

Original Article

$\alpha(\text{gg})^*$ -Closed Sets in Topological Spaces

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Received Date: 14 February 2022
Revised Date: 24 March 2022
Accepted Date: 26 March 2022

Abstract - In this paper we introduce the concept of alpha generalization of generalized star closed sets in topological spaces. Also, we investigated its relation among other closed sets. Further, we derived some of its characteristics.

Keywords - $\alpha(\text{gg})^*$ -closed sets, $(\text{gg})^*$ -open sets, α -closure.

2020 Subject Classification: 54A05.

I. INTRODUCTION

In 1970, Levine [34] introduced the concept of generalized closed sets in topology. In 2017, Basavaraj, M. Ittangi and H. G. Govardhana Reddy [5] introduced gg -closed sets in topological spaces. In 2018, I. Christal Bai and T. Shyla Isac Mary [10] introduced $(\text{gg})^*$ -closed sets in topological spaces. In this paper, a new class of closed set called alpha generalization of generalized star-closed set is introduced. The concept of alpha generalization of generalized star-closed set and its characteristics are analyzed in this paper.

II. PRELIMINARIES

Throughout this paper (X, τ) represents the topological spaces on which no separation axioms are assumed unless otherwise mentioned. A being a subset of a topological space (X, τ) , $cl(A)$, $int(A)$ denote the closure of A and interior of A respectively. Replace (X, τ) with X if there is no chance of confusion.

Definition 2.1. A subset A of a topological space (X, τ) is called

- (i) pre-open [18] if $A \subseteq int\ cl(A)$ and pre-closed if $cl\ int(A) \subseteq A$.
- (ii) semi-open [34] if $A \subseteq cl\ int(A)$ and semi-closed if $int\ cl(A) \subseteq A$.
- (iii) semi-pre-open [2] if $A \subseteq cl\ int\ cl(A)$ and semi-pre-closed if $int\ cl\ int(A) \subseteq A$.
- (iv) α -open [20] if $A \subseteq int\ cl\ int(A)$ and α -closed if $cl\ int\ cl(A) \subseteq A$.
- (v) regular-open [30] if $A = int\ cl(A)$ and regular-closed if $cl\ int(A) = A$.
- (vi) b-open [3] if $A \subseteq cl\ int(A) \cup int\ cl(A)$ and b-closed if $cl\ int(A) \cap int\ cl(A) \subseteq A$.
- (vii) π -open [20] if A is the union of regular open sets and π -closed if A is the intersection of regular closed sets.

the alpha-closure (resp. semi-closure, resp. semi-pre-closure, resp. pre-closure, resp. b-closure) of a subset of A of a topological space (X, τ) is the intersection of all alpha-closed (resp. semiclosed, resp. semi-pre-closed, resp. pre-closed, resp. b-closed) sets containing A and is denoted by $\alpha cl A$ (resp. $scl A$, resp. $spcl A$, resp. $pcl A$, resp. $bcl A$).

Definition 2.2. A subset A of a topological space (X, τ) is called

- (i) regular semi-open [9] if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.
- (ii) generalized-closed [16] (briefly g -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
- (iii) generalization of generalized closed [5] (briefly gg -closed) if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open.
- (iv) generalization of generalized star closed [10] (briefly $(\text{gg})^*$ -closed) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is gg -open.
- (v) generalized semi-closed [4] (briefly gs -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
- (vi) α -generalized-closed [17] (briefly αg -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.



- (vii) $g^\#$ -closed [34] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg -open.
- (viii) generalized semi-pre-closed [11] (briefly gsp -closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
- (ix) generalized b-closed [1] (briefly gb -closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
- (x) semi generalized-closed [8] (briefly sg -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.
- (xi) strongly generalized semi-pre-closed [24] (briefly $(gsp)^*$ closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gsp -open.
- (xii) weakly-closed [29] (briefly w -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.
- (xiii) \hat{g}^* -closed [25] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is \hat{g} -open.
- (xiv) strongly generalized closed [32] (briefly g^* -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open.
- (xv) *g closed [32] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.
- (xvi) weakly semi-closed [6] (briefly ws -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is w -open.
- (xvii) generalized alpha regular-closed [25] (briefly gar -closed) if $acl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open.
- (xviii) generalized pre-regular closed [13] (briefly gpr -closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open.
- (xix) generalized semi-pre-regular closed [19] (briefly $gspr$ -closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open.
- (xx) regular weakly-closed [7] (briefly rw -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open.
- (xxi) generalised star semi-closed [26] (briefly g^*s -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs -open.
- (xxii) generalized star b-closed [21] (briefly g^*b -closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open.
- (xxiii) gs^* -closed or $(gs)^*$ -closed [12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs -open.
- (xxiv) R^* -closed [14] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open.
- (xxv) generalized star b-omega closed [22] (briefly $g^*b\omega$ -closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs -open.
- (xxvi) α -generalized regular closed [33] (briefly αgr -closed) if $acl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.

