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Original Article

The Geodesic Polynomials in a Graph

Swamy[1], N. D. Soner[2], B. M. Chandrashekara[3]

[1,2] *Department of Studies in Mathematics, University of Mysore, Karnataka, India.*
[3] *Department of Mathematices, Dr. G. Shankar Govt. Women's First Grade College and P G Study Centre, Ajjarkadu, Udupi, India*

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Abstract - *A shortest path between two vertices in a graph G is a geodesic in G A graph polynomial is a graph invariant. There are many graph polynomials can be found in the literature including, the characteristic polynomial, the chromatic polynomial and so on. This paper aims at the study of two new graph polynomials associated with the geodesics in a graph introduced by R. Rajendra and P.S.K. Reddy, namely, the geodesic polynomial of a graph and the geodesic polynomial at a vertex in a graph. We obtain some results involving geodesic polynomials of graphs and geodesic polynomials at the vertices*

Keywords *- Graph, geodesic, polynomial.*

I. INTRODUCTION

For standard terminology and notion in graph theory, we follow the textbook of Harary [10]. The non-standard will be given in this paper as and when required.

Let $G = (V, E)$ be a graph (finite, simple, connected and undirected). The degree of a vertex v in G is denoted by $\deg(v)$. The number of edges in a path P is its length $l(P)$. A shortest path between two vertices u and w in G is called a $u-w$ geodesic. We say that a geodesic P is passing through a vertex v in G if v is an internal vertex of P. The length of a longest geodesic in G is called the diameter of G, denoted by $\mathrm{diam}(G)$. The eccentricity of a vertex v in G is the maximum distance between v and any other vertex in G. A vertex with maximum eccentricity in G is called a peripheral vertex in G. The vertices whose eccentricities are equal to diameter G are peripheral vertices of G. The set of all peripheral vertices of G is denoted by $PV(G)$. The distance between two vertices u and v in G, denoted by $d(u,v)$ is the number of edges in a shortest path (geodesic) connecting them. For two vertices u and v in G, $g(u,v)$ denotes the number of geodesics whose end vertices are u and v. The number of geodesics in G is denoted by f_G (or simply f), and f_i denotes the number of geodesics of length i in G. Clearly, $f_1 = |E|$ and for a graph with n vertices, we have,

$$f = \sum_{i=1}^{n-1} f_i = \sum_{i=1}^{d} f_i \qquad (1)$$

where d = diam(G).

For a vertex v in G, f_{iv} denotes the number of geodesics of length i having v as an end-vertex in G. The number of geodesics having v as an end-vertex in G is denoted by f_{Gv}. Clearly, $f_{1v} = \deg(v)$, $f_{iv} = 0$, $\forall\ i > e(v)$ and we have

$$f_{Gv} = \sum_{i=1}^{e(v)} f_{iv} \qquad (2)$$

We write $v_i \sim v_j$ to indicate that the vertices v_i and v_j are adjacent in a graph. The adjacency matrix of a graph G with vertices $v_1,\dots,v_n$ is the $n\times n$ matrix $A_G = (a_{ij})$, where

$$\boldsymbol{a}_{\boldsymbol{ij}}^{(\mathbf{1})} = \begin{cases} \mathbf{1}, \boldsymbol{if}\ vi\ \sim\ vj \\ \mathbf{0}, \text{otherwise} \end{cases}$$

Let G be a graph and v be a vertex in G. The stress of v, denoted by $\mathrm{str}_G(v)$ or simply $\mathrm{str}(v)$, is defined as the number of geodesics in G passing through v [10, A. Shimbel]. The total stress of G (See [8]), denoted by $N_{\mathrm{str}}(G)$, is defined as,

$$N_{str}(G) = \sum_{v \in V} Str(v) \tag{3}$$

The concept of stress of a vertex in a graph was defined by Alfonso Shimbel in 1953 [18]. The concept has many applications in the study of social and biological networks. Some related works are carried in [13], [14], [15] and [19].

