International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373/doi:10.14445/22315373/IJMTT-V68I2P503
Volume 68 Issue 2, 12-18, February, 2022

πJ*-Closed Sets in Topological Spaces

Hamant Kumar

Department of Mathematics, Veerangana Avantibai Government Degree College, Atrauli, Aligarh, U. P. (India)

Abstract *- In this paper, a new class of sets called πJ*-closed sets is introduced and its properties are studied. The relationships among closed, r-closed, r*-closed, δ-closed, πJ*-closed and other generalized closed sets are investigated. Several examples are provided to illustrate the characterizations of these new classes of sets.*

Keywords - *η*-closed, r*-closed, J*-closed, J**-closed and πJ*-closed.*

2020 Subject Classification: *54A05, 54C08*

I. INTRODUCTION

In 1937, Stone [18] introduced the notion of regular open sets. In 1963, Levine [7] initiated s-open sets. In 1965, Njastad [13] introduced the concept of α-open sets. In 1968, the notion of π-open sets were introduced by Zaitsev [20] which are weaker form of regular open sets. Velicko [16] proposed δ-open sets which are stronger than open sets. In 1970, Levine [8] initiated the study of generalized closed (briefly g-closed) sets. In 1982, Mashour [9] introduced the concept of p-open sets. In 1983, Abd EI-Monsef et al [1] introduced the notion of β-open sets. In 1996, Dontchev and Ganster [4] introduced the notion of δg-closed sets. In 2000, Dontchev and Noiri [5] introduced the concept of πg-closed sets. In 2006, Park [14] introduced the notion of πgp-closed sets. Aslim et al. [3] introduced the concept of πgs-closed sets. In 2009, Arockiarani and Janaki [2] introduced the notion of πgα-closed sets. In 2010, Sarsak and Rajesh [17] introduced the concept of πgsp-closed sets. In 2012, Sudha and Sivakamasundari [19] introduced the notion of δg*-closed sets. In 2016, Pious and Annalakshmi [15] introduced the notion of regular*-open sets. In 2019, Meenakshi et. al [10, 11] introducedthe concepts of η*-open and J-closed sets. In 2020, Meenakshi et. al [12] introduced a new class of sets namely J*-closed and J**-closed sets. Recently, Kumar [6] introduced a new class of sets namely πgη-closed sets and obtained their properties.

II. PRELIMINARIES

Throughout this paper, spaces (X, ℑ), (Y, σ), and (Z, γ) (or simply X, Y and Z) always mean topological spaces. Let A be a subset of a space X. The closure of A and interior of A are denoted by cl(A) and int(A) respectively. A subset A is said to be regular open [18] (resp. regular closed [18]) if A = int(cl(A)) (resp. A = cl(int(A)). The finite union of regular open sets is said to be π-open [20]. The complement of a π-open set is said to be π-closed [20].

Definition 2.1. A subset A of a topological space (X, ℑ) is said to be
(i) pre-open [**9**] (briefly **p-open**) if A ⊂ cl(int(A)).
(ii) **semi-open** [**7**] (briefly **s-open**) if A ⊂ int(cl(A)).
(iii) **α-open** [**13**] if A ⊂ int(cl(int(A))).
(iv) **β-open** [**1**] if A ⊂ cl(int(cl(A))).

The complement of a pre-open (resp. semi-open α-open, β-open) set is called pre-closed (briefly p-closed) (resp. semi-closed (briefly s-closed), α-closed, β-closed). The intersection of all p-closed (resp. s-closed, α-closed, β-closed) sets containing A, is called p-closure (s-closure, α-closure, β-closure) of A, and is denoted by p-cl(A) (resp. s-cl(A), α-cl(A), β-cl(A)). A subset A of a topological space (X, ℑ) is said to be clopen if it is both open and closed in (X, ℑ).

Definition 2.2. A subset A of a topological space (X, ℑ) is said to be generalized closed (briefly g-closed) [8] if cl(A) ⊂ U whenever A ⊂ U and U ∈ ℑ. The complement of a g-closed set is called g-open.

The generalized closure of A is defined as the intersection of all g-closed sets in X containing A and is denoted by cl*(A). The generalized interior of A is defined as the union of all g-open sets in X contained in A and is denoted by int*(A).