III. $\alpha(GG)^*$ -CLOSED SETS

Definition 3.1. A set A of a topological space (X, τ) is called alpha generalization of **generalized star closed** (briefly $\alpha(gg)^*$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(gg)^*$ - open in (X, τ) . the set of all $\alpha(gg)^*$ -closed sets in (X, τ) is denoted by $\alpha(GG)^*C$.

Example 3.2. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$. Then $\emptyset, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}$ and X are $\alpha(gg)^*$ -closed.

Theorem 3.3.

- (i) Every closed set is $\alpha(gg)^*$ -closed
- (ii) Every α -closed set is $\alpha(gg)^*$ -closed.
- (iii) Every regular closed set is $\alpha(gg)^*$ -closed.
- (iv) Every π -closed set is $\alpha(gg)^*$ -closed.
- (v) Every g^* -closed set is $\alpha(gg)^*$ -closed.
- (vi) Every $(gs)^*$ -closed set is $\alpha(gg)^*$ -closed.
- (vii) Every $(gsp)^*$ -closed set is $\alpha(gg)^*$ -closed.

(viii) **Proof.**

- (i) Let A be any closed subset of a space X . Let $A \subseteq U$ and U is $(gg)^*$ -open in X . Since A is closed, $cl(A) = A \subseteq U$. But $\alpha cl(A) \subseteq cl(A) \subseteq U$. Therefore $\alpha cl(A) \subseteq U$. Hence A is $\alpha(gg)^*$ -closed set in X .
- (ii) Let A be any α -closed subset of a space X . Let $A \subseteq U$ and U is $(gg)^*$ -open in X . Since A is α -closed, $\alpha cl(A) = A \subseteq U$. Therefore $\alpha cl(A) \subseteq U$. Hence A is $\alpha(gg)^*$ -closed set in X .
- (iii) Let A be any regular closed subset of a space X . Let $A \subseteq U$ and U is $(gg)^*$ -open in X . Since A is regular closed, $rcl(A) = A \subseteq U$. But $\alpha cl(A) \subseteq rcl(A) \subseteq U$. Therefore $\alpha cl(A) \subseteq U$. Hence A is $\alpha(gg)^*$ -closed set in X .
- (iv) Let A be any π -closed subset of a space X . Let $A \subseteq U$ and U is $(gg)^*$ -open in X . Since A is π -closed, $\pi cl(A) = A \subseteq U$. But $\alpha cl(A) \subseteq \pi cl(A) \subseteq U$. Therefore $\alpha cl(A) \subseteq U$. Hence A is $\alpha(gg)^*$ -closed set in X .
- (v) Let A be any g^* -closed subset of a space X . Let $A \subseteq U$ and U is $(gg)^*$ -open in X . Since every $(gg)^*$ -open is g -open [10] and since U is g -open, $cl(A) \subseteq A \subseteq U$. But $\alpha cl(A) \subseteq cl(A) \subseteq U$. Therefore $\alpha cl(A) \subseteq U$. Hence A is $\alpha(gg)^*$ -closed set in X .
- (vi) Let A be any $(gs)^*$ closed subset of a space X . Let $A \subseteq U$ and U is $(gg)^*$ -open in X . Since every $(gg)^*$ -open is gs -open [10] and since A is $(gs)^*$ closed, $cl(A) \subseteq A \subseteq U$. But $\alpha cl(A) \subseteq cl(A) \subseteq U$. Therefore $\alpha cl(A) \subseteq U$. Hence A is $\alpha(gg)^*$ -closed set in X .

