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On τ^*-Generalized γ Continuous Multifunctions in Topological Spaces

A. Ponsuryadevi[1], R. Selvi[2]

[1]*Research Scholar, Department of Mathematics, Sri Parasakthi College for Women, Courtallam, Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu, India.*
[2]*Assistant Professor, Department of Mathematics, Sri Parasakthi College for Women, Courtallam, Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu, India.*

Abstract *- In this paper, we introduce the concept of τ^*-generalized γ continuous multifunctions in topological spaces and study some of their properties where τ^* is defined by $\tau^* = \{G : cl^*(G^C) = G^C\}$.*

Keywords - *τ^*-gγ open set, τ^*-gγ closed set, τ^*-gγcontinuous, τ^*-gγ frontier.*

I. INTRODUCTION

In 1996, Andrijevic [4] has introduced a weak form of open sets called b-open sets. This notion was also called γ - open set in the sense of El-Atik [10] and called sp-open sets in the sense of Dontchev and przemski [8]. In 1996, El-Atik [10] introduced the concept of γ-continous functions as a generalization semi-continous functions due to Levine [14] and pre-continuous functions due to Mashhour *et al*. [15]. A weak form of continuous multifunctions called upper (lower) γ-continuous multifunctions was introduced by Abd-El-Monsef and Nasef [2]. Most of these weaker form of continuity, in ordinary topology such as α-continuity, pre-continuity, quasi-continuity and β-continuity have been extended to multifunctions [16,19-22].

Dunham [9] introduced the concept of the closure operator Cl^* and a new topology τ^* where τ^*={G:$Cl^*(G^c)$= G^c} and studied some of their properties. Pushpalatha *et al*., [23] introduced and studied. τ^*-generalized closed sets, Eswaran and Pushpalatha [12] introduced and studied. τ^*-generalized continuous functions. Several authors have introduced and studied various function in topological spaces.

For a multifunction F: X → Y, we shall denote the upper and lower inverse of a set G of Y by F^+(G) and F^-(G), respectively, that is F^+(G)={$x\in$ X:F(x)⊂G} and F^-(G)={$x \in$ X:F(x)∩ G≠ ∅}. For a multifunction $F: X \to Y$, the graph multifunction $G_F : X \to X \times Y$ is defined as follows: $G_F(x) = \{x\} \times F(x)$ for every $x \in X$.

II. PREMILINARIES

Definition: 2.1

Let X be a topological space and A be a subset of X. Then A is called α–open if A⊂ cl(int(cl (A))).[17]

Definition: 2.2

Let X be a topological space and A be a subset of X. Then A is called Semi open if A⊂cl(int(A)).[18]

Definition: 2.3

Let X be a topological space and A be a subset of X. Then A is called Pre-open if A⊂ int (cl (A)).[15]

Definition: 2.4

Let X be a topological space and A be a subset of X. Then A is called β-open [1] or semi pre open if A⊂ cl (int (cl(A)).[3]

Definition: 2.5

Let X be a topological space and A be a subset of X. Then A is called b-open [4] or γ open if A⊂ cl (int (A)∪ int (cl (A))).[10]

Definition: 2.6

For the subset A of a topological space X, the generalized closure operator Cl^* is defined by the intersection of all g-closed sets containing A. [9]

Definition: 2.7

For the subset A of a topological space X, the topology τ^* is defined by τ^*={G:$Cl^*(G^c)$= G^c}. [9]

Definition: 2.8

A subsets A of a topological space (X,τ^*) is called τ^*-generalized γ closed set(briefly τ^*-g γ closed) if γ Cl^*(A)⊆U whenever A⊆U and U is τ^* open in X. The complement of τ^*-generalized γ closed set is called the τ^*-generalized γ open set.

Lemma: 2.9

For a multifunction F : X→Y , the following hold:

(i) $G^+(A \times B) = A \cap F^+(B)$ and

(ii) $G^-(A \times B) = A \cap F^-(B)$ for any subsets $A \subset X$ and $B \subset Y$ [19].

Lemma 2.10:

Let A and X_0 be subsets of a space (X, τ). If A$\epsilon\gamma$O(X) and $X_0 \in \tau^\alpha$, then A⋂$X_0 \in \gamma$O(X_0).[4]

Lemma 2.11:

Let A⊂ $X_0 \subset X$, $X_0\epsilon\gamma$O(X) and $A\epsilon\gamma$O(X_0), then A$\epsilon\gamma$O(X).[10]

Definition: 2.12

A function f : X→Y from a topological space X into a topological space Y is called b- continuous if the inverse image of an open set in Y is b-open in X.

