International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373/doi:10.14445/22315373/IJMTT-V68I2P508

Volume 68 Issue 2, 47-51, February 2022

Original Article

Perfectly W - α - Irresolute Functions

P. Thamil Selvi[1], P. Kavitha[2], G. M. Kohila Gowri[3]

[1,2,3] *Assistant professor, Department of Mathematics, Parvathy's Arts and science College, Madurai kamaraj university, Dindigul, Tamil Nadu, India.*

Abstract **-** *Perfectly w-α-irresolute functions in weak structure are introduced, and their characterizations and properties are investigated.*

Keyword - *w-α-irresolute, Perfectly w-α-irresolute, Perfectly w-continuous.*

I. INTRODUCTION

Å,Csåsår [1] has intoduced a vew notion of structures called weak structure.In [1] Csåsår defined some strctures and opetators under more general conditions. In this paper, some structures, some new structures with respect to a weak structure on X are defined and their properties are discussed. In 1980 maheswari and Thakur [2] introduced and investigated the notion of w-α-irresoluteness of functions between topological spaces. After then some strong forms of this notions are introduced by Lo Faro [3], Navalagi [4] and recently Zorluntuna [5] as strongly w-α-irresoluteness and perfectly w-α-irresoluteness respectively. This devoted to the investigation of a class of function called perfectly w-α-irresolute functions.

II. PRELIMINARIES

Throughout the present paper, spaces always mean weak structure topological spaces on which no separation axiom is assumed (or simply f: X→Y) denote a function f from a weak structure topological spaces (x,w) into a weak structure topological spaces (Y,W_1). Let X be a nonempty set w ϵ ρ(x) where ρ(x) is the power set of X. Then w is called weak structure [1] (briefly WS) on X if φ ϵ w. A nonempty set X with a weak structure w, is denoted by the pair (X,W) and is called simply a space (X,W) and is called simply a space (X,W). The elements of w is called w-open sets[1] and the complements w-open sets are called w-closed sets.[1] for a weak structure w on X, the intersection of all w-closed sets containing a subset A of X is denoted by $c_w(A)$ and the union of all w-open sets contained in A is denoted by $i_w(A)$.A subset A is said to be w-regular open (resp.w-regular closed) if A=$i_w c_w(A)$ (resp.A=$c_w i_w(A)$. A subset A of space X is called w-α-open[6] (resp. w-pre-open[7]) if A ⊂ $i_w c_w i_w(A)$ (resp. A ⊂ $i_w c_w$(A)). The complement of an w-α-open is said to be w-α-open is said to be w-α-closed.The family of all w-α-open subset of (X,W) is denoted by T^α.It is known that is weak structure topology for X by Njastad [6].For a subset a of (X,W).The w-closure of A with respect to $T^\alpha$is denoted by T^α-$c_w(A)$.

III. FUNDAMENTAL PROPERTIES

Definition:3.1

A function f: X→Y is called perfectly w-continuous[8] (resp.contra w-α-continous[9]) of $f^{-1}(W)$ is w-clopen.(resp.w-α-open)on X for every w-open set W of Y.

Definition:3.2

A function f:X→Y is called w-α-irresolute [2] (resp . contra w-α-irresolute [11] , w-α-precontinous) if $f^{-1}(W)$ is w-α-open (resp.w-α-closed,w-pre-open) in X for every w-α-open set W of Y

Definition:3.3

A function f: X→Y is called slightly w-α-continous[12], $f^{-1}(W)$ is w-α-open in X for every w-clopen set W of Y.

Definition:3.4

A function f: X→Y is said to be perfectly w-α-irresolute if $f^{-1}(W)$ is w-α-clopen in X for every w-α-open set W of Y.

Definition:3.5

For a function f: (X,W)→(Y,W_1) the folowinng are equivalent

(i) f is w-α-irresolute.

(ii) For every w-α-closed subset W of closed subset W of , $f^{-1}(W)$ is w-clopen in X.

(iii) f: (X,W)→(Y,$\alpha(W_1)$) is perfectly w-continous, where W1α is the family of all w-α-open subset of (Y, W_1).

Proof:

The following implications are obvious.

(1) => (2) => (3) =>(4).

Definition:3.6

A space X is said to be w-locally indiscrete if every w-open subset of X is W – closed.

Definition:3.7

It is easily shown that every w-α-open set in a w-locally indiscrete space is w-clopen.

Theorem:3.8

A space X is w-locally indiscrete if and only if the identity map of X is perfectly
w-α-irresolute.

Proof:

Let f: X→X be a perfectly w-α-irresolute. Let w be a w-α-open set of X. Therefore $f^{-1}(W)$=w is w-clopen in X. By the remark :3.7, w is w-locally indiscrete space.

(=>) let X be a w- locally indiscrete space. let w be a w-α-open set in X. since f: X→X is a identity function. Therefore $f^{-1}(W)$ = W is a w-α-open set X. Therefore $f^{-1}(W)$=W is w-clopen in X. then f is be a perfectly w-α-irresolute.

Lemma:3.9

The following properties are equivalent for a subset A of a space X:

(i) A is w-clopen.

