International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373/ https://doi.org/10.14445/22315373/IJMTT-V68I2P514
Volume 68 Issue 2, 89-94, February 2022

Original Article

On Support Strongly Irregular Interval-valued Fuzzy Graphs

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Abstract — *In this paper, support strongly irregular interval-valued fuzzy graphs and support totally strongly irregular interval-valued fuzzy graphs are defined. A necessary and sufficient condition under which they are equivalent is provided.*

Keywords — *Support(2-degree) of a vertex in fuzzy graph, Average (pseudo degree of a vertex in fuzzy graph, Support strongly irregular fuzzy graph, Support strongly totally irregular fuzzy graph).*

I. INTRODUCTION

In this paper, we consider only finite, simple, connected graphs. We denote the vertex set and the edge set of a graph G by V (G) and E(G) respectively. The degree of a vertex v is the number of edges incident at v, and it is denoted by d(v). A graph G is regular if all its vertices have the same degree. The notion of fuzzy sets was introduced by Zadeh as a way of representing uncertainity and vagueness [26]. The first definition of fuzzy graph was introduced by Haufmann in 1973. In 1975, A. Rosenfeld introduced the concept of fuzzy graphs [8]. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas. Irregular fuzzy graphs play a central role in combinatorics and theoretical computer science.In 1975, Zadeh introduced the notion of interval-valued fuzzy sets as an extension of fuzzy set[27] in which the values of the membership degree are intervals of numbers instead of the numbers. In 2011, Akram and Dudek[1] defined interval-valued fuzzy graphs and give some operations on them.

II. PRELIMINARIES

Nagoorgani and Radha introduced the concept of degree, total degree, regular fuzzy graphs in 2008 [5]. Nagoorgani and Latha introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2012 [6]. N.R.Santhi Maheswari and C.Sekar introduced (2, k)-regular fuzzy graphs and totally (2, k)-regular fuzzy graphs, (r,2,k)-regular fuzzy graphs,(m, k)-regular fuzzy graphs and (r, m, k)-regular fuzzy graphs [9,13,14,15]. N.R.Santhi Maheswari and C. Sekar introduced 2-neighbourly irregular fuzzy graphs and m-neighbourly irregular fuzzy graphs [20,12]. N.R.Santhi Maheswari and C.Sekar introduced an edge irregular fuzzy graphs, neighbourly edge irregular fuzzy graphs and strongly edge irregular fuzzy graph [16,10,17]. D.S.Cao, introduced 2-degree of vertex v is the the sum of the degrees of the vertices adjacent to v and it is denoted by t(v)[3]. A.Yu, M.Lu and F.Tian, introduced pseudo degree (average degree) of a vertex v is (t (v)) /d(v), where d(v), is the number of edges incident at the vertex v [2]. N.R.Santhi Maheswari and C.Sekar introduced 2-degree of a vertex in fuzzy graphs , pseudo degree of a vertex in fuzzy graph and pseudo regular fuzzy graphs[11]. N.R Santhi Maheswari and M.Sutha introduced concept of pseudo irregular fuzzy graphs and highly pseudo irregular fuzzy graphs[18]. N.R.Santhi Maheswari and M.Rajeswari introduced the concept of strongly pseudo irregular fuzzy graphs [19]. N.R.Santhi Maheswari and V.Jeyapratha introduced the concept of neighbourly pseudo irregular fuzzy graphs[21]. N.R.Santhi Maheswari and K.Amutha introduced support neighbourly edge irregular graphs and 1-neighbourly edge irregular graphs, Pseudo Edge Regular and Pseudo Neighbourly edge irregular graphs [22,23,24]. J.Krishnaveni and N.R.Santhi Maheswari introduced support and total support of a vertex in fuzzy graphs, support neighbourly irregular fuzzy graphs and support neighbourly totally irregular fuzzy graphs[4]. K.Priyadharshini and N.R.Santhi Maheswari introduced support highly irregular graphs[7].N.R.Santhi Maheswari and K.Priyadharshini introduced support highly irregular fuzzy graphs[25]. These ideas *motivate us to introduce support highly irregular interval-valued fuzzy graphs and support totally highly irregular interval-valued fuzzy graphs and discussed some of its properties.*

III. PRELIMINARIES

We present some known definitions and results for ready reference to go through the work presented in this paper. By graph, we mean a pair G*=(V,E), where V is the set and E is a relation on V. The elements of V are vertices of G* and the elements of E are edges of G*.

Definition 3.1 2-degree (support) of v is defined as the sum of the degrees of the vertices adjacent to v and it is denoted by t(v)[3].

