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Original Article

Conformally Flat Spherically Symmetric Model in Time – Independent Gravitational Field

Prashant R. Dhongle

Assistant Professor, Department of Mathematics, Seth Kesarimal Porwal College, Kamptee, Nagpur, Maharashtra, India

Abstract — *The conformally flat spherically symmetric charged perfect fluid distribution model in time – independent gravitational field is obtained in this paper. The electromagnetic field is present in the model and it is an accelerating universe with negative pressure. The other physical properties of the model are also discussed.*

Keywords — *Charged Perfect Fluid, Conformally Flat, Electromagnetic Field, Spherically Symmetric, Stationary Space-time.*

I. INTRODUCTION

The time – independent gravitational fields are the one that does not change in time. In this field we can choose a system of reference in which all the components of the metric tensor are independent of the time coordinate. Time – independent gravitational fields are also known as stationary space-times. Stationary space-times play an important role in general relativity, since they represent time – independent gravitational fields which may arise as final states of multipole astrophysical processes (collapsing stars, . . .) [1] - [3]. The Schwarzschild metric of black hole and the Kerr metric of a rotating black hole are common examples of stationary space-times.

Many researchers are working on the geometrical and physical aspects of the models of the universe in stationary space-times. Patel et al. [4] have obtained a one parameter class of stationary rotating string cosmological models. Chakraborty et al. [5] have obtained a one parameter family of solutions for the string dust cosmological model in a stationary cylindrically symmetric space-time. Beig et al. [6] have discussed the far – field behavior of stationary space-times. Nayak [7] has investigated the relations between the inertial forces and the Einstein equations in axially symmetric stationary space-times. Borkar et al. [8] have deduced the Bianchi type I string dust model with magnetic field in stationary space-time. Dhongle [9] has obtained the de-Sitter model with dark energy in stationary space-time. Many other authors like Xanthopoulos [10], Ferrando et al. [11], Marklund et al. [12], Garcia et al. [13], Wu et al. [14] and Borkar et al. [15], [16] have discussed various models of universe in stationary space-time with different parameters and conditions.

Number of authors has shown interest in the study of physical properties of space-times which are conformal to certain well known gravitational fields. Many conformally flat physically significant space-times are known such as Schwarzschild internal solution and Lemaitre cosmological universe. Pandey et al. [17] have deduced the conformally flat spherically symmetric charged perfect fluid distribution in general relativity. Conformally flat spherically symmetric cosmological models representing a charged perfect fluid as well as a bulk viscous fluid distribution have been obtained by Pradhan et al. [18]. Moradpour et al. [19] have discussed the dynamical conformal spherically symmetric solutions in an accelerated background. Takisa et al. [20] have investigated the spherical conformal models for compact stars.

In this paper we have taken up the study of conformally flat spherically symmetric model having the charged perfect fluid distribution in stationary space-time. We have investigated the geometrical and physical properties of the model and explained the behavior of it. Our model is accelerating universe with negative pressure in which the negative pressure can prevent the collapse of mass distribution.

II. THE METRIC AND THE FIELD EQUATIONS

We consider the conformal metric in spherical polar coordinates

$$ds^2 = e^{\lambda}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2 - dt^2), \qquad (1)$$

where λ is a function of *r* and *t* alone. We number the coordinates as $x^1 = r$, $x^2 = \theta$, $x^3 = \phi$ and $x^4 = t$. Assume that the space-time is field with matter consisting of charged perfect fluid distribution with electromagnetic field. Einstein – Maxwell's equations for a distribution of charged perfect fluid are

$$R_{ij} - \frac{1}{2} g_{ij} R = 8\pi T_{ij} . \tag{2}$$

The energy momentum tensor T_{ij} is given by

$$T_{ij} = (\in + p) v_i v_j + p g_{ij} + E_{ij} , \tag{3}$$

in which

$$E_{ij} = \frac{1}{4\pi}\left[F_{ai} F_{bj} g^{ab} - \frac{1}{4} g_{ij} F_{ab} F^{ab} \right] \tag{4}$$