(vii) Let A be any $(gsp)^*$ closed subset of a space X . Let $A \subseteq U$ and U is $(gg)^*$ -open in X . Since every $(gg)^*$ -open is gsp -open [10] and since A is $(gsp)^*$ closed, $cl(A) \subseteq A \subseteq U$. But $\alpha cl(A) \subseteq cl(A) \subseteq U$. Therefore $\alpha cl(A) \subseteq U$. Hence A is $\alpha(gg)^*$ -closed set in X .

Remark 3.4. the converses of the above theorems need not be true in general as seen from the following example.

Example 3.5. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$. Then

- (i) $\{c\}$ is $\alpha(gg)^*$ -closed but not closed.
- (ii) $\{a, d\}$ is $\alpha(gg)^*$ -closed but not α -closed.
- (iii) $\{c\}$ is $\alpha(gg)^*$ -closed but not regular closed.
- (iv) $\{c\}$ is $\alpha(gg)^*$ -closed but not π -closed.
- (v) $\{c\}$ is $\alpha(gg)^*$ -closed but not g^* -closed.
- (vi) $\{c\}$ is $\alpha(gg)^*$ -closed but not $(gs)^*$ -closed.
- (vii) $\{c\}$ is $\alpha(gg)^*$ -closed but not $(gsp)^*$ closed.

Theorem 3.6.

- (i) Every $\alpha(gg)^*$ -closed set is gs -closed.
- (ii) Every $\alpha(gg)^*$ -closed set is gb -closed.
- (iii) Every $\alpha(gg)^*$ -closed set is gsp -closed.
- (iv) Every $\alpha(gg)^*$ -closed set is gpr -closed.
- (v) Every $\alpha(gg)^*$ -closed set is $gspr$ -closed.
- (vi) Every $\alpha(gg)^*$ -closed set is gar -closed.

Proof.

- (i) Let A be any $\alpha(gg)^*$ -closed subset of a space X . Let $A \subseteq U$ and U is open in X . Since every open set is $(gg)^*$ open, U is $(gg)^*$ open. Also, since A is $\alpha(gg)^*$ -closed, $\alpha cl(A) \subseteq U$. But $scl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore, $scl(A) \subseteq U$. Hence A is gs -closed set in X .
- (ii) Let A be any $\alpha(gg)^*$ -closed subset of a space X . Let $A \subseteq U$ and U is open in X . Since every open set is $(gg)^*$ -open, U is $(gg)^*$ -open. Also, since A is $\alpha(gg)^*$ -closed, $\alpha cl(A) \subseteq U$. But $bcl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore, $bcl(A) \subseteq U$. Hence A is gb -closed set in X .
- (iii) Let A be any $\alpha(gg)^*$ -closed subset of a space X . Let $A \subseteq U$ and U is open. Since every open set is $(gg)^*$ -open, U is $(gg)^*$ -open. Also, since A is $\alpha(gg)^*$ -closed, $\alpha cl(A) \subseteq U$. But $spcl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore, $spcl(A) \subseteq U$. Hence A is gsp -closed set in X .
- (iv) Let A be any $\alpha(gg)^*$ -closed subset of a space X . Let $A \subseteq U$ and U is regular-open in X . Since every regular-open set is $(gg)^*$ -open, U is $(gg)^*$ -open. Also, since A is $\alpha(gg)^*$ -closed, $\alpha cl(A) \subseteq U$. But $pcl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore, $pcl(A) \subseteq U$. Hence A is gpr -closed set in X .
- (v) Let A be any $\alpha(gg)^*$ -closed subset of a space X . Let $A \subseteq U$ and U is regular-open in X . Since every regular-open set is $(gg)^*$ -open, U is $(gg)^*$ -open. Also, since A is $\alpha(gg)^*$ -closed, $\alpha cl(A) \subseteq U$. But $spcl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore, $spcl(A) \subseteq U$. Hence A is $gspr$ -closed set in X .
- (vi) Let A be any $\alpha(gg)^*$ -closed subset of a space X . Let $A \subseteq U$ and U is regular-open in X . Since every regular-open set is $(gg)^*$ -open, U is $(gg)^*$ -open. Also, since A is $\alpha(gg)^*$ -closed, $\alpha cl(A) \subseteq U$. Hence A is gar -closed set in X .

Remark 3.7. the converses of the above theorems need not be true in general as seen from the following example.