Definition 3.2 Average (pseudo) degree of v is defined as (t(v))/(d(v)), where t(v) is the 2-degree of v and d(v) is the degree of v and it is denoted by da(v)[2].

Definition 3.3 A graph is called pseudo-regular if every vertex of G has equal (pseudo) average-degree [2] .

Definition 3.4 A fuzzy graph $G:(\sigma,\mu)$ is a pair of functions (σ,μ), where $\sigma:V \to [0,1]$ is a fuzzy subset of a non-empty set V and $\mu: VXV \to [0,1]$ is a symmetric fuzzy relation on σ such that for all u,v in V, the relation $\sigma(uv) \leq \sigma(u)\Lambda\sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\sigma(uv) = \sigma(u)\Lambda\sigma(v)$ is satisfied [5].

Definition 3.5 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. The degree of a vertex u in G is denoted by d(u) and is defined as $d(u) = \sum \mu(uv)$, for all $uv \in E$[5].

Definition 3.6 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. The total degree of a vertex u in G is denoted by td(u) and is defined as $td(u) = d(u) + \sigma(u)$, for all $u \in V$[5].

Definition 3.7 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be an irregular fuzzy graph, if there is a vertex which is adjacent to the vertices with distinct degrees[6].

Definition 3.8 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be a totally irregular fuzzy graph if there is a vertex which is adjacent to the vertices with distinct total degrees[6].

Definition 3.9 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be a neighbourly irregular fuzzy graph if every two adjacent vertices of G have distinct degrees[6].

Definition 3.10 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be a neighbourly totally irregular fuzzy graph if every two adjacent vertices have distinct total degrees[6].

Definition 3.11 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be a highly irregular fuzzy graph if every vertex of G is adjacent to vertices with distinct degrees[6].

Definition 3.12 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be a highly totally irregular fuzzy graph if every vertex of G is adjacent to vertices with distinct total degrees[6].

Definition 3.13 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be a regular fuzzy graph if all the vertices of G have same degree[5].

Definition 3.14 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be a totally regular fuzzy graph if all the vertices of G have same total degree[5].

Definition 3.15 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The support (2-degree) of a vertex v in G is defined as the sum of degrees of the vertices adjacent to v and is denoted by s(v). That is, $s(v) = \sum dG(u)$, where $dG(u)$ is the degree of the vertex u which is adjacent with the vertex v[4].

Definition 3.16 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. The total support of a vertex v in G is denoted by $ts(v)$ and is defined as $ts(v)=s(v)+\sigma(v)$, for all $v\in V$[4].

Definition 3.17 A graph G is said to be a support neighbourly irregular fuzzy graph if every two adjacent vertices of G have distinct supports[4].

Definition 3.18 A graph G is said to be a support neighbourly totally irregular graph if every two adjacent vertices of G have distinct total supports[4].

Definition 3.19 A graph G is said to be a support highly irregular fuzzy graph if every vertex of G is adjacent to the vertices having distinct supports[4].

Definition 3.20 A graph G is said to be a support highly totally irregular graph if every vertex of G is adjacent to the vertices having distinct total supports[25].

Definition 3.21 An interval-valued fuzzy graph with an underlying set V is defined to be the pair (A,B), where $A=(\mu_A^-,\mu_A^+)$ is an interval-valued fuzzy set on V such that $\mu_A^-(x)\le\mu_A^+(x)$, for all $x\in V$ and $B=(\mu_B^-,\mu_B^+)$ is an interval-valued fuzzy set on E such that $\mu_B^-(x,y)\le min((\mu_A^-(x),\mu_A^-(y)))$ and $\mu_B^+(x,y)\le min((\mu_A^+(x),\mu_A^+(y)))$, for all edge $xy\in E$. Hence A is called an interval-valued fuzzy vertex set on V and B is called an interval-valued fuzzy edge set on E.

Definition 3.22 Let $G:(A,B)$ be an interval-valued fuzzy graph. The negative degree of a vertex $u\in G$ is defined as $d_{-G}(u)=\sum\mu_B^-(u,v)$, for $uv\in E$. The positive degree of a vertex $u\in G$ is defined as $d_G^+(u)=\sum\mu_B^+(u,v)$, for $uv\in E$ and $\mu_B^+(uv)=\mu_B^-(uv)=0$ if uv not in E. The degree of a vertex u is defined as $d_G(u)=(d_G^-(u),d_G^+(u))$.

Definition 3.23 Let $G:(A,B)$ be an interval-valued fuzzy graph on $G^*(V,E)$. The total degree of a vertex $u\in V$ is denoted by $td_G(u)$ and is defined as $td_G(u)=(td_G^-(u),td_G^+(u))$, where $td_G^-(u)=\sum\mu_B^-(u,v)+(\mu_A^-(u))$ and $td_G^+(u)=\sum\mu_B^+(u,v)+(\mu_A^+(u))$.