Definition 2.3. The δ-interior of a subset A of X is the union of all regular open sets of X contained in A and is denoted by δ-int(A). The subset A is called δ-open [16] if δ-int(A) = A. i.e. a set is δ-open if it is the union of regular open sets, the complement of a δ-open set is called δ-closed. Alternatively, a set $A \subset X$ is δ-closed if A = δ-cl(A), where δ-cl(A) is the intersection of all regular closed sets of (X, ℑ) containing A.

Definition 2.4. Let (X, ℑ) be a topological space. A subset A of (X, ℑ) is called regular*-open [15] (or r*-open) if A = int(cl*(A)). The complement of a regular*-open set is called regular*-closed. The union of all regular*-open sets of X contained in A is called regular*-interior and is denoted by r*-int(A). The intersection of all regular*-closed sets of X containing A is called regular*-closure is denoted by r*-cl(A).

Definition 2.5. A subset A of a topological space (X, ℑ) is called η*-open [10, 11] set if it is a union of regular*-open sets (r*-open sets).The complement of a η*-open set is called η*-closed. A subset A of a topological space (X, ℑ) is called η*-interior of A is the union of all η*-open sets of X contained in A and is denoted by η*-int(A). The intersection of all η*-closed sets of X containing A is called as the η*-closure of A and is denoted by η*-cl(A).

Remark 2.6.
(i) regular open ⇒ π-open ⇒ δ-open ⇒ η*-open ⇒ open ⇒ α-open ⇒ s-open ⇒ β-open
(ii) regular closed ⇒ π-closed ⇒ δ-closed ⇒ η*-closed ⇒ closed ⇒ α-closed ⇒ s-closed ⇒ β-closed
(iii) regular open ⇒ π-open ⇒ δ-open ⇒ η*-open ⇒ open ⇒ α-open ⇒ p-open ⇒ β-open
(iv) regular closed ⇒ π-closed ⇒ δ-closed ⇒ η*-closed ⇒ closed ⇒ α-closed ⇒ p-closed ⇒ β-closed

Remark 2.7. For every subset U of X,
(i) β-cl(U) ⊂ s-cl(U) ⊂ α-cl(U) ⊂ cl(U) ⊂ η*-cl(U) ⊂ δ-cl(U) ⊂ π-cl(U) ⊂ r-cl(U).
(ii) g-cl(U) ⊂ cl(U)) ⊂ η*-cl(U) ⊂ δ-cl(U) ⊂ π-cl(U) ⊂ r-cl(U).
(iii) β-cl(U) ⊂ p-cl(U) ⊂ α-cl(U) ⊂ cl(U) ⊂ η*-cl(U) ⊂ δ-cl(U) ⊂ π-cl(U) ⊂ r-cl(U).
(iii) η-cl(U) ⊂ α-cl(U) ⊂ cl(U) ⊂ η*-cl(U) ⊂ δ-cl(U) ⊂ π-cl(U) ⊂ r-cl(U).

Definition 2.8. A subset A of a topological space (X, ℑ) is said to be
(1) **J-closed** [**10**, **11**] if cl(A) ⊂ U whenever A ⊂ U and U is η*-open in X.
(2) **J*-closed** [**12**] if η*-cl(A) ⊂ U whenever A ⊂ U and U ∈ ℑ.
(3) **δ-generalized closed** (briefly **δg-closed**) [**4**] if δ-cl(A) ⊂ U whenever A ⊂ U and U ∈ ℑ.
(4) **δg*-closed**) [**19**] if δ-cl(A) ⊂ U whenever A ⊂ U and U is g-open in X.
(5) **J**-closed** [**12**] if η*-cl(A) ⊂ U whenever A ⊂ U and U is η*-open in X.
(6) **πg-closed** [**5**] if cl(A) ⊂ U whenever A ⊂ U and U is π-open in X.
(7) **πgp-closed** [**14**] if p-cl(A) ⊂ U whenever A ⊂ U and U is π-open in X.
(8) **πgs-closed** [**3**] if s-cl(A) ⊂ U whenever A ⊂ U and U is π-open in X.
(9) **πgα-closed** [**2**] if α-cl(A) ⊂ U whenever A ⊂ U and U is π-open in X.
(10) **πgsp-closed** [**17**] if sp-cl(A) ⊂ U whenever A ⊂ U and U is π-open in X.
(11) **πgη-closed** [**6**] if η-cl(A) ⊂ U whenever A ⊂ U and U is π-open in X.
(12) **J-open** (resp**. J*-open, δg-open, δg*-open, J**-open, πg-open, πgp-open, πgs-open, πgα-open, πgsp-open, πgη-open**) set if the complement of A is J-closed (resp. J*-closed, δg-closed, δg*-closed, J**-closed, πg-closed, πgp-closed, πgs-closed, πgα-closed, πgsp-closed, πgη-closed).