Example 3.8. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a\}, \{c, d\}, \{a, c, d\}, X\}$. Then

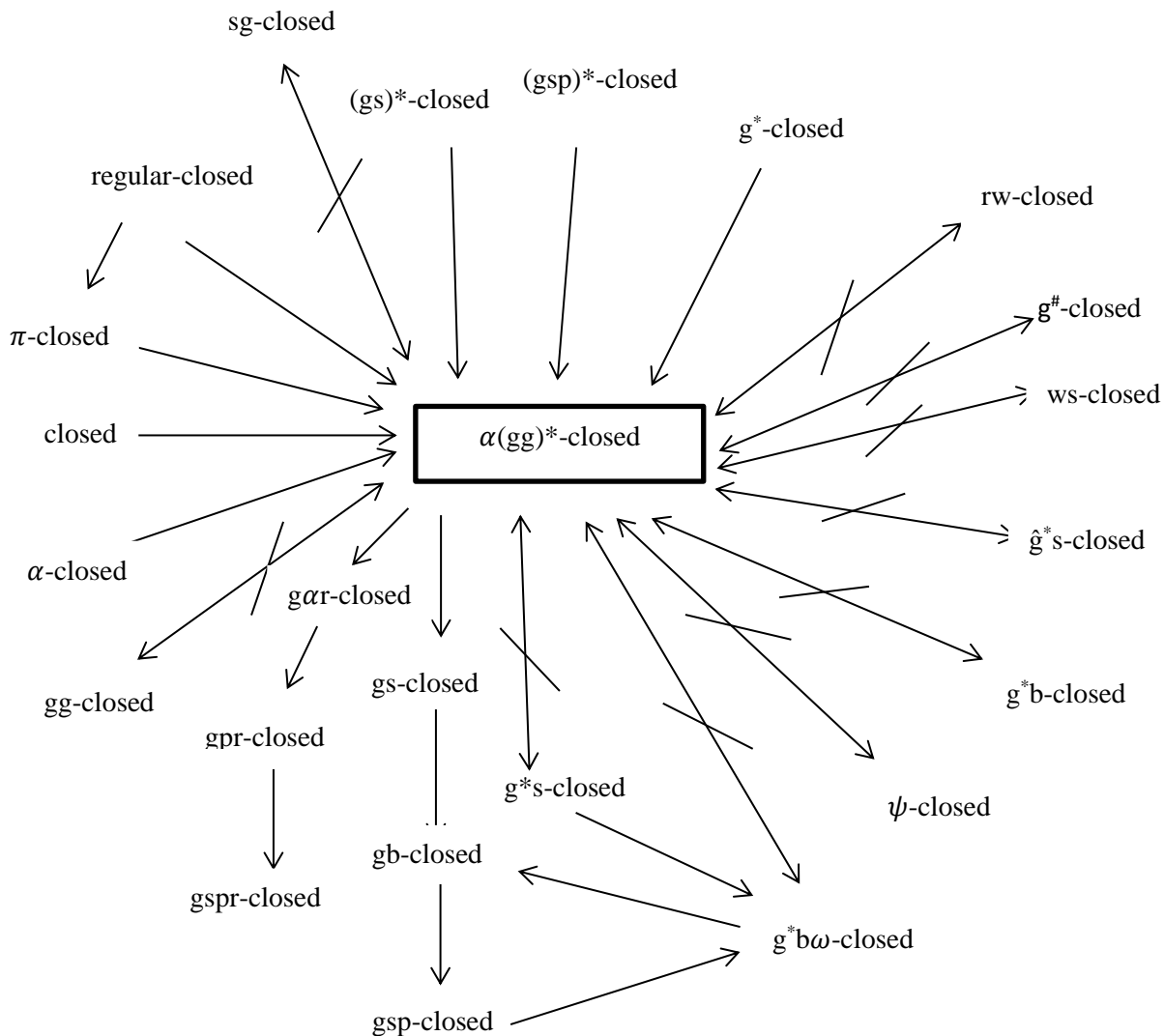
- (i) $\{a\}$ is gs -closed but not $\alpha(gg)^*$ -closed.
- (ii) $\{a\}$ is gb closed but not $\alpha(gg)^*$ -closed.
- (iii) $\{a\}$ is gsp -closed but not $\alpha(gg)^*$ -closed.
- (iv) $\{c\}$ is gpr -closed but not $\alpha(gg)^*$ -closed.
- (v) $\{a\}$ is $gspr$ -closed but not $\alpha(gg)^*$ -closed.
- (vi) $\{c\}$ is gar -closed but not $\alpha(gg)^*$ -closed.

the concept “ $\alpha(\text{gg})^*$ -closed” is independent from the concepts “rw-closed”, “sg-closed”, “ $\text{g}^\#$ -closed”, “ R^* -closed”, “ g^*s -closed”, “ g^*b -closed”, “gg-closed”, “ $\text{g}^*\text{b}\omega$ -closed”, “ $\hat{\text{g}}^*\text{s}$ -closed”, “ ψ -closed”, “ws-closed”.

