia using a Runge-Kutta method of order four for solving systems in (1). The estimated initial population values given above were used for the simulation. The effect of the inclusion of vaccines as precautionary measures in order to control the spread of Covid 19 in Nigeria, was shown in figure 2 - 14. The rate of vaccination (κ), immunity rate (θ) and Precautionary measure rate (τ) are used to govern the value of R_0. In the above, if $(\kappa, \theta, \tau) < 0.5$, $R_0 > 1$ and if $(\kappa, \theta, \tau) > 0.5$, $R_0 < 1$. The simulations have been done for various values of κ, θ and τ as the key parameters and are shown in the figures below.





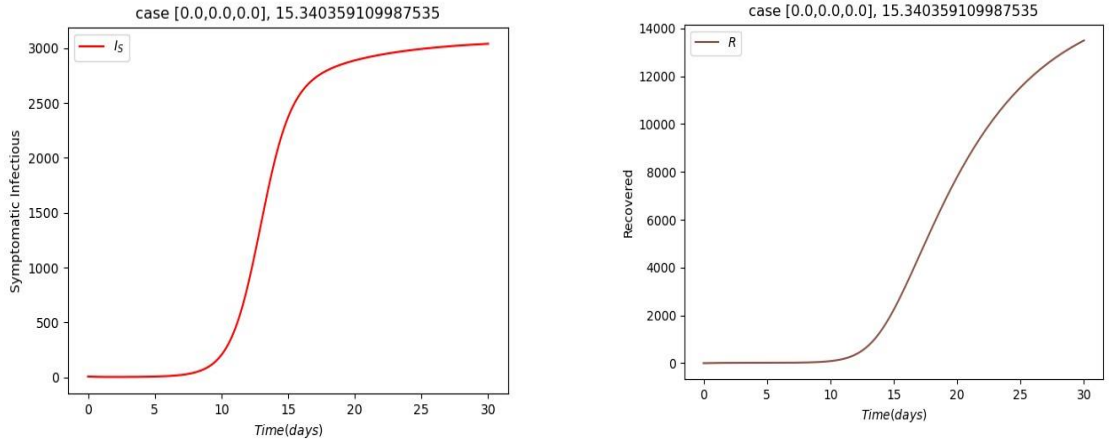


Fig. 2 The plot display the simulation with time using: $\kappa = 0.0, \theta = 0.0$ and $\tau = 0.0$ $R_0 = 15.34035911 > 1$

Figure 2 shows that if none of the new recruits is vaccinated or having immunity gained or follow the precautionary measures against the spread of Covid-19 infection, that is, $\kappa = 0\%$, $\theta = 0\%$ and $\tau = 0\%$, the graph shows that both Symptomatic and Asymptomatic infected will increase drastically, then $R_0 > 1$.

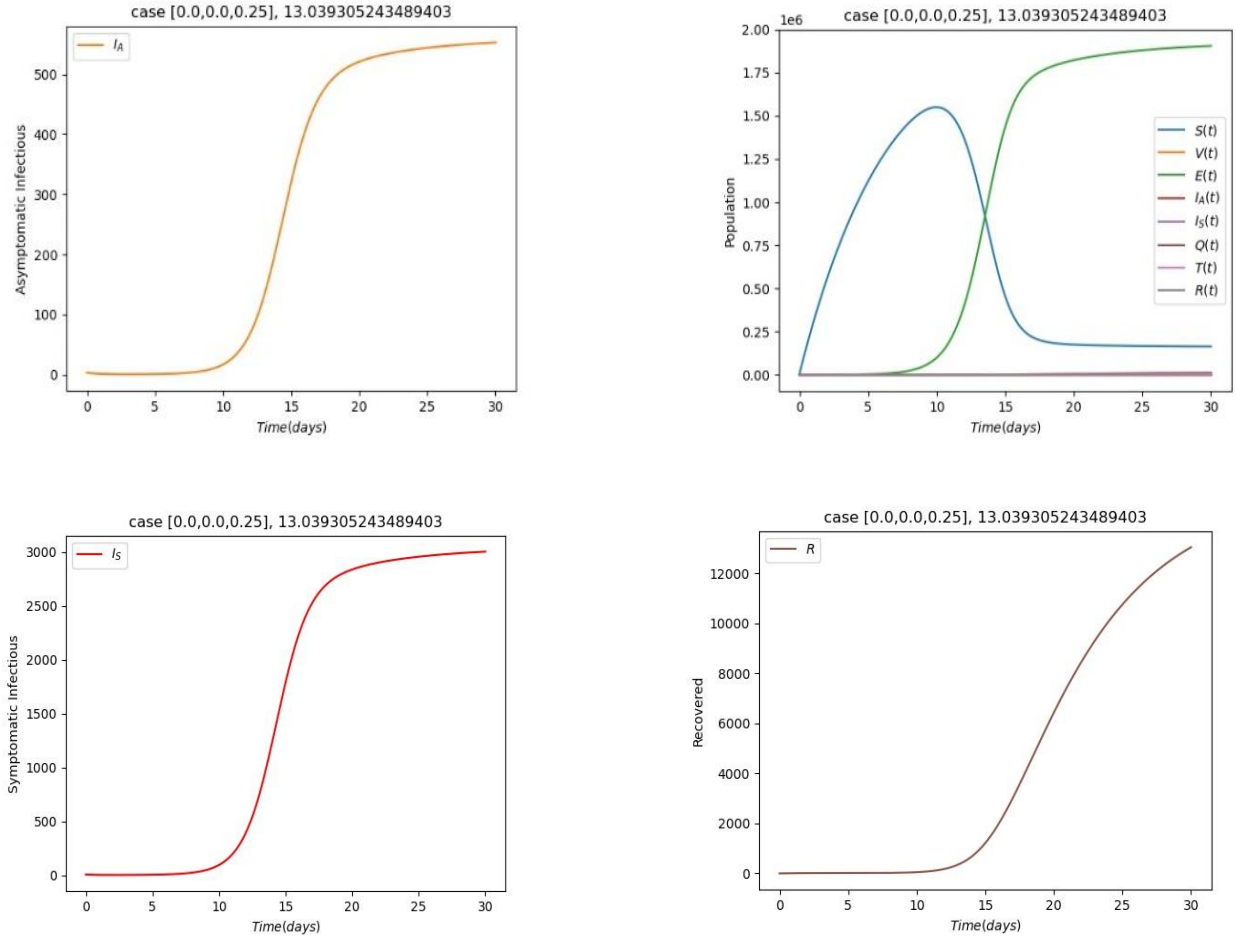


Fig. 3 The plot display the simulation with time using: $\kappa = 0, \theta = 0$ and $\tau = 0.25$ $R_0 = 13.03930524 > 1$

Figure 3 shows that if the rate of vaccination is at 0%, the immunity gain rate at 0% and the rate at which people take other precautionary measures is 25%, the graph shows that both Symptomatic and Asymptomatic infected will increase drastically, then $R_0 > 1$.