The concept of Tension on an edge in a graph was introduced by Bhargava, Dattatreya and Rajendra [8]. The tension on an edge e in a graph G, denoted by $\tau_G(e)$ or simply $\tau(e)$, is defined as the number of geodesics in G passing through e. The total tension of G, denoted by $N_\tau(G)$, is defined as,

$$N_T(G) \sum_{e \in E} T(e) \tag{4}$$

In [14], Rajendra et al. introduced two new topological indices for graphs called the first stress index and the second stress index by means of the stresses of vertices. Further, they established some inequalities and compute both stress indices for some standard graphs.

Rajendra et al. [15] introduced another topological index for graphs called Peripheral Geodesic Index: the peripheral geodesic index $Pg(G)$ of a graph G is defined as the number of geodesics between peripheral vertices of G. They have computed Peripheral Geodesic Index for some standard graphs. Further, established formulae for computing the number of graph geodesics in a graph and the peripheral geodesic index using the adjacency matrix.

A graph polynomial is a graph invariant. There are many graph polynomials can be found in the literature including, the characteristic polynomial, the chromatic polynomial, the Martin polynomial, the matching polynomials, and the Tutte polynomial, etc.,. R. Rajendra and P.S.K. Reddy have defined two graph polynomials associated with the geodesics, namely, the geodesic polynomial of a graph and the geodesic polynomial at a vertex in a graph (see Definitions 2.2 and 4.2). In this paper, we obtain some results involving geodesic polynomials of graphs and geodesic polynomials at the vertices. All graphs considered in this paper are simple and connected. For more information about polynomial of graph and its application see [1-6-] and [11-12]

II. GEODESIC POLYNOMIAL OF A GRAPH

Definition 2.1 (R. Rajendra and P.S.K. Reddy)**.** The geodesic sequence of a graph G with n vertices is the sequence $f_1, f_2, \ldots, f_d$,

where f_i is the number of geodesics of length i in G, $1 \le i \le d$ and $d = \text{diam}(G)$.

Definition 2.2 (R. Rajendra and P.S.K. Reddy)**.** The geodesic polynomial of a graph G with n vertices, denoted by $g_G(x)$ is the generating function(polynomial) of the geodesic sequence $f_1, f_2, \ldots, f_d$ of G; i.e.,

$$g_G(x) = f_1 + f_2 x + f_3 x^2 + \ldots + f_d x^{d-1} = \sum_{i=1}^{d} f_i x^{i-1} \tag{5}$$

Observation:

1. For a graph G with n vertices, $g_G(x) \in \mathbb{Z}_{\geq 0}[x]$ and

   $$\deg(g_G(x)) = \text{diam}(G) - 1 \le n - 1,$$

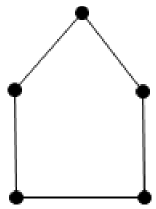
   where diam(G) is the diameter of the graph and $\mathbb{Z}_{\geq 0}$ is the set of nonnegative integers. Note that not all polynomials with coefficients in $\mathbb{Z}_{\geq 0}$ are geodesic polynomials of graphs; for instance, there is no graph with the polynomial $1 + x$ as its geodesic polynomial.

2. For a graph G with n vertices, $g_G(0) = f_1$ and $g_G(1) = f$.

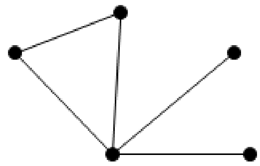
3. 3. The derivative of the geodesic polynomial (5) is

$$\bar{g}_G(x) = \sum_{i=1}^{d}(i-1)f_i x^{i-2} \tag{6}$$

If G_1 and G_2 are two isomorphic graphs, then $g_{G1}(x) = g_{G2}(x)$. The converse of this statement is not true. For example, consider the following graphs C_5 and G:



C_5



G

The geodesic polynomials of C_5 and G are

$$g_{c5}(x) = 5 + 5x \quad \text{and } g_G(\mathrm{x}) = 5 + 5x.$$

The graphs C_5 and G are not isomorphic but they have the same geodesic polynomial $5 + 5x$.

