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Original Article

Higher order Burgers and Non Linear Equation with Different Boundary Condition by HPM

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Abstract — *In this paper we are studying importance of Homotopy Perturbation Method. To check the efficacy of this method three problem of Partial Differential Equation (PDE) will be solved using HPM. The numerical results shall prove that this method is a useful and impressive for explanation of PDE and advantageousness of the new capability. The HPM is defined as a new idea, new developments techniques certainly provided the conclusion.*

Keywords — *Analytical Solution, Higher Order Differential Equation, Homotopy Perturbation Method.*

I. INTRODUCTION

Many physical problems are being described by PD equation in mathematical physics along with more range of application in science and engineering [3, 5, 8, 12, 13,15]. The inspection of correct or approximate explanation to this PD equation will advise out to know these physical developments are being exceptional. In this article we will provide correct explanation of a few linear partial differential equation[16,17.18,19,24,25]. It is evident to examine that this formula delivers the correct explanation.

The present method is simple, skilful as well as broadly helpful to explain non liner differential equation. In HPM a homotopy is formulated by suggested and installed parameter P∈(0, 1).

The HPM is very easy to implement, effective, and convenient over available analytic methods for obtaining complex mathematical solutions of linear and nonlinear expressions. In recent decades, HPM was first presented by Ji Huan He [1, 2, 4,14]. The HPM is capable in studying various linear and nonlinear physical systems[21,22,23]. HPM is a combination of classical perturbation methods and homotopy in topology. This coupling of methods provides a new tool of finding analytical solutions of problems occurring in many branches of science and technology.

II. HOMOTOPY PERTURBATION FORMULA (HPM)

Suppose

$$B(u) - g(s) = 0, \quad s\in\xi \qquad (2.1.1)$$

along the initial condition of

$$C(u, \frac{\partial u}{\partial n}) = 0, \quad s\in \lambda, \qquad (2.1.2)$$

point B is a general operative g(s) is an accepted analytic action, C is an initial operative, also λ is the boundary of the domain ξ. The operative B as it may be generally divided toward two operative, K and M, point K is a linear while M a nonlinear operative.

Equation (2.1.1) can be further generalized as

$$K(u) + M(u) - g(s) = 0 \qquad (2.1.3)$$

proving the homotopy formula, we hence create a homotopy

$$v(s, p): \xi \times [0,1] \rightarrow S,$$

$$(v, p) = (1 - p)[K(v) - K(u_0)] + p[B(v) - g(s)] = 0, \qquad (2.1.4)$$

Or

H(v, p) = K(v) - K(u_0) + pK(u_0) + pK(u_0) + p[M(v) - g(s)] = 0 (2.1.5)

point p∈[0,1] is called homotopy limitation and u_0 is an original proximate as the explanation about (2.1.1), and that delight the initial condition. certainly, about (2.1.4) or (2.1.5), we get,

H(v, 1) = K(v) - K(u_0) = 0 (2.1.6)

H(v, 1) = B(v)–g(s) = 0 (2.1.7)

Along with altering action like p from zero to unity is just that of H(v, p) from K(v) - K(u_0) to B(v) - g(s).

In topology, here is called deformative K(v) - K(u_0) and B(v) – g(s) are called homotopy

We suppose that the explanation as (2.1.4) or (2.1.5) we write as a series in p as pursue:

V = $v_0 + \mathrm{p}v_1 + p^2v_2 + p^3v_3+\ldots$ (2.1.8)

Putting p =1 result in the proximate explanation as

U = $\lim_{p\to 1} v = v_0 + v_1 + v_2 + v_3+\ldots$

For application of this method below some examples are given.

A. Example 1

Let's consider (3+1) dimension Burger equation [7,9]

$$\frac{\partial\phi}{\partial t} + \mathrm{a}[\phi\frac{\partial\phi}{\partial q} + \phi\frac{\partial\phi}{\partial r} + \phi\frac{\partial\phi}{\partial s}] = \mathrm{b}[\frac{\partial^2\phi}{\partial q^2} + \frac{\partial^2\phi}{\partial r^2} + \frac{\partial^2\phi}{\partial s^2}] \quad (1.1)$$

Initial condition

ϕ (q, r, s, 0) = e^{q+r+s}; (1.2)