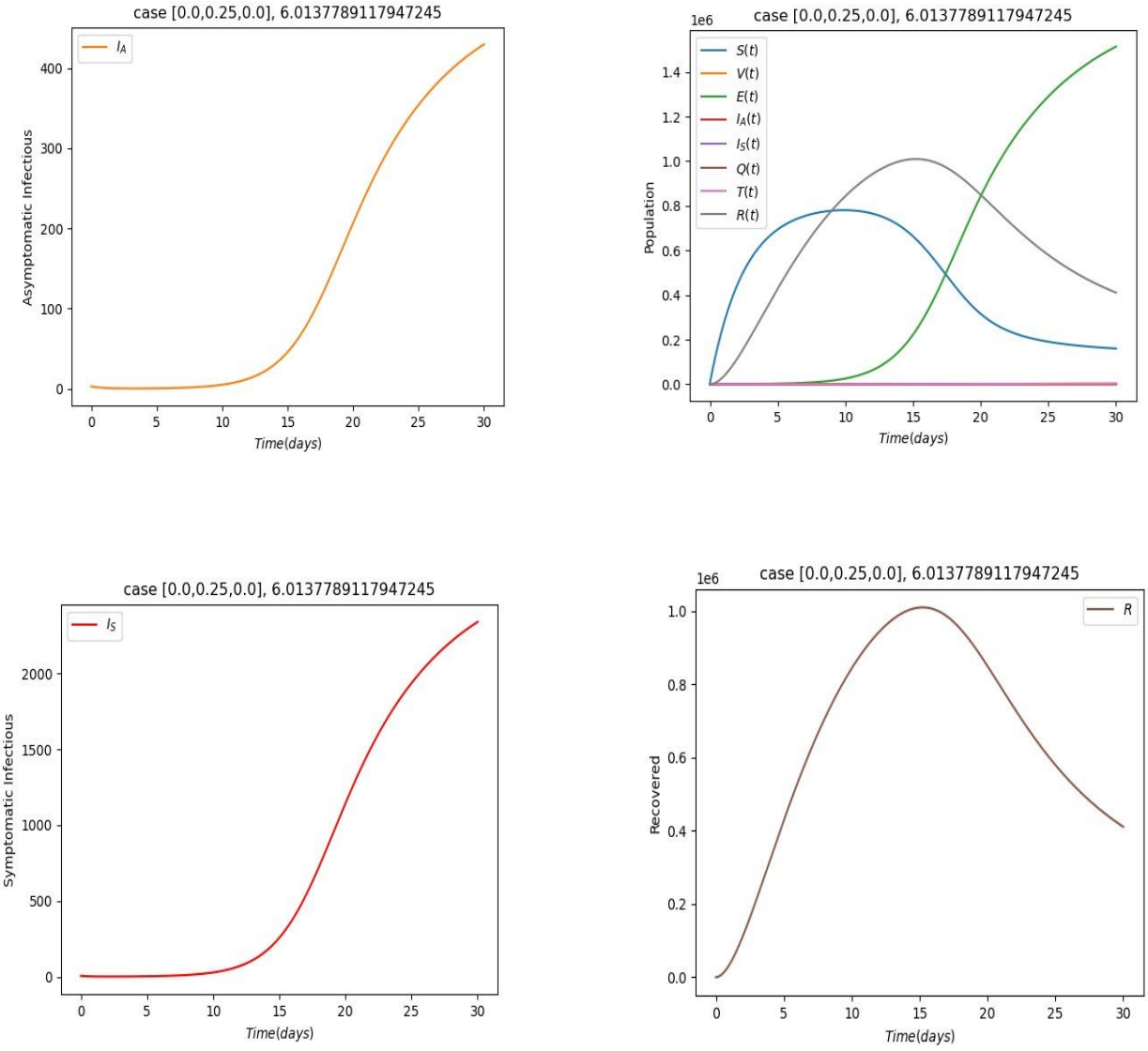


Fig. 4 The plot display the simulation with time using: $\kappa = 0.0, \theta = 0.25$ and $\tau = 0.0$ $R_0 = 6.013778912 > 1$

Figure 4 shows that if vaccination rate is at 0%, immunity gain rate is 25% and other precautionary measures rate is at 0% against Covid-19 infection, the graph shows that both Symptomatic and Asymptomatic infected will still increase drastically in Nigeria, then, $R_0 > 1$.

Figure 5 shows that if not more than 25% of susceptible and new recruits are vaccinated with 0% immunity gain rate and 0% rate of taking precautionary measures against Covid infection, the graph shows that both the Symptomatic and Asymptomatic infected will continue to be in existence, the $R_0 > 1$.