Definition: 2.11

A function f : X → Y from a topological space X into a topological space Y is called g-continuous if the inverse image of a closed set in Y is g-closed in X.[5]

Definition: 2.12

A function f : X → Y from a topological space X into a topological space Y is called generalized γ-continuous (briefly gγ -continuous) if the inverse image of a closed set in Y is gγ -closed in X.

Definition: 2.13

A function f : X → Y from a topological space X into a topological space Y is called τ^*-g continuous if the inverse image of a g-closed set in Y is τ^*-gclosed in X. [12]

Definition: 2.14

A function f : X → Y from a topological space X into a topological space Y is called τ^*-generalized γ continuous function (briefly τ^*-gγ continuous) if the inverse image of every gγ open set in Y is τ^*-g open in X.

Definition: 2.15

A of a topological space X is said to be,

(i) α-paracompact if every cover of A by open sets of X is refined bya cover of A which consists of open sets of X and is locally finite in X; [24]

(ii) α-regular if for each $a \in A$ and each open set U *of* X containing a there exists an open set G *of* x such that $a \in G \subset Cl$ $(G) \subset U$. [13]

Definition: 2.16

A topological space X is said to be γ- compact if every γ-open cover of X has a finity subcover [10].

Note: 2.17

(D,>) is a directed set, (F_λ) is a net of multifunction F_λ:X→Y. $\lambda \in D$ and F is multifunction on X into Y.

Definition: 2.18

Let $(F_\lambda)_{\lambda \in D}$ be a net of multifunctions on X into Y. A multifunction $F^*: X \to Y$ is defined as follows: for each $x\epsilon X$, $F^*(x) = \{y \in Y$: for each open neighbourhood of y and each $\eta \in D$, there exists $\gamma \in D$ such that $\gamma > \eta$ and V⋂$F_\lambda(x) \neq \emptyset\}$ is called the upper topological limit of the net F_λ. [6]

Definition: 2.19

A net $(F_{\lambda})_{\lambda\in D}$ is said to be equally upper γ- continuous at $x_0 \in X$ if for every open set V_{λ} containing $F_{\lambda}(x_0)$ there exists a γ-open set U containing x_0 such that $F_{\lambda}(U) \subset V_{\lambda}$ for all $\lambda \in D$.[2]

III. ON τ^*-GENERALIZED γ CONTINUOUS MULTIFUNCTIONS

Definition 3.1

A multifunction $F : (X,\tau^*) \to (Y,\sigma)$ is said to be

(i) Upper τ^* -generalized γ continuous at a point $x \in X$ if for each open set V of Y such that $F(x) \subset V$, there exists $\cup \in \tau^*$-g $\gamma(X,x)$ such that $F(U) \subset V$;

(ii) Lower τ^* -generalized γ continuous at a point $x \subset$X if for each open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists $\cup \in \tau^*$g $\gamma(X,x)$ such that $F(u) \cap V \neq \emptyset$ for every $u \in U$

(iii) Upper (lower) τ^* -generalized γ continuous if F has this property at each point of X.

Theorem 3.2

The following are equivalent for a multifunction $F : (X,\tau^*) \to (Y,\sigma)$;

(i) F is upper τ^*-g γ continuous;

(ii) F^+(V) $\in \tau^*$-g γO(X) for any open set V of Y;

(iii) $F^-(V)$is τ^*-g γ closed in X for any closed set V of Y;

(iv) τ^*-g $\gamma cl(F^-(B)) \subset F^-(cl\,(B))$ for any B$\subset Y$;

(v) For each point x of X and each neighbourhood V of F(x), $F^+(V)$ is τ^*-g γ neighbourhood U of x such that $F(U) \subset$V;

(vi) For each point x of X and each neighbourhood V of $F(x)$, there exists an τ^*-g γ neighbourhood U of x such that $F(U) \subset$V;

(vii) $Cl(int\,(B)) \subset int\,(F^-(B)))$ for every subset B of Y;

(viii) $F^+(int\,(B)) \subset int(cl(F^-(B)))\cup cl(int(F^+(B)))$ for every subset B of Y

Proof :

(i)⇒(ii) Let V be any open set of Y and $x \in F^+(V)$. There exists U$\in \tau^*$-g γ (X,x) such that F(U) $\subset$V. Then, we obtain $x \in U \subset cl(int\,(U)) \cup int(cl(U) \subset cl(int(F^+(V))) \cup int\,(cl(F^+(V)))$. We have $F^+(V) \subset cl(intF^+(V))) \cup int\,(clF^+(V)))$ and hence $F^+(V) \in \tau^*$-g γ O(X).