Definition 3.24 Let $G:(A,B)$ be an interval-valued fuzzy graph on $G^*(V,E)$, where $A=(\mu_A^-,\mu_A^+)$ and $B=(\mu_B^-,\mu_B^+)$ be two interval-valued fuzzy sets on a non-empty set V and $E\subseteq V\times V$ respectively. Then G is said to be regular interval-valued fuzzy graph if all the vertices of G has same degree (c_1,c_2).

Definition 3.25 Let $G:(A,B)$ be an interval-valued fuzzy graph on $G^*(V,E)$, then G is said to be totally regular interval-valued fuzzy graph if all the vertices of G has same total degree (c_1,c_2).

VI. SUPPORT STRONGLY IRREGULAR INTERVAL-VALUED FUZZY GRAPHS

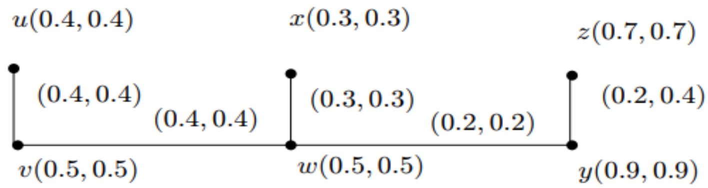
In this section, we define support strongly irregular interval-valued fuzzy graph and totally support strongly irregular interval-valued fuzzy graph and discussed about its properties.

Definition 4.1 Let $G:(A,B)$ be an interval-valued fuzzy graph on $G^*:(V,E)$. Then G is said to be support strongly irregular interval-valued fuzzy graph if every pair of vertices in G have distinct support.

Definition 4.2 Let $G:(A,B)$ be an interval-valued fuzzy graph on $G^*:(V,E)$. Then G is said to be totally support strongly irregular interval-valued fuzzy graph if every pair of vertices in G have distinct total support..

Remark 4.3 A support strongly irregular interval-valued fuzzy graph need not be support strongly totally irregular interval-valued fuzzy graph.

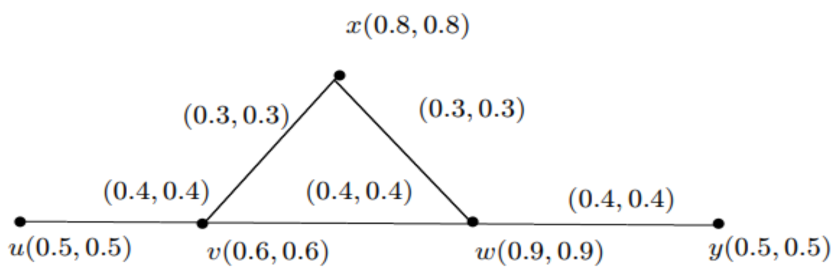
Example 4.4 Consider an interval-valued fuzzy graph $G:(A,B)$ on graph $G^*(V,E)$.



Here, $s_G(u) = (0.8,0.8), s_G(v) = (1.3,1.3), s_G(w) = (1.7,1.7), s_G(x) = (0.9,0.9), s_G(y) = (1.1,1.3), s(z) = (0.4,0.6)$. Also $ts_G(u) = (1.2,1.2), ts_G(v) = (1.8,1.8), ts_G(w) = (1.2,1.2), ts_G(x) = (1.2,1.2), ts_G(y) = (2,1.2), ts_G(z) = (1.1,1.3)$. Here, every pair of vertices of G have distinct support, but u and x have same total support. Therefore G is support strongly irregular interval-valued fuzzy graph but not totally support strongly irregular interval-valued fuzzy graph.

Remark 4.5 A totally support strongly irregular interval-valued fuzzy graph need not be support strongly irregular interval-valued fuzzy graph.

Example 4.6 Consider an interval-valued fuzzy graph $G:(A,B)$ on graph $G^*(V,E)$.



Here, $s_G(u) = (0.9,0.9), s_G(v) = (2.1,2.1), s_G(w) = (2.1,2.1), s_G(x) = (2.2,2.2), s_G(y) = (1.1,1.1)$. Also, $ts_G(u) = (1.6,1.6), ts_G(v) = (2.7,2.7), ts_G(w) = (2.9,2.9), ts_G(x) = (3.1,3.1), ts_G(y) = (1.6,1.6)$. Here, every pair of vertices have distinct total support but v and w have same support. Therefore G is totally support strongly irregular but not support strongly irregular interval-valued fuzzy graph.