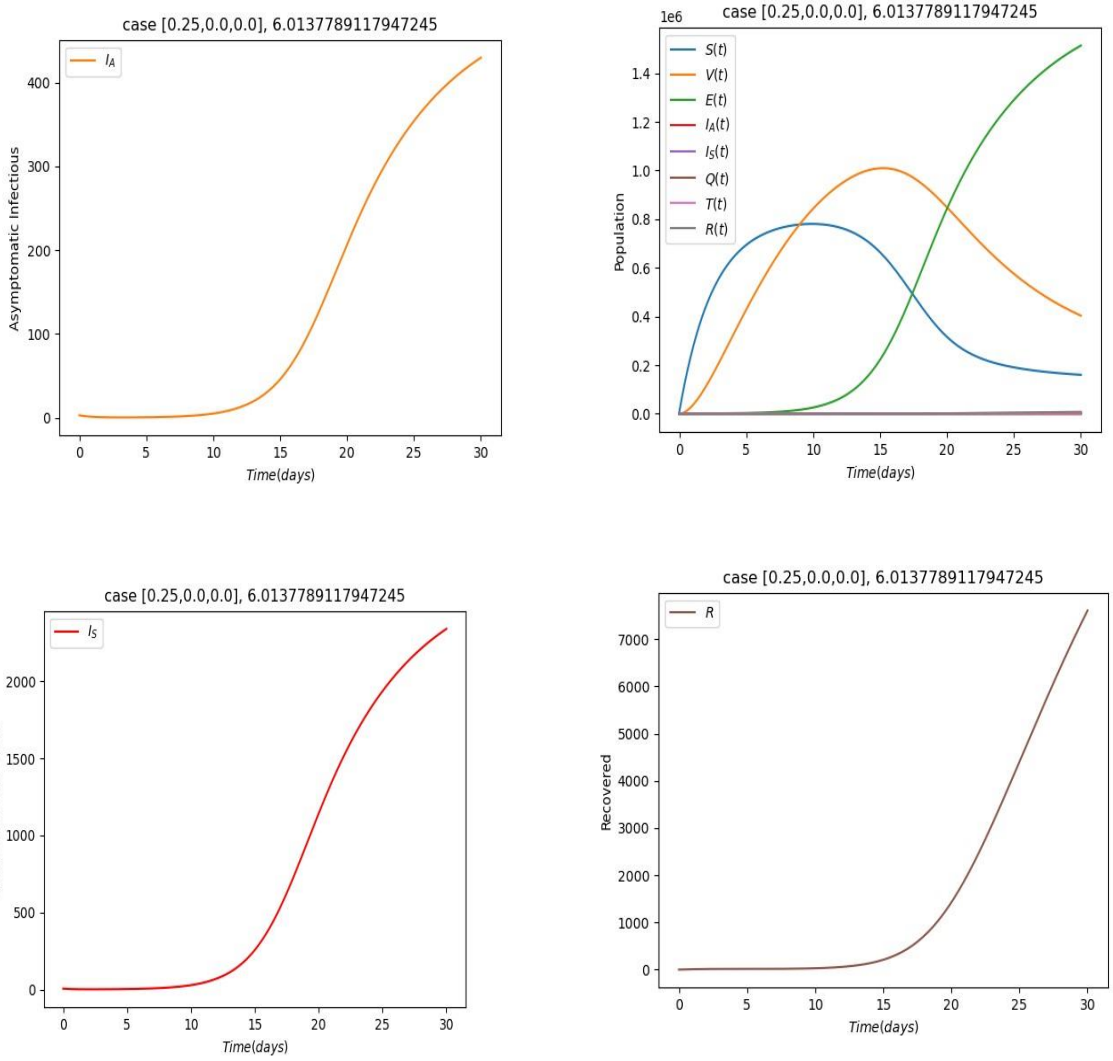
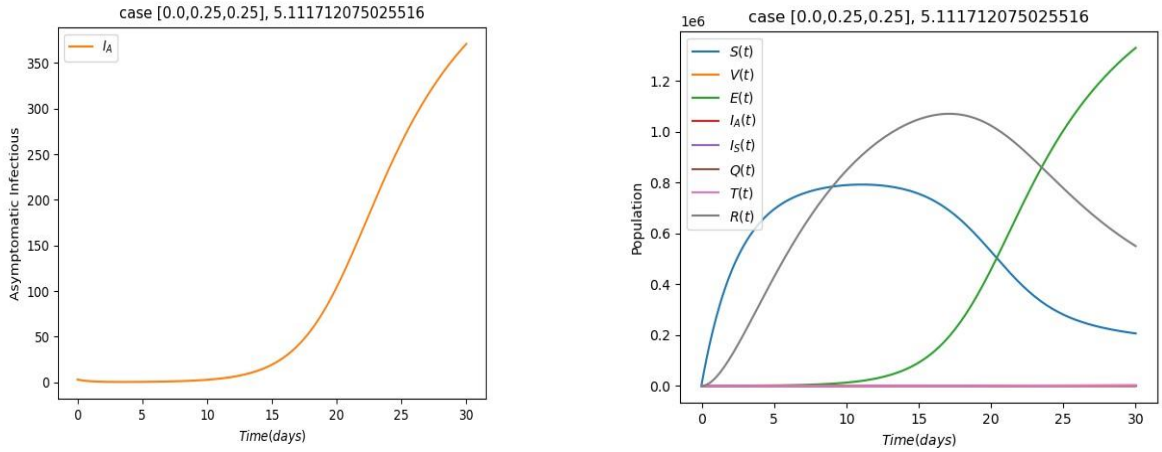


Fig. 5 The plot display the simulation with time using: $\kappa = 0.25, \theta = 0.0$ and $\tau = 0.0$ $R_0 = 6.013778912 > 1$



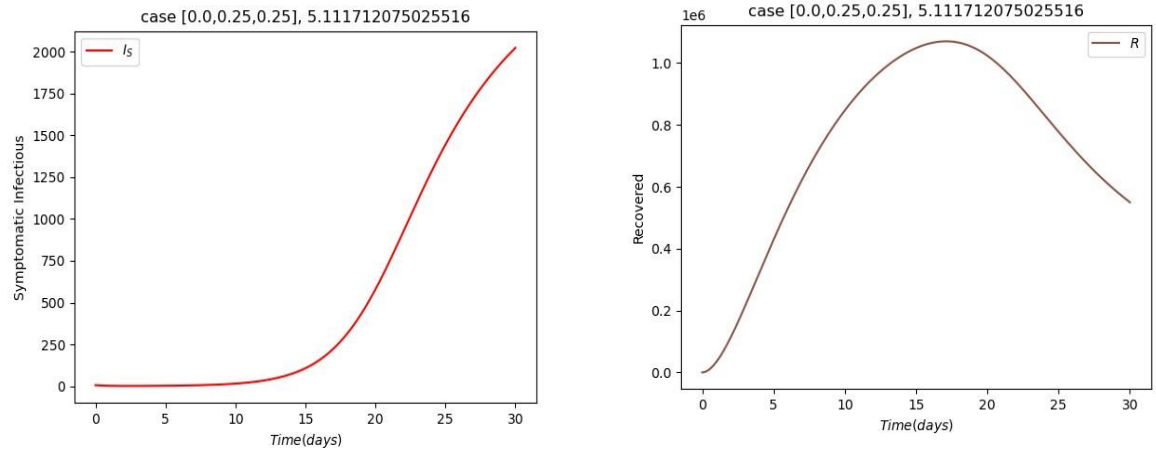


Fig. 6 The plot display the simulation with time using: $\kappa = 0, \theta = 0.25$ and $\tau = 0.25$ $R_0 = 5.111712076 > 1$