The homotopy equation are:

$$\frac{\partial\phi}{\partial t} = \mathrm{p}[\frac{\partial\phi}{\partial t} - \mathrm{a}[\phi\phi_q + \phi\phi_r + \phi\phi_s] + b[\phi_{qq} + \phi_{rr} + \phi_{ss}] - D_t\phi] \quad (1.3)$$

The explanation of equation (1.3), (1.2) as power series in p

$$\phi = \phi_0 + p\phi_1 + p^2\phi_2 + p^3\phi_3 + \cdots \quad (1.4)$$

Therefore, putting (1.4), and the initial conditions (1.2) into the (1.3) and comparing the indication by exact power of p. we get the henceforth set of linear PDE.

$$\frac{\partial\phi_0}{\partial t} = 0, \phi_0 = \mathrm{q + r + s} \quad (1.5)$$

$$\frac{\partial\phi_1}{\partial t} = [\frac{\partial\phi_0}{\partial t} - a\phi_0(\phi_0)_q - a\phi_0(\phi_0)_r - a\phi_0(\phi_0)_s + b(\phi_0)_{qq} + b(\phi_0)_{rr} + b(\phi_0)_{ss} - D_t\phi_0]; \phi_1(\mathrm{q, r, s}, 0) = 0$$

$$\frac{\partial\phi_1}{\partial t} = -\mathrm{a}(e^{q+r+s})(e^{q+r+s}) - \mathrm{a}(e^{q+r+s})(e^{q+r+s}) - \mathrm{a}(e^{q+r+s})e^{q+r+s} + \mathrm{b}e^{q+r+s} + \mathrm{b}e^{q+r+s} + \mathrm{b}e^{q+r+s}$$

$$\phi_1 = (-3\mathrm{a}+3\mathrm{b})(e^{q+r+s})\mathrm{t} \quad (1.6)$$

$\frac{\partial \phi_2}{\partial t}$ = $[\frac{\partial \phi_1}{\partial t}$ $-a\phi_0(\phi_1)_q - a\phi_1(\phi_0)_q - a\phi_1(\phi_0)_r - a\phi_0(\phi_1)_r - a\phi_1(\phi_0)_s - a\phi_0(\phi_1)_s + b(\phi_0)_{qq} + b(\phi_0)_{rr} + b(\phi_0)_{ss} - D_t\phi_1]$, ϕ_2(q, r, s, 0) = 0

$\frac{\partial \phi_2}{\partial t}$ = -a(e^{q+r+s}) (-3a + 3b)t – (-3at+3bt)(e^{q+r+s}))1 – a(q + r + s)(-3at) -a(-3at(q + r + s) -a(q + r + s) (-3at) – (-3at(q + r + s))1

ϕ_2 = $9a^2 + 9b^2t^2$ (e^{q+r+s}) (1.7)

ϕ_0= e^{q+r+s}

ϕ_1= (-3a + 3b)(e^{q+r+s})t

ϕ_2 = (9a^2 + 9b^2)(e^{q+r+s})t^2

:

:

Therefore, explanation of the Burger's standard is obtained by Homotopy Perturbation Method.

ϕ (q, r, s, t) = $\sum_{i=0}^{n-1} \phi_i$(q, r, s, t)

= $\phi_0 + \phi_1 + \phi_2 + \ldots$..

= e^{q+r+s}- (3a - 3b) (e^{q+r+s})t +(9$a^2 - 9b^2$) (e^{q+r+s})t^2-……

= e^{q+r+s} (1-3at +9a^2t^2- ………) +e^{q+r+s} $(3bt - 9b^2t^2 + \cdots..)$

$= \frac{e^{q+r+s}}{1-3at} + \frac{e^{q+r+s}(3bt)}{1-3bt}$

Here we have taken (3+1) D Burger's equation. This is the accurate explanation of Burger's models.

B. Example 2

Consider Non-linear PDE [10,20]

ϕ_t + ϵϕ_x+ ϕ = 1 (2.1)

ϵ_t -ϕϵ_x- ϵ = 1 (2.2)

initial condition.