(ii)⇔(iii) In fact that $F^+(Y-B) = X - F^-(B)$ for every subset B of Y;

(iii) ⇒(iv) For any subset B of Y, $cl(B)$ is closed in Y and $F^-(cl(B))$ is τ^*-g γ closed in X.
Hence τ^*-g γcl$F^-(B) \subset F^-(cl(B)))$.

(iv)) ⇒(iii) Let V be any closed set of Y. Then τ^*g $\gamma cl(F^-(V)) \subset F^-(cl(V)) = F^-(V)$. Hence $F^-(V)$ is τ^*-g γ closed in Y.

(ii) ⇒(v) Let $x \in X$ and V be a neighbourhood of F(x). Then, there exists an open set G of Y such that F(x) $\subset$G$\subset$V. Since $F^+(G) \in \tau^*$-g $\gamma O(X), F^+(V)$ is τ^*-g γ neighbourhood of x.

(v) ⇒(vi) Let $x \in X$ and V be a neighbourhood of F(x) put U=$F^+(V)$, then U is an τ^*-g γ neighbourhood U of x such that F(U) $\subset$V.

(vi) ⇒(i) Let $x \in X$ and V be any open set of Y such that F(x) $\subset$V. Then exists an τ^*g γ neighbourhood U of x such that F(U) $\subset$V. Then $A \in \tau^*$-g γ O(X) such that $x \in A \subset \cup$ hence F(A) $\subset$V.

(iii) ⇒(vii) For any subset B of Y, $cl(B)$ is closed in Y by (iii), $F^-(cl(B))$ is τ^*-g γ closed in X. Then $F^-(cl(B)) \supset int(cl(F^-(cl(B))) \cap cl(int(F^-(B)))$.

(vii) ⇒(viii) By replacing Y-B instead of B in (vii), we have $cl(intF^+(Y-B))) \cap int\,(cl\,F^+(Y-B))) \subset F^+(cl(Y-B))$ and $F^+(int(B)) \subset int(cl(F^+(B)))\cup cl(int(F^+(B)))$.

(viii) ⇒(ii) Let V be any open set of Y. Then by using (viii) we have $F^+(V)\tau^*$-g γ O(X);

Theorem. 3.3

The following are equivalent for a multifunction $F : (X,\tau^*) \to (Y,\sigma)$:

(i) F is lower τ^*-g γ continuous;

(ii) $F^-(V) \in \tau^*$-g γ O(X) for any open set V of Y;

(iii) $F^+(V)$ is τ^*-g γ closed in X for any closed set V of Y.

(iv) τ^*-g $\gamma cl(F^+(cl(B))$ for any B$\subset$ Y;

(v) F(τ^*-g $\gamma cl(A)) \subset cl(F(A))$ for any A$\subset$ Y;

(vi) $cl(intF^+(cl(B))) \cap int\,(cl\,F^+(V)))$ for every subset B of Y;

(vii) $F^-(int\,(B)) \subset int(cl(F^-(B)))\cup cl(intF^-(B)))$ for every subset B of Y.

Proof:

The proof is similar to the theorem 3.2

Theorem. 3.4

Let $F : (X,\tau^*) \to (Y,\sigma)$ be a multifunction such that F(x) is compact for each $x \in$X. Then F is upper τ^*-gγ continuous if and only if G_F: X→X × Y is upper τ^*-gγ continuous.