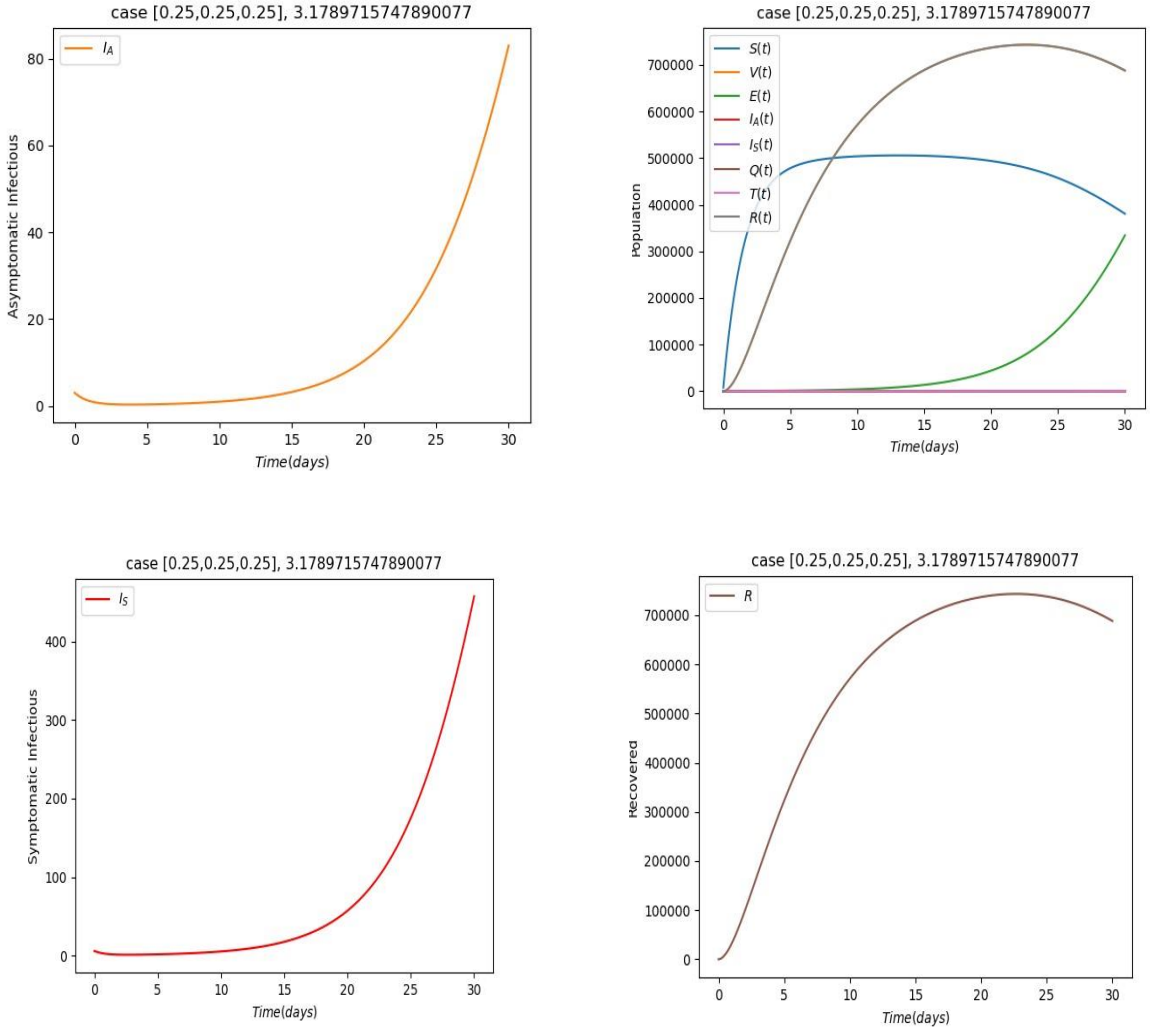


Fig. 7 The plot display the simulation with time using: $\kappa = 0.25$, $\theta = 0.25$ and $\tau = 0.25$ $R_0 = 3.178971575 > 1$

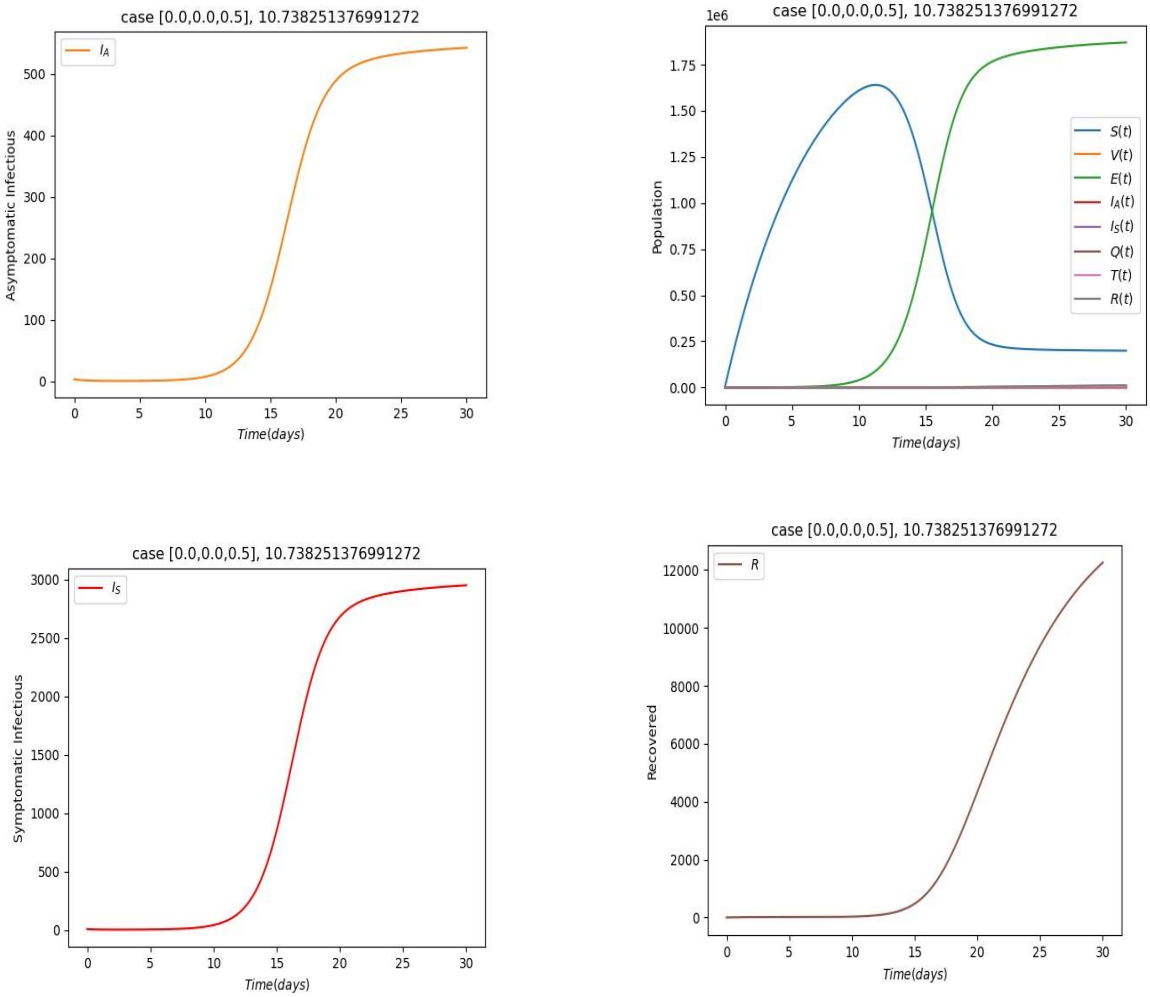
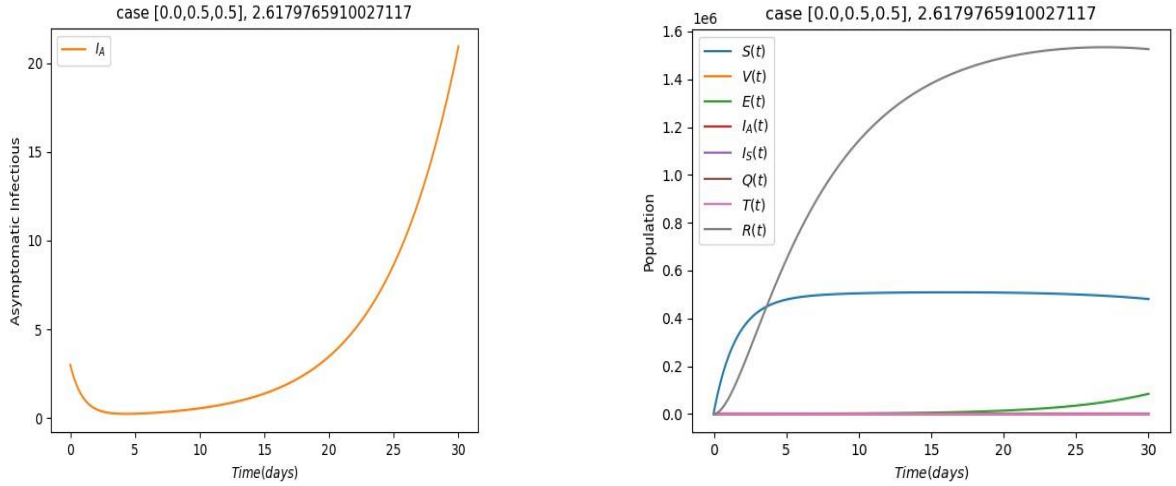