III. πJ*-CLOSED SETS

In this section a new class of generalized closed sets, called πJ*-closed sets are introduced. The relations between πJ*-closed sets and various existing generalized closed sets are investigated.

3.1 Definition A subset U of a topological space (X, ℑ) is said to be **πJ*-closed** if η*-cl(U) ⊂ M whenever U ⊂ M and M is π-open in (X, ℑ).

3.2 Theorem. Every J*-closed set is πJ*-closed but not conversely.
Proof. Let U be a J*-closed set and M be any π-open set containing U in X. Since every π-open set is open, so M is open. Given that U is J*-closed, η*-cl(U) ⊂ M. Hence U is πJ*-closed.

3.3 Theorem. Every J**-closed set is πJ*-closed but not conversely.
Proof. Let U be a J**-closed set and M be any π-open set containing U in X. Since every π-open set is η*-open, so M is η*-open. Given that U is J**-closed, η*-cl(U) ⊂ M. Hence U is πJ*-closed.

3.4 Theorem. Every η*-closed set is πJ*-closed but not conversely.
Proof. Let U be a η*-closed set and M be any π-open set containing U in X. Given that U is η*-closed, η*-cl(U) = U ⊂ M. Hence U is πJ*-closed.

3.5 Corollary. Every r*-closed set is πJ*-closed but not conversely.
Proof. Since every r*-closed set is η*-closed and every η*-closed set is πJ*-closed.

3.6 Corollary. Every regular-closed set is πJ*-closed but not conversely.
Proof. Since every regular-closed set is η*-closed and every η*-closed set is πJ*-closed.

3.7 Corollary. Every π-closed set is πJ*-closed but not conversely.
Proof. Since every π-closed set is η*-closed and every η*-closed set is πJ*-closed.

3.8 Theorem. Every δ-closed set is πJ*-closed but not conversely.
Proof. Let U be a δ-closed set and M be any π-open set containing U in X. Given that U is δ-closed, δ-cl(U) = U ⊂ M. Therefore η*-cl(U) ⊂ δ-cl(U) ⊂ M. Hence U is πJ*-closed.

3.9 Theorem. Every δg*-closed set is πJ*-closed but not conversely.
Proof. Let U be a δg*-closed set and M be any π-open set containing U in X. Since every π-open set is g-open, so M is g-open. Given that U is δg*-closed, δ-cl(U) ⊂ M. Therefore η*-cl(U) ⊂ δ-cl(U) ⊂ M. Hence U is πJ*-closed.

3.10 Theorem. Every δg-closed set is πJ*-closed but not conversely.
Proof. Let U be a δg-closed set and M be any π-open set containing U in X. Since every π-open set is open, so M is open. Given that U is δg-closed, δ-cl(U) ⊂ M. Therefore η*-cl(U) ⊂ δ-cl(U) ⊂ M. Hence U is πJ*-closed.

3.11 Theorem. Every θ-closed set is πJ*-closed but not conversely.
Proof. Let U be a θ-closed set and M be any π-open set containing U in X. Given that U is θ-closed, θ-cl(U) = U ⊂ M. Therefore η*-cl(U) ⊂ θ-cl(U) ⊂ M. Hence U is πJ*-closed.

3.12 Theorem. Every πJ*-closed set is πg-closed but not conversely.
Proof. Let U be a πJ*-closed set and M be any π-open set containing U in X. Given that U is πJ*-closed, η*-cl(U) ⊂ M. Therefore cl(U) ⊂ η*-cl(U) ⊂ M. We get cl(U) ⊂ M. Hence U is πg-closed.

3.13 Theorem. Every πJ*-closed set is πgp-closed but not conversely.
Proof. Let U be a πJ*-closed set and M be any π-open set containing U in X. Given that U is πJ*-closed, η*-cl(U) ⊂ M. Therefore p-cl(U) ⊂ η*-cl(U) ⊂ M. We get p-cl(U) ⊂ M. Hence U is πgp-closed.