Example 3.9. Let $X=\{a, b, c, d\}$ with topology $\tau =\{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$.

- (i) $\{c\}$ is $\alpha(\text{gg})^*$ -closed but not rw-closed and $\{a, b\}$ is rw-closed but not $\alpha(\text{gg})^*$ -closed.
- (ii) $\{a, b, c\}$ is $\alpha(\text{gg})^*$ -closed but not sg-closed and $\{a, b\}$ is sg-closed but not $\alpha(\text{gg})^*$ -closed.
- (iii) $\{c\}$ is $\alpha(\text{gg})^*$ -closed but not $\text{g}^\#$ -closed and $\{a, b\}$ is $\text{g}^\#$ -closed but not $\alpha(\text{gg})^*$ -closed.
- (iv) $\{c\}$ is $\alpha(\text{gg})^*$ -closed but not R^* -closed and $\{a, b\}$ is R^* -closed but not $\alpha(\text{gg})^*$ -closed.
- (v) $\{c\}$ is $\alpha(\text{gg})^*$ -closed but not g^*s -closed and $\{a, b\}$ is g^*s -closed but not $\alpha(\text{gg})^*$ -closed.
- (vi) $\{a, b, c\}$ is $\alpha(\text{gg})^*$ -closed but not g^*b -closed and $\{a\}$ is g^*b -closed but not $\alpha(\text{gg})^*$ -closed.
- (vii) $\{c\}$ is $\alpha(\text{gg})^*$ -closed but not gg-closed and $\{a, b\}$ is gg-closed but not $\alpha(\text{gg})^*$ -closed.
- (viii) $\{a, b, c\}$ is $\alpha(\text{gg})^*$ -closed but not $\text{g}^*\text{b}\omega$ -closed and $\{a\}$ is $\text{g}^*\text{b}\omega$ -closed but not $\alpha(\text{gg})^*$ -closed.
- (ix) $\{a, b, c\}$ is $\alpha(\text{gg})^*$ -closed but not $\hat{\text{g}}^*\text{s}$ -closed and $\{a\}$ is $\hat{\text{g}}^*\text{s}$ -closed but not $\alpha(\text{gg})^*$ -closed.
- (x) $\{a, b, c\}$ is $\alpha(\text{gg})^*$ -closed but not ψ -closed and $\{a\}$ is ψ -closed but not $\alpha(\text{gg})^*$ -closed.
- (xi) $\{a, b, c\}$ is $\alpha(\text{gg})^*$ -closed but not ws-closed and $\{a\}$ is ws-closed but not $\alpha(\text{gg})^*$ -closed.

the above discussion lead to the following diagram, “ $A \rightarrow B$ ” means A implies B but not conversely and “ $A \leftrightarrow B$ ” means A and B are independent of each other.



Theorem 3.10. the union of two $\alpha(\text{gg})^*$ -closed subsets of X is $\alpha(\text{gg})^*$ -closed.

Proof. Let A and B be any two $\alpha(\text{gg})^*$ -closed subsets of X. Let $A \subseteq U$ and U is $(\text{gg})^*$ -open, $B \subseteq U$ and U is $(\text{gg})^*$ -open. Then $\alpha\text{cl}(A) \subseteq U$, $\alpha\text{cl}(B) \subseteq U$. This implies that $\alpha\text{cl}(A) \cup \alpha\text{cl}(B) \subseteq U$. We know that, $\alpha\text{cl}(A \cup B) = \alpha\text{cl}(A) \cup \alpha\text{cl}(B) \subseteq U$ [27]. Therefore $\alpha\text{cl}(A \cup B) \subseteq U$, whenever $A \cup B \subseteq U$ and U is $(\text{gg})^*$ -open in X. Thus $A \cup B$ is $\alpha(\text{gg})^*$ -closed set in X.