Fig. 8 The plot display the simulation with time using: $\kappa = 0$, $\theta = 0$ and $\tau = 0.5$ $R_0 = 10.73825138 > 1$



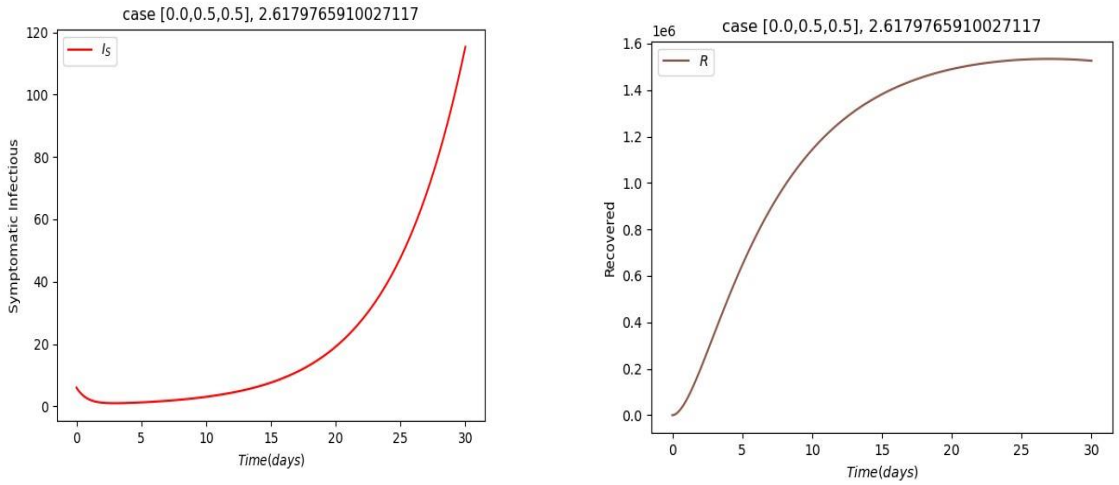


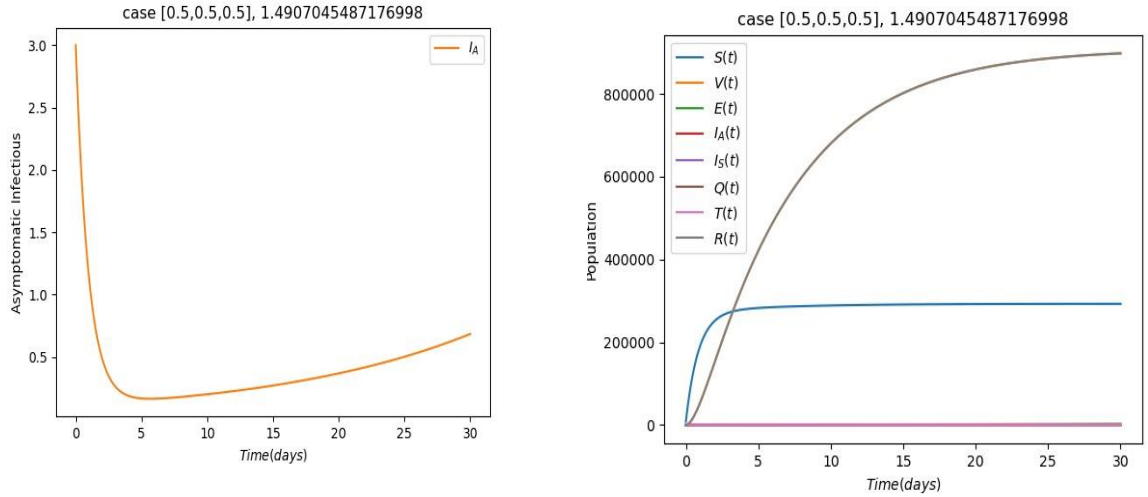
Fig. 9 The plot display the simulation with time using: $\kappa = 0$, $\theta = 0$ and $\tau = 0$ $R_0 = 2.617976591 > 1$

Figure 6 shows that if the vaccination rate is at 0% with about 25% population possesses immunity gain and also 25% take precautionary measure against covid infection, the graph shows that both Symptomatic and Asymptomatic infected will still occur, therefore, $R_0 > 1$

Figure 7 shows that if at least 25% of the population is vaccinated, 25% is having immunity gain and 25% is taking precautionary measure against Covid infection, the graph shows that both Symptomatic and Asymptomatic infected will reduced drastically, but the $R_0 > 1$.

Figure 8 shows that if the rate of vaccination is 0%, immunity gain rate to be 0% and precautionary measure rate is at 50% against Covid infection, the graph shows that instead of the infection to reduce in the population, both Symptomatic and Asymptomatic infected increases drastically, then $R_0 > 1$.

Figure 9 shows that if the rate in which susceptible and new recruits is being vaccinated is still at 0%, but immunity gain rate is 50% and precautionary measure is about 50% against Covid infection the graph shows that the rate of infection spreading will reduced but both Symptomatic and Asymptomatic infected will remain in the population due to the $R_0 > 1$ at this stage.