(ii) A is w-α-closed and w-α-open.

(iii) A is w-α-closed and W-pre-open.

Theorem:3.10

For a function f:x→y, the following conditions are equivalent.

(i) f is perfectly w-α-irresolute

(ii) f is contra w-α-irresolute and w-α-irresolute

(iii) f is contra w-α-irresolute and w-α-precontinuous

Proof:

The follows immediately from lemma 3.10

Definition:3.11

A space X is called strongly w-α-regular [13] if for any w-α-closed F ⊆ X and any point x ϵ X-F, there exist disjoint w-α-open U and V such that xϵU and F ⊆ V.

Theorem:3.12

A space(X,W) is strongly w-α-regular if and only if for every point x of X and every w-α-open V containing x, there exist an w-α-open set U such that

x ϵ U⊆ T α-c_w(U) ⊆ V

Theorem:3.13

Let (Y , W_1)be a strongly w-α-regular space for a WS function f: (X,W)→(Y,W_1),the following conditions are equivalent.

(i) F is perfectly w-α-irresolute.

(ii) For every w-α-open subset V of Y,$f^{-1}(V)$ is regular w-closed in X.

(iii) For every w-α-open subset V of Y,$f^{-1}(V)$ is w-closed in X.

(iv) F is contra w-α-irresolute.

Proof:

The following implications are obvious.

(1) => (2) => (3) =>(4). We show the implication (4)=>(1)

Definition:3.14

Let f:X→Y be a WS function from a topological space X to a topological space Y.then the WS function g:X→X xY defined by g(x_ϵ)= (x_ϵ,f(x_ϵ)) is called the w-graph function of f.

Theorem:3.15

A function f:X→Y is perfectly w-α-irresolute if the graph function g:X→X xY defined by g(x) =(x,f(x)), for each xϵX is perfectly w-α-irresolute.

Proof:

Let w be a w-α-open set of Y. then X xW is w-α-open set of XxY. Since g is perfectly w-α-irresolute. $f^{-1}(W)$= $g^{-1}(X\ x\ W)$ is w-clopen in X. thus f is perfectly w-α-irresolute.

Theorem:3.16

The following properties hold for WS function f:X→Y and g:Y→Z.

(i) If f:X→Y is perfectly w-α-irresolute and g:Y→Z is w-α-irresolute, then g o f:X→Z is perfectly w-α-irresolute.

(ii) If f:X→Y is perfectly w-α-irresolute and g:Y→Z is w-α-continuous, then g o f:X→Z is perfectly w-continuous.

(iii) If f:X→Y is slightly w-α-continuous and g:Y→Z is perfectly w-α-irresolute, Then g o f:X→Z is w-α-irresolute.

(iv) If f:X→Y is perfectly w-α-irresolute and g:Y→Z is contra w-α-irresolute, then g o f:X→Z is perfectly w-α-irresolute.

Proof:

The follow from definitions.

IV. FURTHER PROPERTIES

Definition:4.1

[2] A space (X,W) is said to be w-α-T_0if (X,W^α) is T_0.

Theorem:4.2

Let f:X→Y be w a perfectly w-α-irresolute function from a space X into an w-α-T_0 space Y. then f is a constant on each component of X.

Proof:

Let a and b be a two points of X that lies in the same component of X.Assume that f(a)≠f(b).

Since Y is α-T_0-space. There exists an w-α-open set W containing say f(a) but not f(b).by perfectly –α-irresoluteness of f, if $f^{-1}(U)$ and X-$f^{-1}(V)$ are disjoint w-clopen sets containing and respectively, which is a contraction in view of the fact that b belongs to the component of a.

Remark:4.3

A WS function f:X→Y to be perfectly contra w-α-irresolute$f^{-1}(W)$ is an w-α-open and w-α-closed set of X for each w-α-open set of Y and prove that a WS function f:X→Y is perfectly contra w-α-irresolute if and only if $f^{-1}(W)$ is w- clopen set of X for each w-α-open set of Y.Thus , f is perfectly w-α-irresoluteness is equivalent to perfectly contra w-α-irresoluteness.

Corollary:4.4

Let f:X→Y be w a perfectly w-α-irresolute function and Y be an w-α-T_0-space. If A is non-empty connected subset of X, then f(A) is single point.

Theorem:4.5

A space X is connected if and only if perfectly w-α-irresolute function from space X into any w-α-T_0-space Y is constant.

Proof:

We only prove the " if " part . Suppose that X is not connected .then there exists a proper nonempty w-clopen subset A of X. let Y={x,y} and σ be w-discrete topology on Y, Let f:X→Y be a WS function such that f(A)={x} and f(X-A)={y}.then f is non-constant, perfectly w-α-irresolute and Y is w-α-T_0,which is a contradiction to the theorem 4.3. Hence X must be connected.

Theorem:4.6

If f: (X,W)→(Y,W_1) is perfectly w-α-irresolute surjection and if (X,W) is a connected space. Then (Y, α (w_1)) is an w-indiscrete space.