The following two propositions are immediate from the Definition 2.2 with simple computations

Proposition 2.3. For a graph G with n vertices,

(i) $g_G(x)$ is a constant polynomial if and only if diam(G) = 1;

(ii) $g_G(x)$ is a linear polynomial if and only if diam(G) = 2.

Proposition 2.4.

1. For the complete graph K_n with n vertices,

   $G_{Kn}(X) = f_1$ $\qquad$ $G_{Kn}(X) = \binom{n}{2}$

2. For the complete bipartite graph Km,n

$$gk_{m,n}(x) = f_1 + f_2 x \qquad i.e\ gk_{m,n}(x) = mn + \frac{1}{2}mn(m+n-2)x$$

a linear polynomial

3. For a path P_n with n vertices, deg($g_{Pn}(x)$) = $n - 2$ and

$$gp_n(x) = (n-1) + (n-2)x + (n-3)x^2 + \ldots + 2x^{n-3} + x^{n-2}$$

4. For a cycle C_n with n vertices,

$$deg(gc_n(x)) = \begin{cases} n + nx + nx^2 + \ldots + nx^{\frac{n}{2}-1}, & if\ n\ is\ even \\ n + nx + nx^2 + \ldots + nx^{\frac{n-1}{2}-1}, & if\ \ n\ is\ odd \end{cases}$$

We recall the following proposition from [1]:

Proposition 2.5. [1] For any graph G with diameter d, the total stress of G, is given by

$$N_{str}(G) = \sum_{i=1}^{d}(i-1)f_i \tag{7}$$

Proposition 2.6. For any graph G with diameter d, the total stress of G, is given by

$$N_{str}(G) = g_G^{/}(1) \tag{8}$$

Where $g_G^{/}(1)$ is the derivative of the geodesic polynomial $g_G(x)\ of\ G$

Proof : The derivative of the geodesic polynomial $g_G(x) = \sum_{i=1}^{d} f_i x^{i-2}$ of G is

$g'_G(x) = \sum_{i=1}^{d} (i-1) f_i x^{i-2}$

$$\Rightarrow g'_G(1) = \sum_{i=1}^{d} (i-1) f_i \tag{9}$$

Using (9) in (7) of Proposition 2.5, we have

$$N_{str}(G) = g'_G(1).$$

□

We recall the following proposition from [8]:

Proposition 2.7. For any graph *G* with *n* vertices and diameter *d*, the total tension of *G* is

$$N_T(G) = \sum_{i=1}^{d} i f_i \tag{10}$$

Proposition 2.8. For any graph *G* with diameter *d*, the total tension of *G* is given by

$$N_T(G) = g'_G(1) + g_G(1) = N_{str}(G) + f, \tag{11}$$

Where $g'_G(G)$ is the derivative of the geodesic polynomial $g_G(x)$ of G .

Proof. We have

$$x g'_G(x) + g_G(x) = \sum_{i=1}^{d} i f_i x^{i-1}$$

$$\Rightarrow\ g'_G(1) + g_G(1) = \sum_{i=1}^{d} i f_i \tag{12}$$

Using (12) in (10) of Proposition 2.7 and from (8) of Proposition 2.6, we have

$$N_T(G) = g'_G(1) + g_G(1) = N_{str}(G) + f$$

□

Theorem 2.9. A graph *G* is a path if and only if $g_G(0) = \deg(g_G(x)) + 1$, where $g_G(x)$ is the geodesic polynomial of *G*.