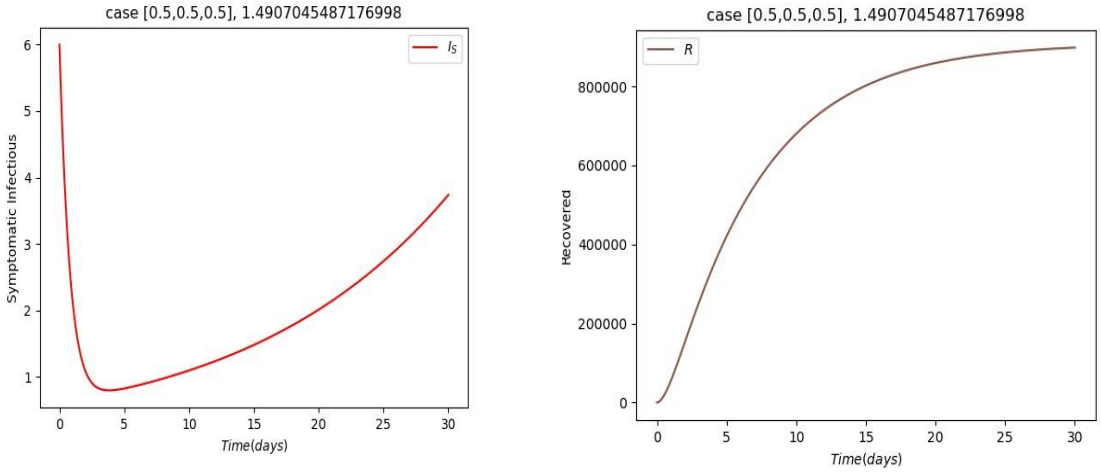


Fig. 10 The plot display the simulation with time using: $\kappa = 0.5$, $\theta = 0.5$ and $\tau = 0.5$ $R_0 = 1.490704549 \geq 1$

Figure 10 shows that if rate of vaccination is 50%, immunity gain is 50% and precautionary measure is 50% against Covid infection, the graphs shows that both Symptomatic and Asymptomatic infected class reduced drastically with time (t), the susceptible class also reduced and recovered class increased more.

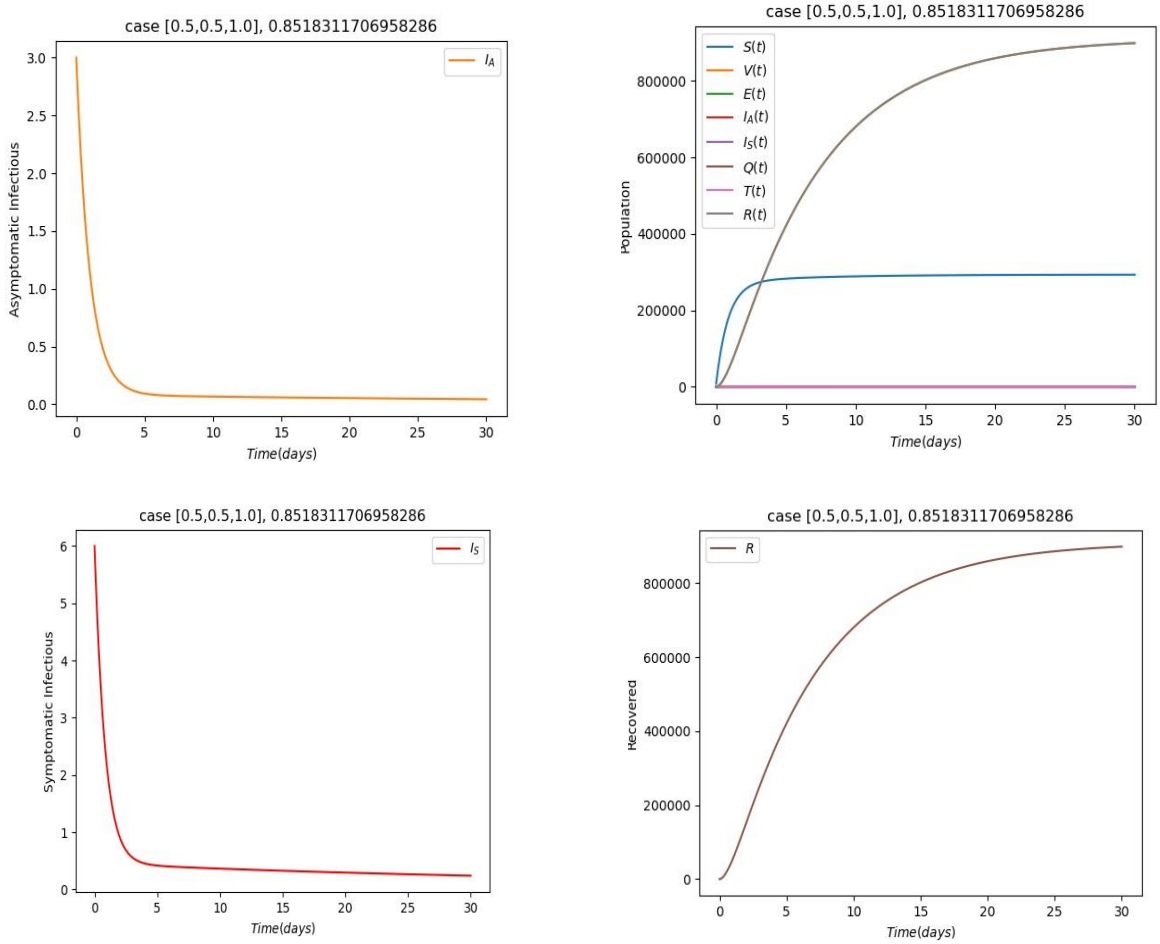


Fig. 11 The plot display the simulation with time using: $\kappa = 0.5$, $\theta = 0.5$ and $\tau = 1.0$ $R_0 = 0.8518311707 < 1$

Figure 11 shows that if at least 50% of the population is been vaccinated, 50% is having immunity gain and almost 100% of the population is taking precautionary measure against Covid infection, the graph shows that both Symptomatic and Asymptomatic infected will reduce drastically, and $R_0 < 1$.