Remark 3.11. Intersection of any two $\alpha(\text{gg})^*$ -closed sets need not be an $\alpha(\text{gg})^*$ -closed as seen from the following example.

Example 3.12. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$. Let $A = \{a, b, c\}$ and $\{a, b, d\}$ be two $\alpha(\text{gg})^*$ -closed sets in X. But $A \cap B = \{a, b\}$ is not an $\alpha(\text{gg})^*$ -closed set.

Theorem 3.13. If a subset A of X is $\alpha(\text{gg})^*$ -closed in X, if and only if $\alpha\text{cl}(A) \setminus A$ does not contain any non-empty $(\text{gg})^*$ -closed set in X.

Proof. Let A be a $\alpha(\text{gg})^*$ -closed set in X and M be a $(\text{gg})^*$ -closed subset of $\alpha\text{cl}(A) \setminus A$. Then $M \subseteq \alpha\text{cl}(A) \cap (X \setminus A) \Rightarrow M \subseteq \alpha\text{cl}(A)$ and $M \subseteq X \setminus A$. $M \subseteq X \setminus A \Rightarrow A \subseteq X \setminus M$. Since A is $\alpha(\text{gg})^*$ -closed and $X \setminus M$ is $(\text{gg})^*$ -open, $\alpha\text{cl}(A) \subseteq X \setminus M$. Then $M \subseteq X \setminus \alpha\text{cl}(A)$. But we have $M \subseteq \alpha\text{cl}(A)$. Therefore, $M \subseteq (X \setminus \alpha\text{cl}(A)) \cap \alpha\text{cl}(A) = \emptyset$. Thus $M = \emptyset$. Hence $\alpha\text{cl}(A) \setminus A$ does not contain any non-empty $(\text{gg})^*$ -closed set in X. Conversely, suppose that $\alpha\text{cl}(A) \setminus A$ does not contain any non-empty $(\text{gg})^*$ -closed set in X. Let $A \subseteq N$ and N be $(\text{gg})^*$ -open. If $\alpha\text{cl}(A)$ is not a subset of N then $\alpha\text{cl}(A) \cap N^c$ is a non-empty $(\text{gg})^*$ -closed subset of $\alpha\text{cl}(A) \setminus A$, which is a contradiction. Therefore, $\alpha\text{cl}(A) \subseteq N$ and hence A is $\alpha(\text{gg})^*$ -closed.

Theorem 3.14. If a subset A is $\alpha(\text{gg})^*$ -closed set in X and $A \subseteq B \subseteq \alpha\text{cl}(A)$, then B is also $\alpha(\text{gg})^*$ -closed set in X.

Proof. Let A be a $\alpha(\text{gg})^*$ -closed set in X such that $A \subseteq B \subseteq \alpha\text{cl}(A)$. To prove B is also a $\alpha(\text{gg})^*$ -closed set in X, it is enough to prove that $\alpha\text{cl}(B) \subseteq U$. Let U be a $(\text{gg})^*$ -open set in X such that $B \subseteq U$. Since $A \subseteq B$ and $B \subseteq U$, $A \subseteq U$. Also, since A is $\alpha(\text{gg})^*$ -closed, $\alpha\text{cl}(A) \subseteq U$. Now, $B \subseteq \alpha\text{cl}(A) \Rightarrow \alpha\text{cl}(B) \subseteq \alpha\text{cl}[\alpha\text{cl}(A)] = \alpha\text{cl}(A) \subseteq U$ [31]. That is, $\alpha\text{cl}(B) \subseteq U$. Therefore, B is a $\alpha(\text{gg})^*$ -closed set in X

Theorem 3.15. For every point x in a space X, $X - \{x\}$ is $\alpha(\text{gg})^*$ -closed or $(\text{gg})^*$ -open.

Proof.

Case (i): Suppose that $X - \{x\}$ is not $(\text{gg})^*$ -open. Then X is the only $(\text{gg})^*$ -open set containing $X - \{x\}$. Then by using Definition: 3.1, $\alpha\text{cl}(X - \{x\}) \subseteq X$. Hence $X - \{x\}$ is $\alpha(\text{gg})^*$ -closed.