3.14 Theorem. Every πJ*-closed set is πgs-closed but not conversely.
Proof. Let U be a πJ*-closed set and M be any π-open set containing U in X. Given that U is πJ*-closed, η*-cl(U) ⊂ M. Therefore s-cl(U) ⊂ η*-cl(U) ⊂ M. We get s-cl(U) ⊂ M. Hence U is πgs-closed.
3.15 Theorem. Every πJ*-closed set is πgα-closed but not conversely.
Proof. Let U be a πJ*-closed set and M be any π-open set containing U in X. Given that U is πJ*-closed, η*-cl(U) ⊂ M. Therefore α-cl(U) ⊂ η*-cl(U) ⊂ M. We get α-cl(U) ⊂ M. Hence U is πgα-closed.

3.16 Theorem. Every πJ*-closed set is πgsp-closed but not conversely.
Proof. Let U be a πJ*-closed set and M be any π-open set containing U in X. Given that U is πJ*-closed, η*-cl(U) ⊂ M. Therefore sp-cl(U) ⊂ η*-cl(U) ⊂ M. We get sp-cl(U) ⊂ M. Hence U is πgsp-closed.

3.17 Theorem. Every πJ*-closed set is πgη-closed but not conversely.

Proof. Let U be a πJ*-closed set and M be any π-open set containing U in X. Given that U is πJ*-closed, η*-cl(U) ⊂ M. Therefore η-cl(U) ⊂ η*-cl(U) ⊂ M. We get η-cl(U) ⊂ M. Hence U is πgη-closed.

3.18 Example. Let X = {a, b, c} and ℑ = {ϕ, X, {a}}. Then the subset A = {b, c} is η*-closed as well as J*-closed but not δ-closed in (X, ℑ).

3.19 Example. Let X = {a, b, c} and ℑ = {ϕ, X, {a}, {a, b}}. Then the subset A = {b} is J**-closed as well as πJ*-closed but not η*-closed in (X, ℑ).

3.20 Example. Let X = {a, b, c} and ℑ = {ϕ, X, {a}, {a, b}, {a, c}}. Then the subset A = {b} is πJ*-closed but not δg*-closed in (X, ℑ).

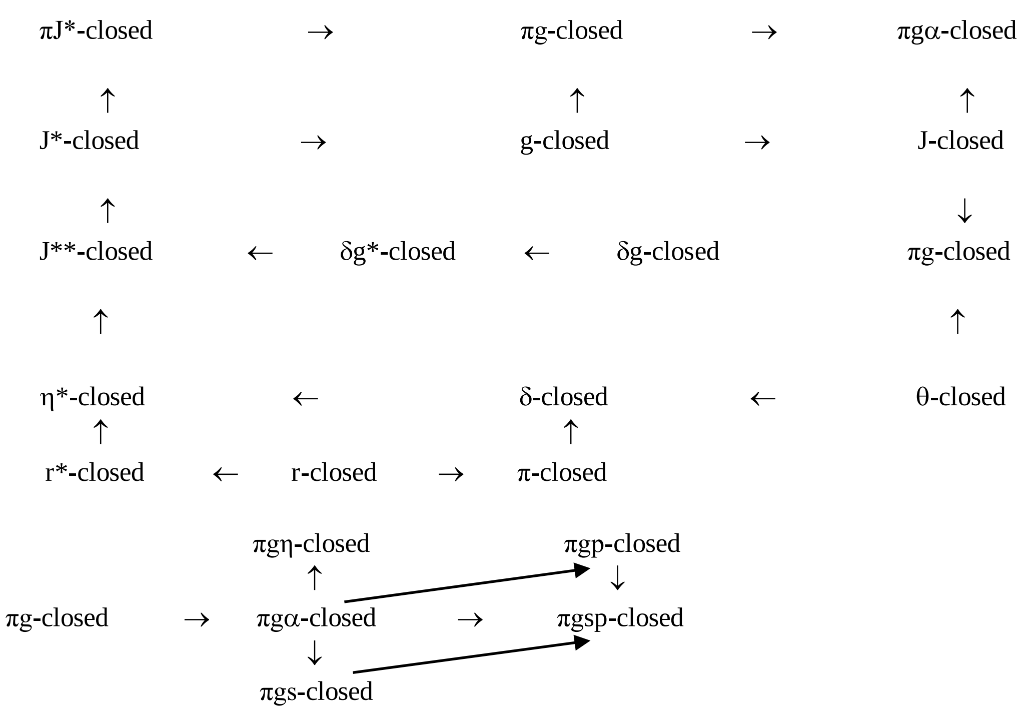
3.21 Example. Let X = {a, b, c, d} and ℑ = {ϕ, X, {a}, {c}, {a, c}, {a, b}, {a, b, c}, {a, c, d}}. Then the subset A = {c} is J**-closed as well πJ*-closed in (X, ℑ).

