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Review Article

Some Distance Measures and Application of Trapezoidal Fuzzy Numbers in Collaborative Filtering Recommendation

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Abstract - *Distance measures of Trapezoidal fuzzy numbers and Application of Trapezoidal Fuzzy numbers in Collaborative filtering recommendation are introduced and evaluated.*

Keywords - *Centroid point, Collaborative filtering, Recommendation, Similarity measure, Trapezoidal fuzzy number.*

I. INTRODUCTION

Real world decision making problems are very often uncertain (or) vague in a number of ways. In 1965, Zadeh [5] introduced the concept of fuzzy set theory to meet those problems. The fuzzyness can be represented by different ways. One of the most useful representation is the membership function. Depending on the nature of the membership function the fuzzy numbers can be classified in different forms, such as Triangular Fuzzy Numbers (TFNs), Trapezoidal Fuzzy Numbers, Interval Fuzzy Numbers etc. Fuzzy matrices play an important role in scientific development. Fuzzy matrices were introduced by M.G.Thomson [4].. Two new operators and some properties of fuzzy matrices over these new operators are given in [1]. Some new operators on triangular fuzzy numbers and triangular fuzzy number matrices are given in [3].

II. PRELIMINARIES

In this paper, some new elementary operators on α-cuts of Trapezoidal Fuzzy Numbers (TrFNs) and some new operators on α-cuts of Trapezoidal Fuzzy Number Matrices (TrFNMs) are defined. Using these operators, some important properties are proved.

Definition 2.1[2]:

A Fuzzy set A in a universe of discourse X is defined as the following set of pairs **A= {(x, μ_A (x)): x ∈ X}**
Here μ_A: X →[0,1] is a mapping called the membership value of x∈X in a fuzzy set A.

Definition 2.2[2]:

A Fuzzy number is an extension of a regular number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each
possible values has its own weight 0 and 1.This weight is called the **membership function**.

Definition 2.3[1]:

A normal fuzzy number A with shape function

$$\mu_A = \begin{cases} \left(\frac{x-a}{b-a}\right)^n & , when\ x \in [a,b), \\ w & , when\ x \in [b,c], \\ \left(\frac{d-x}{d-c}\right)^n & , when\ x \in (c,d] \\ 0 & , \text{Otherwise} \end{cases}$$

where n > 0, will be denoted by A = (a, b, c, d)$_n$.
If A be non-normal fuzzy number, it will be denoted by A = (a, b, c, d; $w)_n$.
If n = 1, we simply write A = (a, b, c, d), which is known as a Normal Trapezoidal fuzzy number.

Definition 2.4[5]:

A Trapezoidal Fuzzy Number (TrFN) denoted by <m,α, β, γ> has the membership function

$$\mu_A(x) = \begin{cases} 0, & \text{for } x \leq m \\ \frac{x-m}{\alpha-m} & m \leq x \leq \alpha \\ 1, & \alpha \leq x \leq \beta \\ \frac{\gamma-x}{\gamma-\beta}, & \beta \leq x \leq \gamma \\ 0, & x \geq \alpha \end{cases}$$

or, $\mu_A(x)$= max (min $\frac{x-m}{\alpha-m}$, 1, $\frac{\gamma-x}{\gamma-\beta}$), 0)

The point m, with membership grade of 1, is called the **mean value** and α, β are the **left hand spreads** of M repectively.

When $\alpha = \beta$, the trapezoidal fuzzy number coincides with triangular one.

Example 2.5:

$\widetilde{M}$= < 3, 6, 7, 9 > is a Trapezoidal Fuzzy number.

III. SOME DISTANCE MEASURES OF TRAPEZOIDAL FUZZY NUMBERS

The proposed ranking method consists of five distance - based components which are the horizontal x component of fuzzy numbers, centroid point, similarity measure, height and spread of fuzzy numbers. Conceptually, the proposed method is essentially an extension of the method given by [20] and [21] where they only consider three components of distances in fuzzy numbers which are the x-value, centroid point and similarity measure. The two extra components i.e .,the spread and height of the fuzzy number are included as they are crucial in discriminating some complex combination of fuzzynumbers.

Let A=(a_1, a_2, a_3, a_4 ;wA) and B=(b_1, b_2, b_3, b_4;wB) be two standardized generalized fuzzy numbers.

Step 1:

Obtain the agreement between A and B by taking the average of the two fuzzy numbers denoted by δ_{AB} where

$\delta_{AB} = \frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}, \frac{a_3+b_3}{2}, \frac{a_4+b_4}{2}$;min($w_A + w_B$),

=($c1$, $c2$,$c3$,$c4$;w) where $C_i \in [-1,1]$.