Proof. Suppose that *G* is a path on *n* vertices. Then

$$g_G(x) = (n-1) + (n-2)x + (n-3)x^2 + \cdots + 2x^{n-3} + x^{n-2}$$

And so $g_G(0) = n - 1 = \deg\big(g_G(x)\big) + 1$

Conversely, suppose that *G* is a graph satisfying

$$g_G(0) = \deg\big(g_G(x)\big) + 1 \tag{13}$$

Since $g_G(0) = f_i$. number of edges in *G* is $e = \deg(g_G(x)) + 1$, which implies that $\deg(g_G(x)) = e - 1$. Since $\deg(g_G(x)) = \mathrm{diam}(G) - 1$, it follows that, $\mathrm{diam}(G) = e$ and hence there is a path in *G* containing all the edges of *G*. The only graph with a path that contain all the edges of the graph is nothing but a path. Hence *G* is a path on $e + 1$ vertices. □

III. COMPUTATION OF GEODESIC POLYNOMIAL USING ADJACENCY MATRIX

Let G be a graph of diameter d with n vertices $v_1 v_2 v_3 \dots . v_n$. Let $A = \left(a_{ij}^{(1)}\right) be$ adjacency matrix of the graph G, where

$$a_{ij}^{(1)} = \begin{cases} 1, & if\ v_i \sim v_j \\ 0, & Otherwise \end{cases}$$

We consider the following powers of A: $A^2, \dots, A^d$, where d is the diameter of G. We denote the $(i,j) - the$ element of $A^t (2 \ll t \ll d), by \quad a_{ij}^{(t)}$, where

$$a_{ij}^{(t)} = \sum_{i=1}^{n} a_{ik}^{(t-1)} a_{kj}^{1}$$

We know that $a_{ij}^{(t)}$ is the number of paths between v_i and v_j of length t. Let g_{ij} be the first non-zero entry in the sequence $a_{ij}^{(1)}, a_{ij}^{(2)}, \dots, a_{ij}^{(d)}$.. Clearly, g_{ij} is the number of geodesics between v_i and v_j, i.e., $g(v_i, v_j) = g_{ij}$. Therefore, the number of geodesics in G is given by

$$f_G = \sum_{1 \le i \le n} g_{ij} \qquad (14)$$

Let us define $\varphi_{ij}^{(t)}, (1 \le t \le d-1)$ as follows:

$$\varphi_{ij}^{(t)} = \begin{cases} 1, & if\ a_{ij}^{(1)} = a_{ij}^{(2)} = \cdots = a_{ij}^{(t)} = 0; \\ 0, & otherwise \end{cases} \qquad (15)$$

Then the number of geodesics of length $l\ (1 \le l \le d)\ in\ G, given\ by$

$$f_1 = \sum_{1 \le i < j \le n} \varphi_{ij}^{(l-1)} a_{ij}^{l} \qquad (16)$$

Hence, the geodesic polynomial of G is

$$g_G(x) = \sum_{i=1}^{d} f_i x^{i-1} = \sum_{i=1}^{d} f_l x^{l-1}$$

$$= \sum_{l=1}^{d} \Big(\sum_{l \le i < j \le n} \varphi_{ij}^{(l-1)} a_{ij}^{(l)} \Big) x^{l-1} \qquad \big(Using(16)\big) \qquad (17)$$

Since $f_G = f_1 + f_2 + \cdots + f_d$ and $f_1 = |E(G)|$, using (16), we have

$$f_G = \sum_{l=1}^{d} \sum_{1 \le i < j \le n} \varphi_{ij}^{(l-1)} a_{ij}^{(l)} = |E(G)| + \sum_{l=2}^{d} \sum_{1 \le i < j \le n} \varphi_{ij}^{(l-1)} a_{ij}^{(l)} \qquad (18)$$

From (17), we have,

$$N_{str}(G) = g_G'(1) = \sum_{l=2}^{d} (l-1) \Big(\sum_{1 \le i < j \le n} \varphi_{ij}^{(l-1)} a_{ij}^{(l)} \quad Thus\ we\ have$$

IV. GEODESIC POLYNOMIAL AT A VERTEX

Definition 4.1 (R. Rajendra and P.S.K. Reddy)**.** Let G be a graph and v be a vertex in G. The geodesic sequence at the vertex v is the sequence

$$f_{iv}, f_{2v}, \dots, f_{e(v)v},$$

Where f_{iv} is the number of geodesics of length i having v as an end-vertex in G, $1 \leq i \leq e(v)$.