Case (ii): Suppose that $X - \{x\}$ is not $\alpha(\text{gg})^*$ -closed. Then there exist a $(\text{gg})^*$ -open set U containing $X - \{x\}$ such that $\alpha\text{cl}(X - \{x\}) \not\subseteq U$. Therefore, $\alpha\text{cl}(X - \{x\})$ is either $X - \{x\}$ or X. If $\alpha\text{cl}(X - \{x\}) = X - \{x\}$, then $X - \{x\}$ is α -closed. By Theorem: 3.3(ii), every α -closed set is $\alpha(\text{gg})^*$ -closed, then $X - \{x\}$ is $\alpha(\text{gg})^*$ -closed. This is a contradiction to our assumption. Therefore, $\alpha\text{cl}(X - \{x\}) = X$. To prove that $X - \{x\}$ is $(\text{gg})^*$ -open. Suppose that $X - \{x\}$ is not $(\text{gg})^*$ -open. By case (i), $X - \{x\}$ is $\alpha(\text{gg})^*$ -closed. This is a contradiction to our assumption. Therefore, $X - \{x\}$ is $(\text{gg})^*$ -open.

Theorem 3.16. A subset A of a space X is $\alpha(\text{gg})^*$ -closed if and only if for each $A \subseteq N$ and N is $(\text{gg})^*$ -open, there exists a α -closed set M such that $A \subseteq M \subseteq N$.

Proof. Suppose A is a $\alpha(\text{gg})^*$ -closed set and $A \subseteq N$ and N is $(\text{gg})^*$ -open. Then $\alpha\text{cl}(A) \subseteq N$. If we put $M = \alpha\text{cl}(A)$, then M is α -closed set and $A \subseteq M \subseteq N$. Conversely, assume that N is a $(\text{gg})^*$ -open set containing A. Then there exists a α -closed set M such that $A \subseteq M \subseteq N$. Since $\alpha\text{cl}(A)$ is the smallest α -closed set containing A, we have $A \subseteq \alpha\text{cl}(A) \subseteq M$. Also, since $M \subseteq N$, $\alpha\text{cl}(A) \subseteq N$. Hence A is a $\alpha(\text{gg})^*$ -closed set in X.

Theorem 3.17. If A is α -closed and B is $\alpha(\text{gg})^*$ -closed subset of a space X then $A \cup B$ is $\alpha(\text{gg})^*$ -closed.

Proof. Let N be a $(\text{gg})^*$ -open set containing $A \cup B$. Then $A \subseteq N$ and $B \subseteq N$. Since B is $\alpha(\text{gg})^*$ -closed and $B \subseteq N$, we have $\alpha\text{cl}(B) \subseteq N$. Then $A \cup B \subseteq A \cup \alpha\text{cl}(B) \subseteq N$. Since A is α -closed, we have $A \cup \alpha\text{cl}(B)$ is α -closed. Hence there exist a α -closed set $A \cup \alpha\text{cl}(B)$ such that $A \cup B \subseteq A \cup \alpha\text{cl}(B) \subseteq N$. Therefore, by Theorem: 3.16, $A \cup B$ is $\alpha(\text{gg})^*$ -closed.

Theorem 3.18.

- (i) If A is closed and B is g^* -closed subset of a space X, then $A \cup B$ is $\alpha(gg)^*$ -closed.
- (ii) If A is closed and B is $(gs)^*$ -closed subset of a space X, then $A \cup B$ is $\alpha(gg)^*$ -closed.
- (iii) If A is closed and B is $(gsp)^*$ -closed subset of a space X, then $A \cup B$ is $\alpha(gg)^*$ -closed.

Proof.

- (i) Since every closed set is α -closed, A is α -closed and by Theorem: 3.3(v), B is $\alpha(gg)^*$ -closed. Therefore by Theorem: 3.17, $A \cup B$ is $\alpha(gg)^*$ -closed.
- (ii) Since every closed set is α -closed, A is α -closed and by Theorem: 3.3(vi), B is $\alpha(gg)^*$ -closed. Therefore by Theorem: 3.17, $A \cup B$ is $\alpha(gg)^*$ -closed.
- (iii) Since every closed set is α -closed, A is α -closed and by Theorem: 3.3(vii), B is $\alpha(gg)^*$ -closed. Therefore by Theorem: 3.17, $A \cup B$ is $\alpha(gg)^*$ -closed.

IV. CONCLUSION

In this paper, we focussed on alpha generalization of generalized star closed sets in topological spaces. Also, we established its relationships among other closed sets and examined some of its properties. In future, this idea can be extended to bitopological and tritopological spaces.

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