Step 2:

Calculate the centroid point x^* , y^* of each standardized generalized fuzzy numbers and their agreement using formula given by

$$x_A^* = \frac{\int_{-1}^{1} x f(x)dx}{\int_{-1}^{1} f(x)dx}, \quad y_A^* = \frac{\int_0^w \alpha \ A^\alpha \ d\alpha}{\int_0^w A^\alpha \ \ d\alpha} \ ,$$

where A^α is the length of the α-cut of A^α and x_A^*, y_A^* are the centroid of fuzzy number A with $x_A^* \in [-1,1]$ and $y_A^* \in [0,1]$.

Step 3:

Calculate the similarity measure between the agreement and δ_{AB} and A given as

$$\varphi^* \delta_{AB}, A = \mu_d \ \delta_{AB}, A \ \ \left(1 - x_{\delta_{AB}}^* - x_A^*\right) \frac{\min\left(y_{\delta_{AB}}^*, y_A^*\right)}{\max\left(y_{\delta_{AB}}^*, y_A^*\right)},$$

Where,

$\mu_d \ \delta_{AB}, A = \frac{1}{4}\left(1 - \frac{c_1-a_1}{0.5} + 1 - \frac{c_2-a_2}{0.5} + 1 - \frac{c_3-a_3}{0.5} + 1 - \frac{c_4-a_4}{0.5}\right)$Essentially, μd is the distance between each point of the fuzzy numbers.

Step 4:

Calculate the spread of each fuzzy numbers using the following formula:

$$\sigma^*(A) = \overline{\frac{\sum_{i=1}^{4}(a_1 - x_A^*)^2}{4-1}},$$

where $\sigma^*(A) \in [0, 1.1547]$. The value of 1.1547 is obtained by considering the maximum of $\sigma^*(A)$ with $A = (-1, -1, 1, 1; wA)$.

Step 5:

Obtain the score values for A and B . Let ΔA be the score value representing A, the ranking value is calculated $\Delta A = x_A^* \cdot h_A^* \cdot \varphi^* \ \delta_{AB}, A^{\sigma^*(A)}$

and $\Delta A \in$ -1,1 .

Step 6:

Rank A and B.

If $\Delta A > \Delta B$ then $A > B$ i.e A is ranked higher than B.

If $\Delta A < \Delta B$ then $A < B$ i.e A is ranked lower than B.

If $\Delta A = \Delta B$ then $A \approx B$ i.e A and B are equally ranked.

IV. APPLICATION OF TRAPEZOIDAL FUZZY NUMBERS IN COLLABORATIVE FILTERING RECOMMENDATION

Collaborative filtering technique has been widely applied [5–7], which recommends items to a user based on the rating information of its neighbours.

The basic principle of collaborative filtering recommendation assumes that users will have similar evaluation on other items if they have similar evaluation on some items. Thus, items can be recommended to a user based on its nearest neighbours' evaluation. Therefore, similarity measurement based on users' evaluation on items is the key problem in collaborative filtering recommendation systems.

Comprehensive measurement, such as evaluation ranking frequency vector and cloud model (CM), can be used to represent users' comprehensive evaluation and to calculate users' similarity [9].

Cloud model is a cognitive model proposed by Liu et al. which can synthetically describe the randomness and fuzziness of concepts and implement the uncertain transformation between a qualitative concept and its quantitative instantiations [10]. Zhang et al. analysed the effect of evaluation ranking frequency vector in collaborative filtering recommendation system and used cloud model to express the users' comprehensive evaluation on items; simulation on MoviesLens dataset was carried out to prove that collaborative filtering recommendation based on cloud model obtained better performance than that of ranking frequency vector [6]. Therefore, the paper compares the performance of collaborative filtering recommendation based on cloud model and trapezoidal fuzzy number, respectively.

If a user's evaluation on traded items is X=$\{x_1, x_2, \ldots, x_n\}$, the user's comprehensive evaluation on items can be represented by cloud model :

$$Ex=\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i,$$

$$He=\sqrt{\frac{\pi}{2}} \times \frac{1}{n}\sum_{i=1}^{n}|x_i - E_x|,$$

$$En=\sqrt{S^2 - \frac{1}{3}H_e^2}.$$

V= (E_x , E_n , H_e)

The trapezoidal fuzzy number shows the fuzzy and uncertainty of information, so it is appropriate to represent the user's comprehensive evaluation on items. The average of the user's comprehensive evaluation on items represents the centre of a trapezoidal fuzzy number, namely, the degree of concentration on items. The absolute value of user's evaluation on item minus the average of user's evaluation on items is called the **Fuzzy Discretization** (FD) of user's evaluation, that is, the uncertainty of user's evaluation.