Definition 4.2 (R. Rajendra and P.S.K. Reddy)**.** The geodesic polynomial at a vertex v in a graph G, denoted by $g_{Gv}(x)$ is the generating function (polynomial) of the geodesic sequence $f_{1v}, f_{2v}, \dots, f_{ev(v)v,}$ at the vertex v; i.e.,

$$g_{Gv}(x) = f_{1v} + f_{2v}x + f_{3v}x^2 + \cdots + f_{e(v)v}x^{e(v)-1} = \sum_{i=1}^{e(v)} f_{iv}x^{i-1} \qquad (19)$$

Observation: For a vertex v in a graph G with n vertices,

1- $g_{Gv}(x)$ is a polynomial with non-negative integer coefficients;
2- $\deg\left(g_{Gv}(x)\right) = \mathrm{e(v)} - 1 \leq \mathrm{diam\ (G)} - 1 = \deg\left(g_G(\mathrm{x})\right) \leq \mathrm{n} - 1$;
3- $g_{Gv}(0) = f_{1v} = \deg(v)\ and\ g_{Gv}(1) = \sum_{i=1}^{e(v)} f_{vi} = f_{Gv}$;
4- $if\ v \in PV(G), then\ \deg\left(g_{Gv}(x)\right) = \deg\left(g_G(x)\right)$;
5- $if\ \mu\ is\ an\ automorphism\ of\ G, then\ g_{Gv}(x) = g_{G\mu(v)}(x)$.

Then following proposition in immediate from the Definition 4.2 with simple computation:

Proposition 4.3

1. For any vertex v in the complete graph k_n with $n \geq 2$

Vertices, $g_{k_nv}(x) = n - 1$, a constant polynomial.

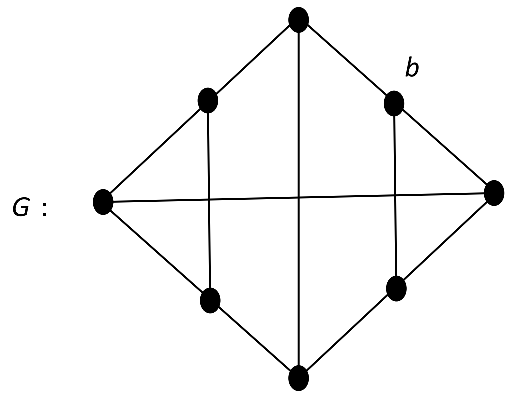
2. Let $\{u_1, \dots, u_n\}$ and $\{v_1, \dots, v_n\}$ be the partite sets of vertices of the complete bipartite graphs K_{mn} Then $g_{k_{mn}v_i}(x) = n + n(m-1)x \quad and\ g_{k_{mn}v_i}(x) = m + m(n-1)x$.
3. For any vertex v in the cycle C_n with $n \geq 3$ vertices,

$$g_{k_nv}(x) = \begin{cases} 2 + 2x + 2x^2 + \cdots + 2x^{\frac{n}{2}-1}\ , if\ n\ is\ even \\ 2 + 2x + 2x^2 + \cdots + 2x^{\frac{n-1}{2}-1.}\ \ if\ n\ is\ odd \end{cases}$$

Proposition 4.4. If the geodesic polynomials at all vertices in a graph G are all same, then G is regular.

Proof. Suppose that the geodesic polynomials at all vertices in a graph G are all same. Let u and v be two vertices in G. Then $g_{Gu}(x) = g_{Gv}(x)$, which implies $g_{Gu}(0) = g_{Gv}(0)\ \ i.e., \deg(u) = \deg\ (v)$ since $u\ and\ v\ are\ arbitrary\ vertices$ in G, it follows that, all vertices in G are of some degree and so G is regular.