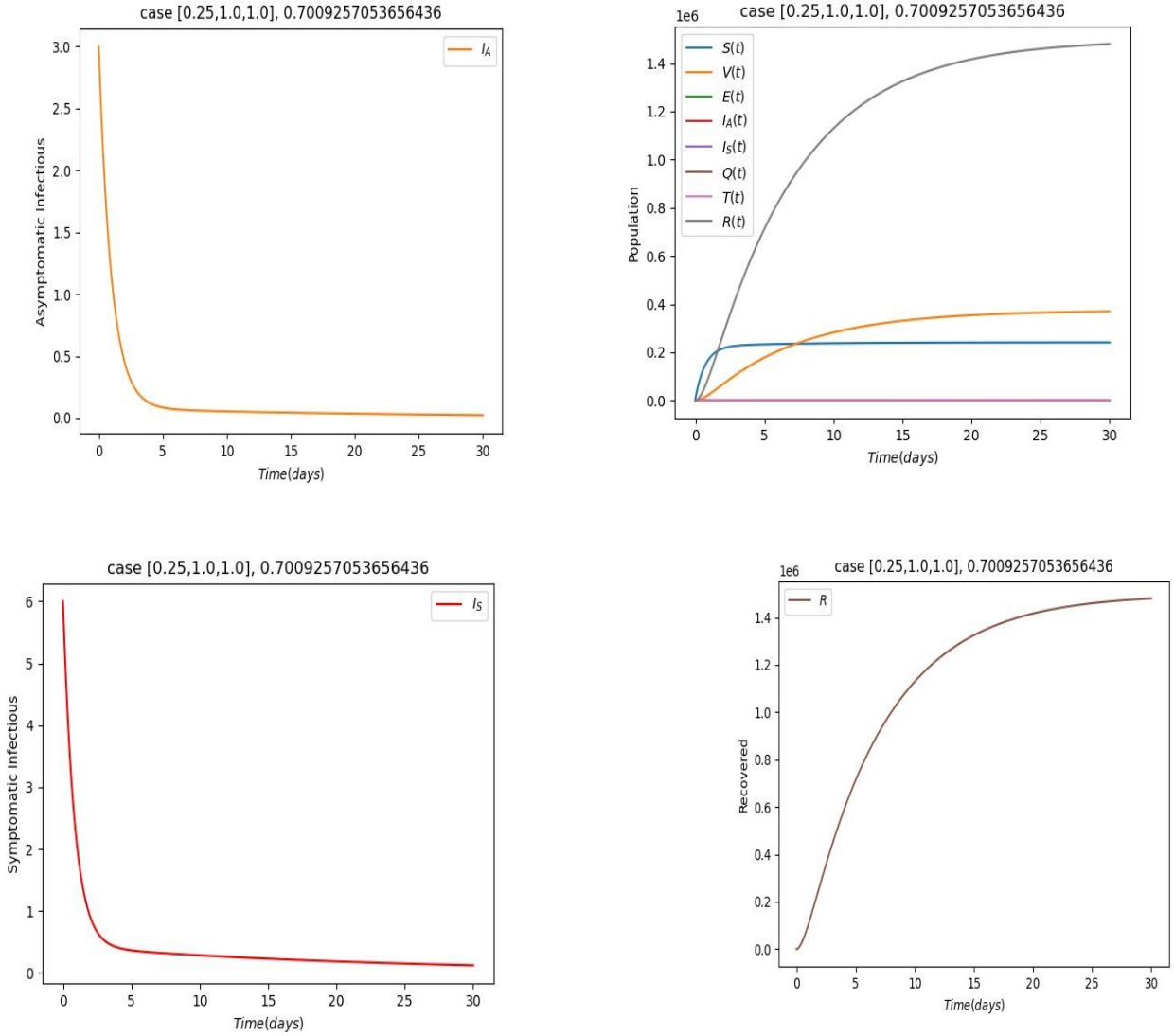


Fig. 12 The plot display the simulation with time using: $\kappa = 0.25$, $\theta = 1.0$ and $\tau = 1.0$ $R_0 = 0.7009257054 < 1$

Figure 12 shows that if at least 25% of the population is been vaccinated, 100% is having immunity gain and almost 100% of the population is taking precautionary measure against Covid infection, the graph shows that both Symptomatic and Asymptomatic infected will be very low to the point of vanishing in the population, and $R_0 < 1$.

Figure 13 shows that if the rate of vaccination is about 50% towards the entire population, then 100% is having immunity gain and almost 100% of the population is taking precautionary measure against Covid infection, the graph shows that both Symptomatic and Asymptomatic infected will more reduced in the population, and $R_0 < 1$

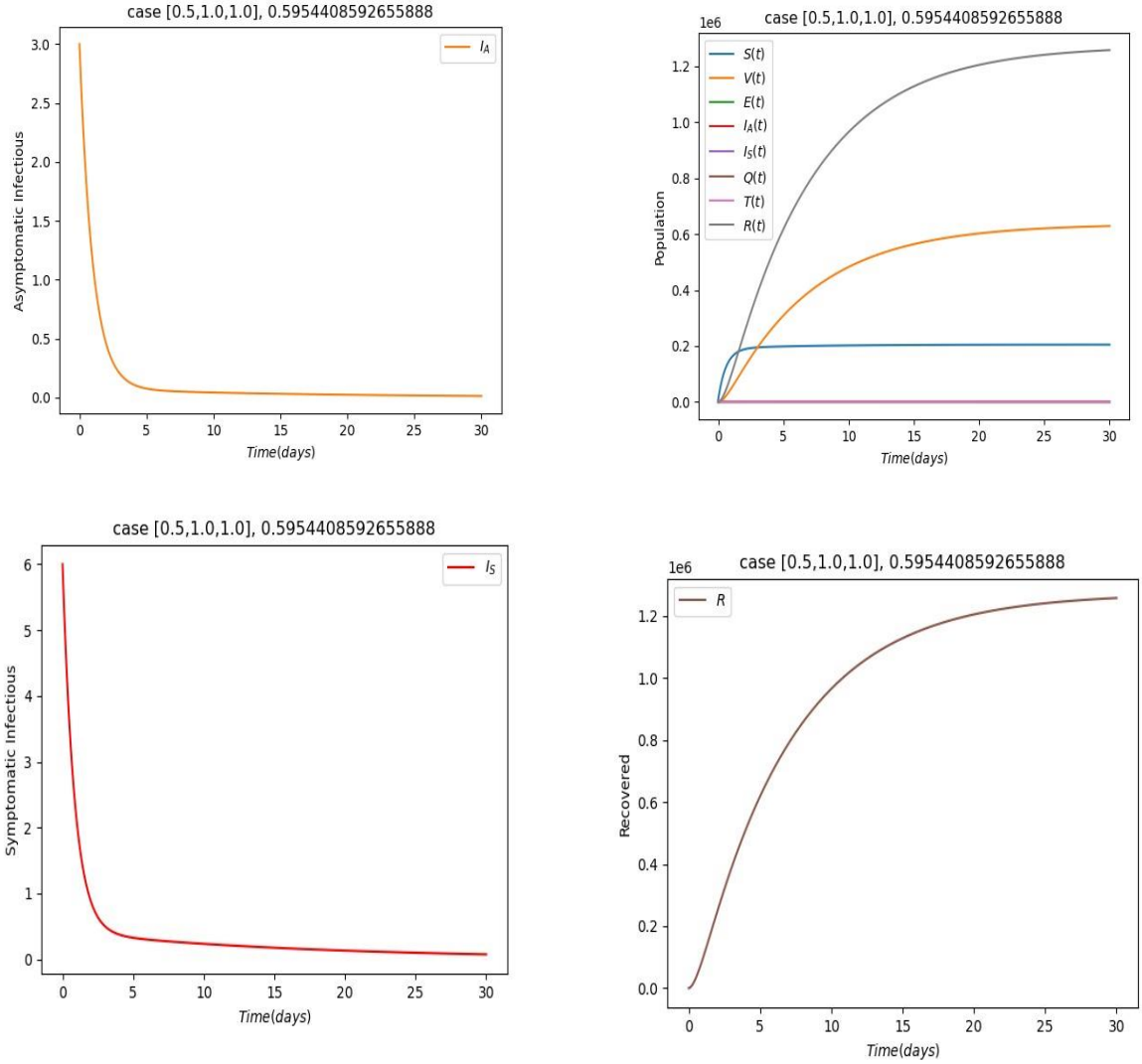
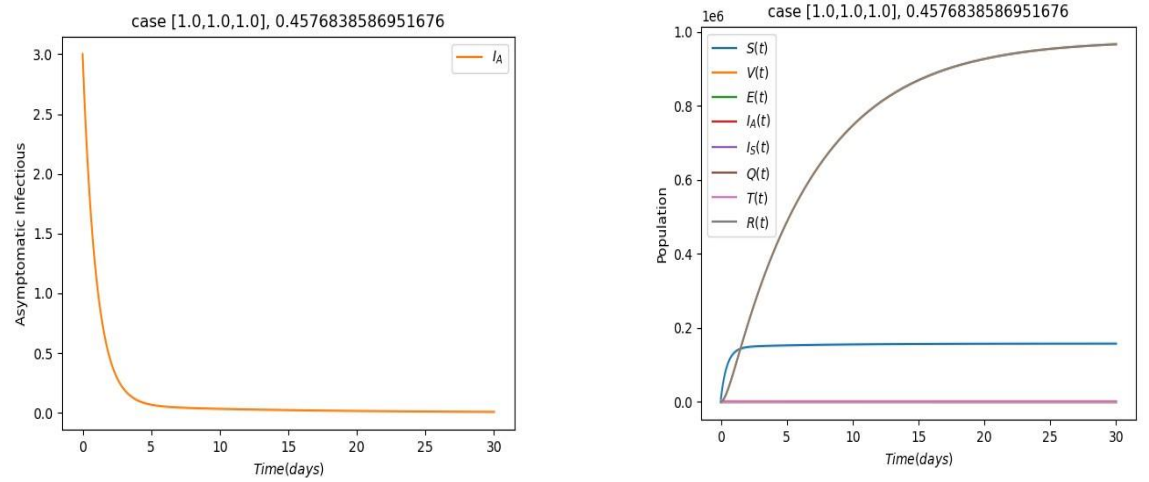