Theorem 4.7 Let $G:(A,B)$ be an interval-valued fuzzy graph on $G^*(V,E)$. Then $A(u) = (\mu_A^-(u), \mu_A^+(u))$, for all $u \in V$ is a constant function then the following are equivalent.

- G is a support strongly irregular interval-valued fuzzy graph.
- G is a totally support strongly irregular interval-valued fuzzy graph.

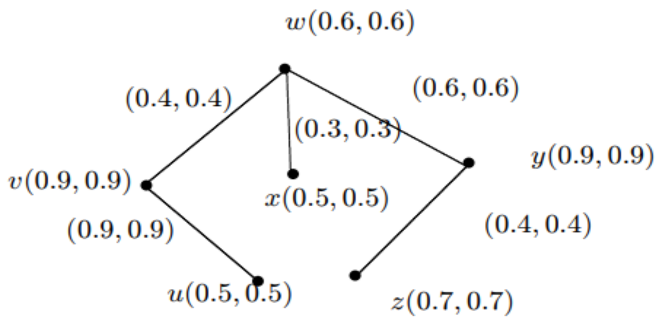
Proof. Assume that $A(u) = (\mu_A^-(u), \mu_A^+(u)) = (c_1, c_2)$, for all $u \in V$, where c_1 and c_2 are constant. Suppose G is a support strongly irregular interval-valued fuzzy graph. Then, every pair of vertices in G have distinct support. Let v_1 and v_2 be any pair of vertices with distinct supports (l_1, l_1) and (m_1, m_2) respectively. Then $(l_1, l_1) \neq (m_1, m_2)$. Suppose G is not a totally support strongly irregular interval-valued fuzzy graph. Then, at least one pair of vertices in G have distinct total support $\Rightarrow ts_G(v_1) = ts_G(v_2)$ $\Rightarrow d_G(v_1) + A(v_1) = d_G(v_2) + A(v_2)$ $\Rightarrow (l_1, l_2) + (c_1, c_2) = (m_1, m_2) + (c_1, c_2)$ $\Rightarrow (l_1, l_2) = (m_1, m_2)$, which is a contradiction to $(l_1, l_2) \neq (m_1, m_2)$. Hence G is totally support strongly irregular interval-valued fuzzy graph . Thus $(ii) \Rightarrow (i)$ is proved. Hence (i) and (ii) are equivalent.

Now, suppose G is a support strongly irregular interval-valued fuzzy graph. Then, every pair of vertices in G have distinct total support. Let u_1 and u_2 be any pair of vertices in G with distinct total support (g_1, g_2) and (h_1, h_2) respectively. Now, $(g_1, g_2) \neq (h_1, h_2) \Rightarrow t_G(u_1) \neq t_G(u_2) \Rightarrow d_G(u_1) + A(u_1) \neq d_G(u_2) + A(u_2)$

$\Rightarrow d_G(u_1) + (c_1, c_2) \neq d_G(u_2) + (c_1, c_2) \Rightarrow d_G(u_1) \neq d_G(u_2)$. Hence G is support strongly irregular interval-valued fuzzy graph. Thus $(ii) \Rightarrow (i)$ is proved. Hence (i) and (ii) are equivalent.

Remark 4.8 Converse of above theorem need not be true.

Example 4.9 Consider an interval-valued fuzzy graph $G: (A, B)$ on graph $G^*(V, E)$.



Here,
$s_G(u) = (0.9,0.9), s_G(v) = (2,2), s_G(w) = (2.2,2.2), s_G(x) = (1.1,1.1), s_G(y) = (1.5,1.5), s(z) = (0.6,0.6)$. Also $ts_G(u) = (1.8,1.8), ts_G(v) = (2.9,2.9), ts_G(w) = (2.8,2.8), ts_G(x) = (1.6,1.6), ts_G(y) = (2.4,2.4), ts_G(z) = (1.3,1.3)$

.Here, every pair of vertices have distinct support and total support. Therefore G is both support irregular and totally support irregular interval-valued fuzzy graph but A is not constant.

Theorem 4.10 Let $G: (A, B)$ be an interval-valued fuzzy graph on $G^*(V, E)$.If G is support strongly irregular interval-valued fuzzy graph, then G is both support neighbourly and support highly irregular interval-valued fuzzy graph.

Proof. Let G be a support strongly irregular interval-valued fuzzy graph. Then by definition, all the vertices of G have distinct support which means every adjacent vertices have distinct support and neighbours of every vertex have distinct support. Hence G is support neighbourly irregular interval-valued fuzzy graph and support highly irregular interval-valued fuzzy graph.

Remark 4.11 Converse of above theorem need not be true.

V. CONCLUSION

In this paper, support strongly irregular and totally support strongly irregular fuzzy graphs have been introduced and discussed some of its properties.

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