Proof:

Suppose that $F : (X,\tau^*) \to (Y,\sigma)$ is upper τ^*-gγ continuous. Let $x \in$X and W be any open set of X × Y containing $G_F(x)$. For each y∈F(x), there exists open sets U(y) ⊂X and V(y) ⊂Y such that $(x$,y) ∈ U(y) × V(y) ⊂W. The family of { V(y): y∈F(x)} is an open cover of F(x) and F(x) is compact. Then, there exists a finite number of points, says,y_1,y_2,…..y_n in F(x) such that F(x) ⊂∪{V(y_i):1≤i≤n}. Set U=∩{U(y_i): 1≤i≤n} and V=∪{V(y_i):1≤i≤n}. Then U and V are open in X and Y,respectively, and $\{x\}$× F(x) ⊂U×V⊂W. Since F is upper τ^*-gγ continuous, there exists $U_0 \in$ τ^*-gγ(X,x) such that F(U_0) ⊂V. By lemma 2.9, we have U∩ U_0 ⊂U∩ $F^+(V)$=G_F^+(U×V) ⊂ G_F^+(W). Here, we obtain U∩ $U_0 \in \tau^*$-gγ(X,x) and G_F(U∩ U_0) ⊂W. So that G_F *is* upper τ^*-gγ continuous.

Assume that G_F: X→X × Y is upper τ^*-gγ continuous. Let $x \in$X and V be any open set of Y containing F(x). Since X × V and G_F ⊂ X × V, there exists U∈ τ^*-gγ(X,x)such that G_F(u) ⊂ X × V. By lemma 2.9, we have U⊂ G_F^+(X×V)= $F^+(V)$ and F(U) ⊂V. Hence F is upper τ^*-gγ continuous.

Theorem. 3.5

Let $F : (X,\tau^*) \to (Y,\sigma)$ be a multifunction such that F(x) is compact for each $x \in$X. Then F is lower τ^*-gγ continuous if and only if G_F: X→X × Y is lower τ^*-gγ continuous.

Proof:

Suppose that F is lower τ^*-gγ continuous. Let $x \in$X and W be any open set of X × Y such that $x \in G_F^-$ (W). Since W∩ ($\{x\}$× F(x))≠ ∅,there exists open sets y∈F(x) such that $(x$,y) ∈ Wand hence$(x$,y) ∈ U×V⊂W for some open sets U ⊂X and V ⊂Y. Since F(x) ∩ V≠ ∅ , there exists G∈ τ^*-gγ(X,x)such thatG⊂ $F^-(V)$. By lemma 2.9, we have ∪ ∩ G⊂∪∩ $F^-(V)$= G_F^- (U×V) ⊂ G_F^- (W). Then$x \in$ ∪∩ G ∈ τ^*-gγO(X) and hence G_F *is low*er τ^*-gγ continuous.

Assume that G_F is lower τ^*-gγ continuous. Let $x \in$X and V be any open set of Y such that $x \in F^-(V)$. Then X × V is open in X × Y and $G_F(x)$ ∩ (X × V)= ($\{x\}$× F(x)) ∩ (X × V)=$\{x\}$× F(x) ∩ V) ≠ ∅ . Since G_F is lower τ^*-gγ continuous, there exists U∈ τ^*-gγ(X,x) such that U⊂ G_F^- (U×V). By lemma 2.9, U⊂ $F^-(V)$. So that F is lower τ^*-gγ continuous.

Lemma: 3.6

If A is an α-regular α-paracompact set of a topological space X and U is an open neighborhood of A, then there exists an open set G of X such that $A \subset G \subset cl(G) \subset U$. [13]

For a multifunction $F : X \to Y$, by *cl*F : $X \to Y$ we denote a multifunction defined as follows: (*cl* F)(x) = *cl*($F(x)$) for each $x \in X$. Similarly, we can define γclF : $X\to Y$, βclF : $X\to Y$, *scl*F : $X \to Y$, *pcl*F : $X \to Y$ or $\alpha\, clF$: $X \to Y$. [7]

Lemma: 3.7

If F: (X,τ^*)→(Y,σ) is a multifunction such that F (x) is α-paracompact α-regular for each x∈X, then for each open set V of Y , G^+(V) = F^+(V), where G denotes γclF, βclF, *sclF, pclF, αclF or clF.*

Proof:

Let V be any open set of Y . Let $x \in G^+(V)$. Then $G(x) \subset V$ and $F(x) \subset G(x) \subset V$. We have $x \in F^+(V)$, and hence $G^+(V) \subset F^+(V)$. Conversely, let $x \in F^+(V)$, then $F(x) \subset V$. By Lemma 3.6, there exists an open set W of Y such that $F(x) \subset W \subset cl(W) \subset V$; hence $G(x) \subset cl(W) \subset V$. Then, we have $x \in G^+(V)$ and $F^+(V) \subset G^+(V)$.