is the electric field, $\in$ is the density and p is the pressure of the fluid. The electromagnetic field tensor F_{ij} satisfies

$$F^{ij}{}_{|j} = 4\pi \rho v^i \tag{5}$$

and $$F_{[ij|k]} = 0 . \tag{6}$$

Here stroke '|' denotes covariant derivative and ρ is the current density. We are choosing the four velocity vector v^i as

$$g_{ij} v^i v^j = -1 . \tag{7}$$

Einstein field equations (2) in stationary space-times take the form

$$P_{\alpha\beta} + \frac{h}{2} f_{\alpha}{}^{\gamma} f_{\beta\gamma} - \frac{1}{2}\gamma_{\alpha\beta} P = 8\pi T_{\alpha\beta} , \tag{8}$$

$$\frac{3}{8} h f_{\alpha\beta} f^{\alpha\beta} + \frac{1}{2} P = \frac{8\pi}{h} T_{44} , \tag{9}$$

and $$\frac{\sqrt{h}}{2} f_{\alpha}{}^{\beta}{}_{|\beta} + \frac{3}{2} f_{\alpha\beta} (\sqrt{\mathrm{h}})^{|\beta} = \frac{8\pi}{\sqrt{h}} T_{4\alpha} , \tag{10}$$

in which

$$\gamma_{\alpha\beta} = - g_{\alpha\beta} + \frac{g_{4\alpha} g_{4\beta}}{h} , \quad \alpha, \beta = 1, 2, 3 \tag{11}$$

is the three – dimensional metric tensor determining the geometry of space, $f_{\alpha\beta}$ is the three – dimensional anti-symmetric tensor given by

$$f_{\alpha\beta} = g_{\beta|\alpha} - g_{\alpha|\beta} = \frac{\partial g_{\beta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha}}{\partial x^{\beta}} , \tag{12}$$

and $$h = g_{44} , \quad g_{\alpha} = \frac{-g_{4\alpha}}{h} , \quad \alpha = 1, 2, 3 . \tag{13}$$

P is the three - dimensional scalar curvature given by

$$P = \gamma^{\alpha\beta} P_{\alpha\beta} , \tag{14}$$

where $P_{\alpha\beta}$ is the three - dimensional Ricci tensor constructed from the three – dimensional metric tensor $\gamma_{\alpha\beta}$ in the same way as R_{ik} is constructed from the g_{ik} [21].

In stationary space-times, the gravitational potentials g_{ij} are independent of time *t*. Hence, for the metric (1) in stationary space-time, λ is a function of *r* only. Assume that F^{14} is the only non – vanishing component of electromagnetic field F^{ij} and the four velocity vector v^i is $v^i = (v^1, 0, 0, v^4)$, then in stationary space-time the Einstein field equations (8), (9) and (10) for the line - element (1) becomes

$$\frac{1}{r}\lambda_1 + \frac{1}{4}\lambda_1^2 = 8\pi\left\{(\in + p) v_1^2 + p e^{\lambda}\right\} - e^{-\lambda}(F_{14})^2 \tag{15}$$

$$\frac{1}{2}\lambda_{11} + \frac{1}{2r}\lambda_1 = 8\pi p e^{\lambda} + e^{-\lambda}(F_{14})^2 \tag{16}$$

$$-\lambda_{11} - \frac{2}{r}\lambda_1 - \frac{1}{4}\lambda_1^2 = 8\pi\left\{(\in + p)v_4^2 - pe^{\lambda}\right\} + e^{-\lambda}(F_{14})^2 \tag{17}$$

$$(\in + p)v_1 v_4 = 0 \tag{18}$$

III. SOLUTION OF THE FIELD EQUATIONS

Now, we obtain the different physical quantities in our model by solving the field equations. From equation (7), we write