Fig. 13 The plot display the simulation with time using: κ = 0.5, θ = 1.0 and τ = 1.0 R_0 = 0.5954408593 < 1



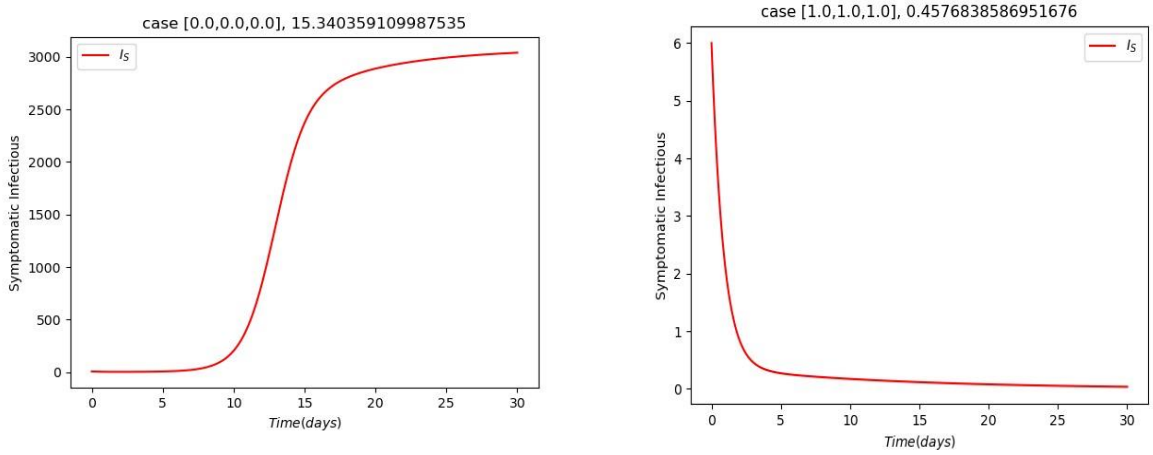


Fig. 14 The plot display the simulation with time using: $\kappa = 1.0$, $\theta = 1.0$ and $\tau = 1.0$ $R_0 = 0.4576838586 < 1$

Figure 14 shows that if possible for 100% of the population is been vaccinated, 100% is having immunity gain and also 100% of the population is taking precautionary measure against Covid infection, the graph shows Covid 19 disease will be totally eradicated from the entire population, both Symptomatic and Asymptomatic infected will reduce drastically, and $R_0 < 1$.

4. Summary and Conclusion

In this work, a deterministic model for novel Covid-19 in an open population to study the dynamics of the spread of the novel Covid-19 with the inclusion of vaccines as one of the precautionary measures in Nigeria was suggested. The work also considered the estimation of the Nigeria data towards Covid-19 as at March 5, 2021 along the cumulative data till September 30, 2022. The stabilities (both local and global) were achieved analytically. The model considered recruitment of new individuals into the S(t) class through birth or immigration, and those immigrant who were exposed to the disease were recruited into the exposed class. The prevention (vaccination, quarantine and treatment) measures and other intervention strategies were included in the model to assess the potential impact of these strategies on the transmission dynamics of the disease. The positivity solutions of the states of each of the variables in the model equations have been shown. The model equilibria were discussed along their stability both locally and globally using Jacobian matrix and Lyapunov approach. It was concluded that the model is stable asymptotically when the determinant of the Jacobian matrix is greater than or equal zero and its trace is less than zero. The basic reproductive ratio was obtained. The simulation of the model was conducted. It was finally concluded that the reproductive ratio will be lesser than one and infection will be reduced if at least the rate of vaccination not lesser than 50% with the entire population taking the precautionary measure seriously. The sensitivity index analysis of the model parameters was used to determine the parameters that were responsible for the eradication of the novel Covid-19 disease in the Nigeria case. The simulations for population classes along with their controls were discussed and shown in figure 2 to 14.

Owing to the severe nature of Covid-19 disease and its high rate of spread, we would recommend that the Nigeria government should make the vaccination exercise a compulsory one in other to achieve a total eradication of the spread of Covid-19 in the country.

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