Theorem: 3.8

Let F: (X,τ^*)→(Y,σ) be a multifunction such that F (x) is α- paracompact and α-regular for each $x \in$ X. Then the following are equivalent:

(i) F is upper τ^*-gγ continuous ;
(ii) γclF is upper τ^*-gγ continuous ;
(iii) βclF is upper τ^*-gγ continuous ;
(iv) sclF is upper τ^*-gγ continuous ;
(v) pclF is upper τ^*-gγ continuous ;
(vi) clF is upper τ^*-gγ continuous .

(vii) αclF is upper τ^*-gγ continuous.

Proof:

By Lemma 3.6, we put $G = \gamma cl$ F, βcl F, scl F, pcl F, or clF. Suppose that F is *upper* τ^*-gγ *continuous* . Let $x \in X$ and V be any open set of Y containing $G(x)$.

By Lemma 3.7 $x \in G^+(V) = F^+(V)$ and there exists $U \in \tau^*$-gγ (X, x) such that $F(U) \subset V$. Since $F(u)$ is α-paracompact and α-regular for each $u \in U$, by Lemma 3.7, there exists an open set W such that $F(u) \subset W \subset cl(W) \subset V$; hence $G(u) \subset cl(W) \subset V$ for each $u \in U$. So that $G(U) \subset V$. Hence G is upper τ^*-gγ continuous.

Conversely, suppose that G is *upper* τ^*-gγ continuous . Let $x \in X$ and V be any open set of Y containing $F(x)$. By Lemma 3.7, $x \in F^+(V) = G^+(V)$ and hence $G(x) \subset V$. There exists $U \in \tau^*$-gγ (X, x) such that $G(U) \subset V$. Thus $U \subset G^+(V) = F^+(V)$, and hence $F(U) \subset V$. So that F is *upper* τ^*-gγ *continuous* .

Lemma: 3.9

If F:$(X,\tau^*) \to (Y,\sigma)$ is a multifunction, then for each open set V of Y , $G^-(V) = F^-(V)$, where G denotes γclF, βclF, $sclF$, $pclF$, αclF *or* clF.

Proof:

Let V be any open set of Y and $x \in G^-(V)$. Then $G(x) \cap V \neq \emptyset$, and hence $F(x) \cap V \neq \emptyset$. Since V is open. Thus, $x \in F^-(V)$ and hence $G^-(V) \subset F^-(V)$. Conversely, assume that $x \in F^-(V)$. Then we have $\emptyset \neq F(x) \cap V \subset G(x) \cap V$ and hence $x \in G^-(V)$. Thus, we have $F^-(V) \subset G^-(V)$. Then $G^-(V) = F^-(V)$.

Theorem: 3.10

Let F: $(X,\tau^*) \to (Y,\sigma)$ be a multifunction then the following are equivalent:

(i) F is lower τ^*-gγ continuous ;
(ii) γclF is lower τ^*-gγ continuous ;
(iii) βclF is lower τ^*-gγ continuous ;
(iv) sclF is lower τ^*-gγ continuous ;
(v) pclF is lower τ^*-gγ continuous ;
(vi) αclF is lower τ^*-gγ continuous.
(vii) lF is lower τ^*-gγ continuous ;

Theorem: 3.11

Let$\{U_\lambda : \lambda \in \Lambda\}$ be an α-open cover of a space X. A multifunction F: $(X,\tau^*) \to (Y,\sigma)$ upper τ^*-gγ continuous(resp. lower τ^*-gγ continuous)if and only if restriction F\$U_\lambda$:$U_\lambda \to (Y,\sigma)$ upper τ^*-gγ continuous(resp. lower τ^*-gγ continuous) for each $\lambda \in \Lambda$.

Proof:

Suppose that let$\lambda \in \Lambda$ and $x \in U_\lambda$. Let V be a n open set of Y such that (F\$U_\lambda$)($x$)⊂V. Since F is upper τ^*-gγ continuous ,and F(x)= (F\$U_\lambda$)($x$)⊂V, there exists G∈ γ (X, x) such that F(G))⊂V . Put U=G∩ U_λ, then by lemma 2.10, we have U∈ τ^*-gγ (U_λ, x) and (F\$U_\lambda$)(U)=F(U)⊂V. Then F\$U_\lambda$ is upper τ^*-gγ continuous .