Therefore, if a user evaluates the items in X=$\{x_1, x_2, \ldots, x_n\}$, its comprehensive evaluation can be expressed by a trapezoidal fuzzy number $\tilde{A} = (a^L, a^M, a^U, a^N)$.

$$a^M = \frac{1}{5n}\sum_{i=1}^{n} x_i$$

$$a^L = a^M - \frac{1}{2}(FD - E),$$

$$a^U = a^M + \frac{1}{2}(FD + E),$$

$$FD = \frac{1}{5n}\sum_{i=1}^{n}|x_i - a^M|,$$

$$E = \frac{1}{5n}\sum_{i=1}^{n} sig(x_i - a^M)$$

A. Comparison of Users' Comprehensive Evaluation

Take the example from Zhang et al. [6]. Four users, A, B, C, and D, evaluated 10 items, respectively. Table 1 shows the detailed evaluation of each user on the items.

Table 1. Users' evaluation on 10 items.

User	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
A	2	1	1	1	2	1	1	2	1	2
B	5	4	5	4	5	4	5	4	5	4
C	4	5	3	4	5	5	4	4	5	3
D	2	1	2	2	1	1	2	2	1	2

Bellogín et al. [20] found that the similarity based on evaluation ranking frequency vector was in [0.98,1] and it is difficult to distinguish its nearest neighbours. They put forward the cloud model to express the user's evaluation on items.
The similarity of users based on the cloud model is as shown in Table 2

Table 2. Users' similarity based on CM.

Similarity	A	B	C	D
A	1	0.956	0.965	0.999
B	0.956	1	0.999	0.967
C	0.965	0.999	1	0.975
D	0.999	0.967	0.975	1

From the case, we can see that the similarity based on trapezoidal fuzzy number has **better** discrimination. Users A and B have different evaluation on items; the similarity based on cloud model is 0.956, which is not consistent with our intuitionistic judging.

While the similarity based on trapezoidal fuzzy numbers is 0.461, which means that there is difference between the evaluation of users A and B. Therefore, it can be concluded that it is reasonable to use a trapezoidal fuzzy number to express users' evaluation on items and calculate their similarity based on it.

B. Simulation in Collaborative Filtering Recommendation System

The simulated system is developed by using Visual Studio.NET. The experiment takes data from Movies Lens (http://www.grouplens.org/), which is a recommendation system for research and provides recommendation list to users based on users' evaluation on movies. The experiment uses the Movies Lens 100 K data set, which contains 100000 records of 943 users' evaluation on 1648 items. 80 percent of the data set is used as history data and 20 percent of it is chosen as test data set.

Accuracy is a metric to evaluate the recommendation quality. And **mean absolute error (MAE)** can be used to represent recommendation accuracy, which calculates the deviation between the user's forecasted score and the user's real score. The smaller the MAE, the better the performance of a recommendation system.
Assume the users' forecasted score set of a recommendation system is F=$\{f_1, f_2, \ldots, f_n\}$, and the users' real score set is R=$\{r_1, r_2, \ldots, r_n\}$. The MAE can be denoted as the following formula: MAE = $\frac{1}{n}\sum_{i=1}^{n}|f_i - r_i|$

A simulated collaborative filtering recommendation system is developed. Cloud model and trapezoidal fuzzy number are used to express the comprehensive users' evaluation on items in the simulated system, similarity of users is calculated based on them, and items are recommended to users based on the nearest neighbours' scores on items. To score the performance of collaborative filtering recommendation, the MAE is used to measure recommendation accuracy.

As a result, the more neighbours are used in collaborative filtering recommendation, the more accuracy the recommendation system can obtain. From the simulation, the MAE of collaborative filtering recommendation based on trapezoidal fuzzy number can be more accurate than that on cloud model.

V. CONCLUSION

Trapezoidal fuzzy number has been applied in many fields such as risk analysis, decision-making, and evaluation. A new method to measure the similarity of two trapezoidal fuzzy numbers is proposed, which measures the similarity based on the shape's indifferent area and centroid of two trapezoidal fuzzy numbers. Comparison with other measurements shows that the similarity measuring method proposed in this paper can be distributed more normally.

Finally, trapezoidal fuzzy number is applied in collaborative filtering recommendation system, in which trapezoidal fuzzy number is used to express the users' comprehensive evaluation on items. Case demonstration and simulation on MoviesLens show that trapezoidal fuzzy number can express users' comprehensive evaluation on items and the collaborative filtering recommendation accuracy can be higher.

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