3.22 Example. Let X = {a, b, c, d} and ℑ = {ϕ, X, {c}, {a, b}, {a, b, c}}. Then the subset A = {a, b} is J*-closed as well as πJ*-closed but not J**-closed in (X, ℑ).

3.23 Example. Let X = {a, b, c} and ℑ = {ϕ, X, {a, b}}. Then the subset A = {a} is J**-closed as well as πJ*-closed but not closed in (X, ℑ).

3.24 Example. Let X = {a, b, c} and ℑ = {ϕ, X, {a}, {b}, {a, b}}. Then the subset A = {c} is η*-closed as well as πJ*-closed but not r*-closed in (X, ℑ).

3.25 Remark. From the above definitions, theorems and known results the relationship between πJ*-closed sets and some other existing generalized closed sets:



Where none of the implications is reversible as can be seen from the above examples:

IV. PROPERTIES Of πJ*-Closed

In this section, we obtained properties of πJ*-closed sets.

Theorem 4.1. Let X be a topological space. If A is π-open and πJ*-closed, then A is η*-closed.
Proof. Let A is π-open and πJ*-closed. Let A ⊂ A where A is π-open. Since A is πJ*-closed, η*-cl(A) ⊂ A. Then A = η*-cl(A). Hence A is η*-closed.

Theorem 4.2. Let X be a topological space. If A is regular-open and πJ*-closed, then A is η*-closed.
Proof. Let A is regular-open and πJ*-closed. Since every regular-open set is π-open and since A is πJ*-closed, η*-cl(A) ⊂ A. Then A = η*-cl(A). Hence A is η*-closed.

Theorem 4.3. Let A be a πJ*-closed set in X. Then η*-cl(A) – A does not contain any nonempty π-closed set.
Proof. Let F be a nonempty π-closed set such that F ⊂ η*-cl(A) – A. Then F ⊂ η*-cl(A) ∩ (X – A) ⊂ (X – A) implies A ⊂ X – F where X – F is π-open. Therefore η*-cl(A) ⊂ X – F implies F ⊂ $(\eta^*\text{-cl}(A)^c$. Now F ⊂ η*-cl(A) ∩ $(\eta^*\text{-cl}(A))^c$ implies F is empty.

Corollary 4.4. Let A be πJ*-closed. A is η*-closed iff η*-cl(A) – A is π-closed.

Proof. Let A be η*-closed set then A = η*-Cl(A) implies η*-cl(A) – A = ϕ which is π-closed.
Conversely, if η*-cl(A) – A is π-closed then A is η*-closed.

Theorem 4.5. If A is πJ*-closed and B is any set A ⊂ B ⊂ η*-cl(A) then B is πJ*-closed.
Proof: Since A is πJ*-closed, η*-cl(A) ⊂ U whenever A ⊂ U and U is π-open. Let B ⊂ U and U is π-open. Since B ⊂ η*-cl(A), η*-cl(B) ⊂ η*-cl(A) ⊂ U. Hence B is πJ*-closed.

Theorem 4.6. If A is π-open and πJ*-closed. Then A is η*-closed and hence clopen.
Proof. Let A be regular open. Since A is πJ*-closed, η*-cl(A) ⊂ A implies A is η*-closed. Hence A is closed. (Since every π-open η*-closed set is closed). Therefore A is clopen.

Remark 4.7. Let (X, ℑ) be a topological space. A finite intersection of πJ*-closed sets may fail to be a πJ*-closed set.

Example 4.8. Let X = {a, b, c, d} and let ℑ = {ϕ, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}, X}. Let A = {a, b, c} and B = {a, b, d} are πJ*-closed sets. But A ∩ B = {a, b} ⊂ {a, b} which is π-open. η*-Cl(A ∩ B) = X ⊄ {a, b}. Hence A ∩ B is not πJ*-closed.