Assume that x ∈X and V be any open setY such that F(x)⊂V , there exists $\lambda \in \Lambda$ such that $x \in U_\lambda$. Since f\$U_\lambda$ is upper τ^*-gγ continuous and (F\$U_\lambda$)($x$)= F($x$) ⊂V , there exists U∈ τ^*-gγ (U_λ, x) such that (F\$U_\lambda$)(U) ⊂V. Then by lemma 2.11, we have U∈ τ^*-gγ (X, x) and F(U)= (F\$U_\lambda$)(U) ⊂V. Hence F is upper τ^*-gγ continuous.

IV. SOME PROPERTIES

Definition: 4.1

The τ^*-*g*γ-frontier of a subset A *of* X, denoted by τ^*-$g\gamma Fr(A)$, is defined by τ^*-$g\gamma Fr(A)$ = τ^*-$g\gamma cl(A) \cap \tau^*$-$g\gamma cl(X \setminus A)$ = τ^*-$g\gamma Cl(A) - \tau^*$- $g\gamma int(A)$.

Theorem: 4.2

The set of all points x of X at which is a multifunction F: $(X,\tau^*) \to (Y,\sigma)$ is not upper τ^*- gγ continuous is identical with the union of τ^*-gγ frontier of the upper inverse images of open sets containing F (x).

Proof:

Let x be a point of X at which F is not upper τ^*-$g\gamma$ continuous. Then there exists an open set V of Y containing $F(x)$ such that $U \cap (X - F^+(V)) \neq \emptyset$ for every $U \in \tau^*$-$g\gamma$ (X, x). Therefore, $x \in \tau^*$-$g\gamma Cl(X - F^+(V)) = X - \tau^*$-$g\gamma$int$(F^+(V))$ and $x \in F^+(V)$. Then, $x \in \tau^*$-$g\gamma$Fr$(F^+(V))$. Conversely, suppose that V is an open set containing $F(x)$ and that $x \in \tau^*$-$g\gamma$Fr$(F^+(V))$. If F is upper τ^*-$g\gamma$ continuous at x, then there exists $U \in \tau^*$-$g\gamma$ (X, x) such that $U \subset F^+(V)$; hence $x \in \tau^*$-$g\gamma$ int$(F^+(V))$. Which is a contradiction, hence F is not upper τ^*-$g\gamma$ continuous at x.

Theorem: 4.3

The set of all points x of X at which is a multifunction F: $(X,\tau^*) \rightarrow (Y,\sigma)$ is not lower τ^*- gγ continuous is identical with the union of τ^*-gγ frontier of the lower inverse images of open sets containing F(x).

Proof:

Let x be a point of X at which F is not lower τ^*-$g\gamma$ continuous. Then there exists an open set V of Y containing $F(x)$ such that $U \cap (X - F^-(V)) \neq \emptyset$ for every $U \in \tau^*$-$g\gamma$ (X, x). Therefore, we have $x \in \tau^*$-$g\gamma Cl(X - F^-(V)) = X - \tau^*$-$g\gamma$int$(F^-(V))$ and $x \in F^-(V)$. Then, $x \in \tau^*$-$g\gamma$Fr$(F^-(V))$. Conversely, suppose that V is an open set containing $F(x)$ and that $x \in \tau^*$-$g\gamma$Fr$(F^-(V))$. If F is lower τ^*-$g\gamma$ continuous, at x, then there exists $U \in \tau^*$-$g\gamma$ (X, x) such that $U \subset F^-(V)$; hence $x \in \tau^*$-$g\gamma$ int$(F^-(V))$. Which is a contradiction, hence F is not lower τ^*-$g\gamma$ continuous at x.

Definition: 4.4

A net $(F_\lambda)_{\lambda \in D}$ is said to be equally upper τ^*-$g\gamma$ continuous at $x_0 \in X$ if for every open set V_λ containing $F_\lambda(x_0)$ there exists a τ^*-$g\gamma$ open set U containing x_0 such that $F_\lambda(U) \subset V_\lambda$ for all $\lambda \in D$.

Theorem: 4.5

Let $(F_\lambda)_{\lambda \in D}$be a net of multifunctions from a topological space (X,τ^*) into a compact topological space (Y,σ) . If the following are satisfied.