Remark 4.5. The converse of the Proposition 4.4 is not true. For instance, consider the following 3-regular graph G where the geodesic polynomials at the indicated vertices a and b are not the same.



In G, $e(a) = 2.$ and $e(b) = 3$ Therefore, the geodesic polynomials at a and b are of degree 1 and 2, respectively. Hence the geodesic polynomials at a and b are different.

Proposition 4.6. The sum of all geodesic polynomials at the vertices in a graph $G$is equal to 2 times the geodesic polynomial of Gi.e.,

$$\sum_{v\in G} g_{Gv}(x) = 2 \cdot g_G(x)$$

Proof. Let G be a graph with n vertices $v_1, v_2, \dots, v_n$. We have

$$\sum_{v\in G} g_{Gv}(x) = \sum_{j=1}^{n}\sum_{i=1}^{e(v_j)} f_{ivj}x^{i-1}$$

$$= \textstyle\sum_{j=1}^{n}[f_{1vj} + f_{2vj}x + f_{3vj}x^2 + \cdots + f_{e(vj)vj}x^{e(vj)-1}]$$

$$=(\textstyle\sum_{j=1}^{n} f_{ivj}) + (\sum_{j=1}^{n} f_{2vj})x + (\sum_{j=1}^{n} f_{3vj})x^2 + \cdots$$

$$=2 \cdot f_1 + 2 \cdot f_2x + 2 \cdot 2f_3x^2 + \cdots + 2 \cdot f_dx^{d-1}, \text{ where } d = diam(G)$$

$$=2 \cdot (f_1 + f_2x + f_3x^2 + \cdots + f_dx^{d-1})$$

$$= 2 \cdot g_G(x)$$

Proposition 4.7. Let G be a graph with at least 2 vertices and v be a vertex in G. Then $g_{Gv}(x)$ is a constant polynomial if and only if v is adjacent to all other vertices in G.

Proof. Suppose that $g_{Gv}(x)$ is a constant polynomial. If possible, let $v$is not adjacent to a vertex$u$. Since $G$is connected, there is a geodesic of length ≥ 2 having end-vertices $u$and$v$. Therefore f_{2v} $6 = 0$ and hence $g_{Gv}(x)$ is not constant, a contradiction. Thus, v is adjacent to all other vertices in$G$.

Conversely, suppose that v is adjacent to all other vertices in G. Then there is no geodesic of length ≥ 2 having an end-vertex $v$and so $fiv = 0$, for all$i \geq 2$. Consequently, $gGv(x) = f_{i1}$ = deg$v$, a constant polynomial.
☐

Corollary 4.8. Let G be a graph with at least 2 vertices and $v$be a vertex in$G$. Then $g_{Gv}(x)$ is a constant polynomial if and only if v is a canter in G and diam$(G) = 1$ or 2.

Proof. We have, v is a canter in $G$and diam$(G) = 1$ or 2 if and only if v is adjacent to all other vertices in G. Hence, by Proposition 4.7, the result follows. ☐

From Corollary 4.8, the following results are immediate:

Corollary 4.9. Let G be a graph with at least 2 vertices and v be a vertex in G. Then $g_{Gv}(x)$ is a constant polynomial if and only if v is a complete graph or a wind-mill graph.

Corollary 4.10. Let G be a graph with at least 2 vertices and v be a vertex in G. If $g_{Gv}(x)$ is a constant polynomial, then

(i) for any vertex u in G, $g_{Gv}(x)$ is either a constant or a linear polynomial; and

(ii) the geodesic polynomial $g_{Gv}(x)$ of G is either a constant or a linear polynomial.

V. CONCLUSION

We obtain some results involving geodesic polynomials of graphs and geodesic polynomials at the vertices.

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