Theorem 4.9. Let (X, ℑ) be a topological space. Every finite union of πJ*-closed sets is always a πJ*-closed set.

Proof. Let A and B be any two πJ*-closed sets. Therefore η*-cl(A) ⊂ U and η*-cl(B) ⊂ U whenever A ⊂ U, B ⊂ U and U is π-open. Let A ∪ B ⊂ U where U is π-open.
Since, η*-cl(A ∪ B) ⊂ η*-cl(A) ∪ η*-cl(B) ⊂ U, we have A ∪ B is πJ*-closed.

V. πJ*-OPEN SETS

In this section, we introduced πJ*-open sets and obtained properties of πJ*-open sets.

Definition 5.1. Let (X, ℑ) be a topological space. A subset A of X is called **πJ*-open** iff its complement is πgη-closed set. We denote the family of all πJ*-open (resp. πJ*-closed) sets of a topological space by **πJ*-O(X)** (resp. **πJ*-C(X))**.

Lemma 5.2. If A be a subset of X, then
(a) η*-cl(X – A) = X – η*-int(A).
(b) η*-int(X – A) = X – η*-cl(A).

Theorem 5.3. A subset A of a space X is πJ*-open iff F ⊂ η*-int(A) whenever F is π-closed and F ⊂ A.
Proof. Let F be π-closed set such that F ⊂ A. Since X – A is πJ*-closed and X – A ⊂ X – F where F ⊂ η*-int(A). Conversely.

Let $F \subset \eta^*$-int(A) where F is π-closed and $F \subset A$. Since $F \subset A$ and $X - F$ is π-open, η^*-cl(X – A) = X –η^*-int(A) $\subset X - F$. Therefore A is $\pi g\eta$-open.

Theorem 5.4. If η^*-int(A) $\subset B \subset A$ and A πJ^*-open then B is πJ^*-open.
Proof: Since η^*-int(A) $\subset B \subset A$, by **Theorem 4.5**, η^*-cl(X – A) $\supset$ (X – B) implies B is πJ^*-open.

Remark 5.5. For any $A \subset X$, η^*-int(η^*-cl(A) – A) = ϕ.

Theorem 5.6. If $A \subset X$ is πJ^*-closed then η^*-cl(A) – A is πJ^*-open.
Proof. Let A be πJ^*-closed and F be a π-closed set such that $F \subset \eta^*$-cl(A) – A. By **Theorem 4.3**, F = $\varnothing$ implies $F \subset \eta^*$-int(η^*-cl(A) – A)). By **Theorem 5.3**, η^*-cl(A) – A is πJ^*-open.

Definition 5.7. A topological space X is called a **πJ^*-$T_{1/2}$ space** if every πJ^*-closed set is η^*-closed.

Theorem 5.8. Let (X, $\Im$) be a topological space.
(a) η^*-O(X) $\subset \pi J^*$-O(X),
(b) A space X is $\pi g\eta$-$T_{1/2}$ iff η^*-O(X) = πJ^*-O(X).
Proof. (a) Let A be a η^*-open set, then X – A is η^*-closed so X – A is πJ^*-closed. Thus A is πJ^*-open. Hence η^*-O(X) $\subset \pi J^*$-O(X).

(b) Necessity: Let (X, $\Im$) be πJ^*-$T_{1/2}$ space. Let A be πJ^*-open. Then X – A is πJ^*-closed. By hypothesis, X – A is η^*-closed. Thus A is η^*-open. Therefore η^*-O(X) = πJ^*-O(X).

Sufficiency: Let η^*-O(X) = πJ^*-O(X). Let A be πJ^*-closed. Then X – A is πJ^*-open. X – A is η^*-open. Hence A is η^*-closed. This implies (X, $\Im$) is πJ^*-$T_{1/2}$ space.

Lemma 5.9. Let A be a subset of X and $x \in X$. Then $x \in \eta^*$-cl(A) iff $V \cap \{x\} \neq \phi$ for every η^*-open set V containing x.

Theorem 5.10. For a topological space X the following are equivalent:
(a) X is πJ^*-$T_{1/2}$ space.
(b) Every singleton set is either π-closed or η^*-open.
Proof. (a) $\Rightarrow$ (b): Let X be a πJ^*-$T_{1/2}$ space. Let $x \in X$ and assuming that {x} is not π-closed. Then clearly X – {x} is not π-open. Hence X – {x} is trivially a πJ^*-closed. Since X is πJ^*-$T_{1/2}$ space, X – {x} is η^*-closed. Therefore {x} is η^*-open.