$$v_4^{\,2} - v_1^{\,2} = e^{\lambda}. \tag{19}$$

In the equation (18) we take $(\in + p) = 0$, *i.e.*, $\in = -p > 0$. Thus, the field equations (15), (16) and (17) becomes

$$\frac{1}{r}\lambda_1 + \frac{1}{4}\lambda_1^2 = 8\pi p e^{\lambda} - e^{-\lambda}(F_{14})^2 , \tag{20}$$

$$\frac{1}{2}\lambda_{11} + \frac{1}{2r}\lambda_1 = 8\pi p e^{\lambda} + e^{-\lambda}(F_{14})^2 , \tag{21}$$

$$-\lambda_{11} - \frac{2}{r}\lambda_1 - \frac{1}{4}\lambda_1^2 = -8\pi p e^{\lambda} + e^{-\lambda}(F_{14})^2 . \tag{22}$$

From equations (20) and (22), we get

$$\lambda_{11} + \frac{1}{r}\lambda_1 = 0 , \tag{23}$$

which on integrating gives

$$\lambda = \log br^a \quad \text{or} \quad e^{\lambda} = br^a , \tag{24}$$

where a and b are constants of integration. Hence the metric (1) reduces to the form

$$ds^2 = br^a(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 - dt^2) . \tag{25}$$

This is the conformally flat spherically symmetric charged perfect fluid distribution model in stationary space-time with negative pressure. Adding equations (21) and (22), we get

$$-\frac{1}{2}\lambda_{11} - \frac{3}{2r}\lambda_1 - \frac{1}{4}\lambda_1^2 = 2e^{-\lambda}(F_{14})^2 , \tag{26}$$

which on solving gives

$$(F_{14})^2 = Ar^{a-2}, \tag{27}$$

where A is a constant. From equation (20), using equations (24) and (27), we obtain the pressure p and density $\in$ for the model (25) as

$$8\pi \in (= -8\pi p) = \frac{A}{b^2} r^{-(a+2)} . \tag{28}$$

Now after simplifying the equation (5), we have

$$4\pi\rho v^1 = 0 \quad \Rightarrow \quad v^1 = 0 \text{, since } \rho \neq 0 \tag{29}$$

and $$4\pi\rho v^4 = e^{-2\lambda}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_{14}) . \tag{30}$$

After solving equation (30) and using the value of $v^4 = e^{\lambda/2}$ from equation (19), the value of ρ is given by

$$4\pi\rho = -A^{\frac{1}{2}} b^{\frac{-3}{2}}\left(\frac{a+2}{2}\right) r^{-(a+2)} . \tag{31}$$

The non – vanishing component of the acceleration vector

$$\dot{v}_i = v_{i\,|\,j}\, v^j \tag{32}$$

is

$$\dot{v}_1 = \frac{a}{2r} . \tag{33}$$

The expressions for expansion Φ , rotation ω_{ij} and shear tensors σ_{ij} (Ellis [22]) are

$$\Phi = v^i{}_{|\,i} = 0 , \tag{34}$$

$$\omega_{ij} = -\frac{1}{2}(v_{i\,|\,j} - v_{j\,|\,i}) - \frac{1}{2}(\dot{v}_i v_j - \dot{v}_j v_i) = 0 , \tag{35}$$

$$\sigma_{ij} = \frac{1}{2}(v_{i\,|\,j} + v_{j\,|\,i}) + \frac{1}{2}(\dot{v}_i v_j + \dot{v}_j v_i) - \frac{\Phi}{3}(g_{ij} + v_i v_j) = 0 . \tag{36}$$

IV. CONCLUSION

We have obtained the conformally flat spherically symmetric charged perfect fluid distribution model with negative pressure in time – independent gravitational field. The model is accelerating universe containing electromagnetic field and the effect of negative pressure can prevent the collapse of a mass distribution. Acceleration is directed towards r – direction. There is no expansion with respect to time, as the model is stationary. There is no rotation and no shear in the model. If A = 0, then our model goes over to vacuum model and it is free from electromagnetic field.

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