(i) $\cap\{Y - F_\eta(x) : \eta > \lambda\} \in \sigma$ for each $\lambda \in$ D and each $x \in$X,

(ii) (F_λ) is equally upper τ^*-$g\gamma$ continuous on X, then F^* is upperτ^*-$g\gamma$ continuous on X.

Proof:

It is well known that $F^*(x) = \cap\{cl\ (\cup F_\eta(x) : \eta > \lambda\} : \lambda \in$ D} from(i),we have $F^*(x) = \cap\{[\ \cup \{F_\eta(x) : \eta > \lambda\}] : \lambda \in$ D}, since the net $(\cup \{F_\eta(x) : \eta > \lambda\})_{\lambda \in D}$ is a family of closed sets having the finite intersection property and Y is compact, it follows that $F^*(x) \neq \emptyset$ for each $x \in$X. Now, let $x_0 \in X$ and let V$\in \sigma$ such that V$\neq$Y and $F^*(x_0) \subset$V. Then $F^*(x_0) \cap$(Y$-$V) $\neq \emptyset$. Then $\cap\{[\ \cup \{F_\eta(x) : \eta > \lambda\}] : \lambda \in$ D}$\cap$(Y$-$V)$= \emptyset$ and hence $\cap\{[\ \cup \{F_\eta(x) : \eta > \lambda\}] : \lambda \in$ D}$=\emptyset$. Since Y is compact and the family $\{[\cup \{F_\eta(x_0) \cap$(Y$-$V)$: \eta > \lambda\} : \lambda \in$ D] is a family of closed sets with the empty intersection, there exists $\lambda \in$ D such that for each $\eta \in$D with $\eta > \lambda$ we have $F_\eta(x_0) \cap$(Y$-$V))$= \emptyset$; hence $F_\eta(x_0) \subset$V. Since the net $(F_\lambda)_{\lambda \in D}$ is equally upper τ^*-$g\gamma$ continuous on X, when the results that there exists a τ^*-$g\gamma$ open set U containing x_0 such that F_η(U) $\subset$V for each $\eta > \lambda$, hence $F_\eta(x)$ $\cap$(Y$-$V) = $\emptyset$ for each $x \in U$. Then, we have $\cup \{F_\eta(x)$ $\cap$(Y$-$V) : $\eta > \lambda$ }$= \emptyset$; hence $\cap\{[\ \cup \{F_\eta(x) : \eta > \lambda\}] : \lambda \in$ D}$\cap$(Y$-$V)$=\emptyset$ which implies that F^*(U)$\subset$V. If V=Y, then it is clear that for each τ^*-$g\gamma$ open set U containing x_0 we have F^*(U)$\subset$V. Since x_0 is arbitrary, hence F^* is upper τ^*-$g\gamma$ continuous on x_0.

Definition: 4.6

A topological space X is said to be τ*-gγ compact if every τ^*-$g\gamma$ open cover of X has a finite subcover.

Theorem: 4.7

Let F: $(X,\tau^*) \rightarrow (Y,\sigma)$ be an upper τ^*-$g\gamma$ continuous surjective multifunction such that F(x) is compact for each $x \in$X. If X is τ*-gγ compact, then Y is compact.

Proof:

Let $\{V_\lambda : \lambda \in \nabla\}$ be an open cover Y. For each $x \in$X, F(x) is compact and there exists a finite subset $\nabla(x)$ of ∇ such that F$(x) \subset \cup\{V_\lambda : \lambda \in \nabla(x)\}$. Set V$(x) = \cup\{V_\lambda \backslash \lambda \in \nabla(x)\}$. Since F is upper τ^*-$g\gamma$ continuous, there exists U$(x) \subset \tau^*$-$g\gamma O(X)$ containing x such that F(U$(x) \subset$ V(x)). The family { U(x): $x \in$X} is an τ*-gγ open cover of X and there exists a finite number of points, say $x_1, x_2, \ldots\ldots, x_n$ in X such that X=$\cup$\{U(x_i):1≤i≤n\}. Then,we have Y=F(X)=F$(\cup_{i=1}^n U(x_i)) = \cup_{i=1}^n F$ (U$(x_i)) \subset \cup_{i=1}^n V(x_i) = \cup_{i=1}^n \cup_{\lambda \in \nabla(x_i)} V_\lambda$.

Hence Y is compact.

V. CONCLUSION

The τ*-generalized γ continuous multifunctions can be used to open map, closed map and then new separation axioms.

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