(b) $\Rightarrow$ (a): Assume every singleton set of X is either π-closed or η^*-open. Let A be a πJ^*-closed set. Let $x \in \eta^*$-cl(A).

Case I: Let {x} be π-closed. Suppose x does not belong to A. Then $x \in \eta^*$-cl(A) – A. By **Theorem 4.3**, $x \in A$. Hence η^*-cl(A) $\subset$ A.

Case II: Let {x} be η^*-open. Since $x \in \eta^*$-cl(A), we have $A \cap \{x\} \neq \phi$ implies $x \in A$. Therefore η^*-cl(A) $\subset$ A. Therefore A is η^*-closed.

VI. CONCLUSION

The concept of new closed set namely πJ^*-closed set using η^*-open sets is introduced and studied. The relationship of πJ^*-closed sets using existing closed sets is established. Finally, some of their fundamental properties are studied. The πJ^*-closed set can be used to derive a new concepts such as πJ^*-closed map and πJ^*-open map, πJ^*-continuous, πJ^*-homeomorphism, πJ^*-closure and πJ^*-interior and πJ^*-separation axioms. This idea can be extended to bitopological and fuzzy topological spaces.

REFERENCES

[1] M. E. Abd EI-Monsef, S. N. EI-Deeb and R. A. Mahamoud, β-open and β-continuous mappings, Bull. Fac. Assut Univ. Sci. 12 (1983) 77.
[2] Arockiarani and C. Janaki, πgα-closed set and Quasi α-normal spaces, Acta Ciencia Indica. 33(2) (2007) 657-666.
[3] A. Aslim, A. Caksu Guler and T. Noiri, On πgs-closed sets in topological spaces, Acta Math. Hungar. 112 (2006) 275-283.
[4] J. Dontchev and M. Ganster., On δ- generalized closed sets and T3/4 spaces, Mem. Fac. Sci. Kochi. Univ.math. 17 (1996) 15-31.
[5] J. Dontchev and T. Noiri, Quasi-normal spaces and πg-closed sets, Acta Math. Hungar. 89(3) (2000) 211-219.

[6] H. Kumar, πgη-closed sets and some related topics, Jour. of Emerging Tech. and Innov. Res. 8(6) (2021) 367-374.
[7] N. Levine, Semi open sets and semi continuity in topological spaces, Amer. Math. Monthly. 70 (1963) 36-41.
[8] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo.19(2) (1970) 89-96.
[9] A. S. Mashour, On pre continuous and weak pre continuous functions, Pro.Math. Phys.Soc. Egypt, 53 (1982) 47-53.
[10] P. L. Meenakshi and K. Sivakamasundari, Unification of regular star open sets, International Journal of research and analytical reviews, Special Issue. 6 (2019) 20-23.
[11] P. L. Meenakshi and K. Sivakamasundari, J-closed sets in topological spaces, JETIR. 6 (2019) 193-201.
[12] P. L. Meenakshi and K. Sivakamasundari, J**-closed sets in topological spaces, Malaya Journal of Matematik, S(1) (2020) 206-213.
[13] O. Njastad, On some class of nearly open sets, Pacific. J. Math. , 15 (1965) 961-970.
[14] J. H. Park, On πgp-closed sets in topological spaces, Indian Journal of Pure and Appl. Math., 112 (2006) 257-283.
[15] S. Pious Missier and M. Annalakshmi, Between Regular Open Sets and Open Sets, IJMA.7(5) (2016) 128-133.
[16] N. V. Velicko, H-closed topological spaces, Amer. Math. Soc. Transl. 78 (1968) 103-118.
[17] M. S. Sarsak and N. Rajesh, π-generalized semi-preclosed sets, International Mathematical Forum. 5(12) (2010) 573-578.
[18] M. H. Stone, Application of the theory of boolean rings to general topology, Trans. Amer. Math. Soc. 41 (1937) 375–481.
[19] R. Sudha and K. Sivakamasundari, δg*-closed sets in topological spaces, International Journal of Mathematical Archive, 3(3) (2012) 1222-1230.
[20] V. Zaitsev, On certain classes of topological spaces and their bicompactifications, Dokl. Akad. Nauk SSSR. 178 (1968) 778-779.