ϕ (x, 0) = x^2 ; ϵ (x, 0) = x^{-2} (2.3)

The homotopy equation (3.1.11), (3.1.12) are;

$\frac{\partial \phi}{\partial t}$ = p[$\frac{\partial \phi}{\partial t}$ +1- (ϵϕ_x + ϕ) $- D_t$ϕ] (2.4)

$\frac{\partial \epsilon}{\partial t}$ = p[$\frac{\partial \epsilon}{\partial t}$ +1+ (ϕϵ_x + ϵ) $- D_t$ϵ] (2.5)

The explanation of equation (2.1), (2.2) as power series in p

ϕ = $\phi_0 + p\phi_1 + p^2\phi_2 + p^3\phi_3 + \cdots$ (2.6)

ϵ = ϵ_0 + pϵ_1 +$p^2\epsilon_2$ +$p^3\epsilon_3$ +…. (2.7)

Therefore, putting (2.6), (2.7) and the initial conditions (2.3) into the (2.4) and (2.5) and comparing the indication by exact power of p, we get the henceforth set of linear PDE.

$\frac{\partial \phi_0}{\partial t} = 0, \phi = x^2$ (2.8)

$\frac{\partial \epsilon_0}{\partial t} = 0, \epsilon = x^{-2}$ (2.9)

$\frac{\partial \phi_1}{\partial t} = [\frac{\partial \phi_0}{\partial t} + 1 - \epsilon_0(\phi_0)_q - \phi_0 - D_t\phi_0]; \phi_1(x,0) = 0$ (2.10)

$\frac{\partial \phi_1}{\partial t} = [1 - x^{-2}x^2 \quad -x^2]$

$\phi_1 = -tx^2$

$\frac{\partial \epsilon_1}{\partial t} = [\frac{\partial \phi_0}{\partial t} + 1 + \phi_0(\epsilon_0)_x + \epsilon_0 - D_t\epsilon_0]; \epsilon_1(x,0) = 0$ (2.11)

$\frac{\partial \epsilon_1}{\partial t} = [1 + (-x^{-2})x^2 \quad +x^{-2}]$

$\epsilon_1 = tx^{-2}$

$\frac{\partial \phi_2}{\partial t} = [\frac{\partial \phi_1}{\partial t} - \epsilon_0(\phi_1)_x - \epsilon_1(\phi_0)_x - \phi_1 - D_t\phi_1], \ \phi_2(x,0) = 0$ (2.12)

$\frac{\partial \phi_2}{\partial t} = [-x^{-2}(-x^2t) - tx^{-2}x^2 - (-t x^2)]$

$\phi_2 = x^2\frac{t^2}{2}$

$\frac{\partial \epsilon_2}{\partial t} = [\frac{\partial \epsilon_1}{\partial t} + \phi_0(\epsilon_1)_x + \phi_1(\epsilon_0)_x + \epsilon_1 - D_t\epsilon_1], \ \epsilon_2(x,0) = 0$ (2.13)

$\frac{\partial \epsilon_2}{\partial t} = [x^2(-x^{-2}t) + (-tx^2)(-x^{-2}) + (t x^{-2})]$

$\epsilon_2 = x^{-2}\frac{t^2}{2}$

$\phi_0 = x^2$

$\epsilon_0 = x^{-2}$

$\phi_1 = -t x^2$

$\epsilon_1 = t x^{-2}$

$\phi_2 = x^2\frac{t^2}{2}$

$\epsilon_2 = x^{-2}\frac{t^2}{2}$

:

:

Therefore, explanation of the Burger's standard is obtained by Homotopy Perturbation Method.

$\phi(x, t) = \sum_{i=0}^{n-1} \phi_i(x, t)$

$= \phi_0 + \phi_1 + \phi_2 + \ldots..$

$= x^2 - t\, x^2 + x^2 \frac{t^2}{2} - \ldots\ldots.$

$= x^2\, e^{-t}$

$\epsilon\,(x, t) = \sum_{i=0}^{n-1} \epsilon_i(x, t)$

$= \epsilon_0 + \epsilon_1 + \epsilon_2 + \ldots..$

$= x^{-2} + t\, x^{-2} + x^{-2} \frac{t^2}{2} - \ldots\ldots.$

$= x^{-2}\, e^{t}$

In this problem we have used the HPM to solve the non-linear equation and obtained the exact solution of the problem. The methods will help us understand the importance of HPM.