Proof:

Suppose that (Y, α (w_1)) is not w- indiscrete. Let A be a proper nonempty w-α-open subset of Y. then $f^{-1}(A)$ is a proper nonempty w-clopen subset of X, which is contraction. Hence (X,W) is a connected.

Corollary:4.7

If f:X→Y is perfectly w-α-irresolute surjection and X is w-connected then Y is w-connected.

Remark:4.8

The topological space consisting of two points with the w-discrete topology is usually denoted by "2".

Theorem:4.9

The following are equivalent for a topological space X

(1) X is w-connected.
(2) Every perfectly w-α-irresolute function from X into an w-α-T_0-space Y is constant.
(3) Every perfectly w-α-irresolute function f:X→2 is constant
(4) There is a no perfectly w-α-irresolute function f:X→2 is w-surjection.

Proof: (1) => (2)

Let x is connected to prove that every perfectly w-α-irresolute function from X into an w-α-T_0-space Y is constant Y is constant by theorem 4.5.

(2)=>(3) and (3)=>(4) are obvious.(4)=>(1) suppose that X is not connected. Then there exists a non empty proper w-clopen open subset w of X. we define the function f:X→({ a,b},$T_{discrete}$) as f(x) =a for x϶W and f(x) = b for x϶X-W.the function f is perfectly w-α-irresolute and w-surjective , which is a contraction with hypothesis (4). Hence X is w-connected if there is no perfectly w-α-irresolute function f:X→2 is w-surjection

Definition:4.10

A space X is called w-α-regular[5] if for any w-closed set F ⊆ X and any point x ϵ X-F, there exist disjoint w-α-open sets U and V such that xϵU and F ⊆ V.

Definition:4.11

[9] A WS function f:X→Y is called
(1) w-α-closed if for each w-closed subset K of X, f (K) is w-closed in Y.
(2) w-α-open if for each w-α-open subset U of X, f (U) is w-α-open in Y.

Theorem:4.12

A WS function f:X→Y is w-α-closed if for each subset X of Y and for each w-open subset U of X with $f^{-1}(S)$ ⊆ U ,there exists an w-α-open set V of Y such that S ⊆ V and $f^{-1}(V)$ ⊆U.

Proof:(=>)

Suppose f is w-α-closed.let S ⊆ Y be any set and U be a w-α-open subset of X with $f^{-1}(S)$ ⊆ U . then Y-f(X-U) is an w-α-open set in Y. set V = Y-f(X-U).then S ⊆ V and $f^{-1}(V)$= $f^{-1}(Y - f(X - U))$=X-$f^{-1}((f(X - U))$ ⊆ U.

(<=) let k be any w-α-closed subset of X and S=Y-f(K).then $f^{-1}(S)$ ⊆ X-K by hypothesis, there exists an w-α-open set V in Y containing S such that $f^{-1}(V)$ ⊆ X-K. then we have K⊆ X-$f^{-1}(V)$ and Y-V=f(K). since Y-V is w-α-closed. f(K) is w-closed. thus f is w-closed map.

V. REFERENCES

[1] Å.Csåsår, Weak Structure,Acta Math.Hunger.,131(2011) 193-195.

[2] Maheswari, S.N and Thakur , S.S., On A-Irresolute Mappings, Tamkang J.Math., 11(1980) 209-204.

[3] Lo Faro, G., On Strongly A-Irresolute Mapping,India J.Pure.Appl.Math., 2(18) (1987) 146-151.

[4] Navalagi G.B, O Completely A-Irresolute Function , Topology Atlas,Preprint, (2001).

[5] Zorlutuna, I ., On Strong Forms Of Completely Irresolute Functions, Chaos Solitons Fractals, 38(2008) 970-979.

[6] Njastad.O., On Some Classes of Nearly Open Sets.,Pacific J.Math., 15(1995) 961-970

[7] Mashhour ., A.S., Abd EL-Monsef,M.E And I Deeb, S.N., On Pre Continuous and Weak Precontinuous Function,Proc., Math.Phys.Soc.Egypt., 53(1982) 47-53.

[8] Noiri, T., Super Continuity and Some Strong Forms of Continuity, Indian. J. Pure Appl. Math., 15(3) (1984) 241-250.

[9] Mashhour ., A.S., Hasanein, I.A., And El-Deeb,S.N., A-Continuous and A-Open Mapping, Acta Math.Hunger., 41(1983) 213-218.

[10] Caldas, M.., Jafari S., Noiri,T., And Saraf,R.K., Weak and Strong Form of A-Irresolute Maps Chaos Solitons Fractals,24 (2005) 223-228.

[11] Chae .G.I ., Noiri , T., And Kim ,J.S., On Slightly A-Continuous Functions, East Asian Math.J.,19 (2) (2003) 241-249.

[12] Maki , H., Devi, R.And Balachandran,K., Generalized A-Closed Sets in Topology ,Bull.Fukuoka Uni.Ed.Part.III, 42(1993) 13-21.

[13] Devi,R.,Balachandran,K., And Mahi, H., Generalized A-Closed Maps and Generalized A-Closed, Indian .J.Pure Appl .Math., 29 (1) (1998) 37-49.