C. Example 3

Consider a Non-linear equation [6,11]

$$u_t + v_q w_r + v_r w_q = -u \tag{3.1}$$

$$v_t + w_q u_r + w_r u_q = v \tag{3.2}$$

$$w_t + u_q v_r + u_r v_q = w \tag{3.3}$$

Initial condition

$$u(q, r, 0) = (q + r) \tag{3.4}$$

$$v(q, r, 0) = (q - r)$$

$$w(q, r, 0) = (-q + r) \tag{3.5}$$

The homotopy equation (3.1), (3.2), (3.3) are.

$$\frac{\partial u}{\partial t} = p[\frac{\partial u}{\partial t} - u - v_q w_r + v_r w_q - D_t u] \tag{3.6}$$

$$\frac{\partial v}{\partial t} = p[\frac{\partial v}{\partial t} + v - w_q u_r - w_r u_q - D_t v] \tag{3.7}$$

$$\frac{\partial w}{\partial t} = p[\frac{\partial w}{\partial t} + w - u_q v_r - u_r v_q - D_t w] \tag{3.8}$$

The explanation of equation (3.6), (3.7), (3.8) as power series in p

$$u = u_0 + pu_1 + p^2 u_2 + p^3 u_3 + \ldots. \tag{3.9}$$

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \ldots. \tag{3.10}$$

$$w = w_0 + pw_1 + p^2 w_2 + p^3 w_3 + \ldots. \tag{3.11}$$

Therefore, putting above equations and the initial conditions and comparing the indication by exact power of p, we get the henceforth set of linear PDE.

$\frac{\partial u_0}{\partial t} = 0,\ u_0 = (q + r)$

$\frac{\partial v_0}{\partial t}$ = 0, v_0 = (q - r)

$\frac{\partial w_0}{\partial t}$ = 0, w_0 = (-q + r)

$\frac{\partial u_1}{\partial t}$ = [-u_0 - $(v_0)_q(w_0)_r + (v_0)_r(w_0)_q$] ;

u_1 (q, r, 0) = 0

= [-(q + r)- (1)(1) + (-1)(- 1)]

$u_1 = -t(q+r)$

$\frac{\partial v_1}{\partial t}$ = [v_0 - $(w_0)_q(u_0)_r - (w_0)_r(u_0)_q$;

v_1 (q, r, 0) = 0

= [q - r) - (-1)(1) - (1)(1)]

v_1 = t(q - r)

$\frac{\partial w_1}{\partial t}$ = [$w_0 - (u_0)_q(v_0)_r - (u_0)_r(v_0)_q$];

w_1 (q, r, 0) = 0

= [(-q + r) - (1)(−1) - (1)(1)]

w_1 = t(-q + r)

$\frac{\partial u_2}{\partial t}$ = [- $(v_0)_q(w_1)_r - (v_1)_q(w_0)_r + (v_0)_r(w_1)_q + (v_1)_r(w_0)_q$]; u_2 (q, r, 0) = 0

$u_2 = \frac{t^2}{2}(q+r)$

$\frac{\partial v_2}{\partial t}$ = [-$(w_0)_q(u_1)_r - (w_1)_q(u_0)_r - (w_0)_r(u_1)_q - (w_1)_r(u_0)_q$] ; v_2 (q, r, 0) = 0

$v_2 = \frac{t^2}{2}(q-r)$

$\frac{\partial w_2}{\partial t}$ = [-$(u_0)_q(v_1)_r - (u_1)_q(v_0)_r - (u_0)_r(v_1)_q - (u_1)_r(v_0)_q$]; w_2 (q, r, 0) = 0

$w_2 = \frac{t^2}{2}(-q+r)$

Continuing in this process we get.

u = $u_0 + u_1 + u_2 + \ldots\ldots$

u = (q + r) - t(q +r) + $\frac{t^2}{2}$(q + r)

u = (q + r) e^{-t}

v = $v_0 + v_1 + v_2 + \ldots\ldots$

v = (q – r) +t(q - r) + $\frac{t^2}{2}$(q − r)+……

v = $(q - r)e^t$

w = $w_0 + w_1 + w_2 + \ldots\ldots$

w = (-q + r) +t(-q + r) $+ \frac{t^2}{2}(-q + r) + \ldots\ldots$

w = (-q + r)e^t

In the above problem we have taken three simultaneous non-linear equations and then applied the homotopy technique to these equations. By applying the technique, we get a new form of solutions for u, v and w

w = (-q +r)e^t.

This is the solution of the equations..

III. CONCLUSION

In this paper, we have considered a nonlinear (3+1) dimension Burger's equation, and many more nonlinear differential equations. We have provided different boundary conditions under which the present problem has a unique solution. We have also observed that this approach may lead to the exact solution of the problems. Furthermore, this method is a powerful tool to solve different types of PDE. Thus, the HPM is capable in observing the comparative or analytic explication of the linear and nonlinear PDE. It is also a helpful and useful method to solve the PDE.

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