International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373 / https://doi.org/10.14445/22315373/IJMTT-V68I3P514

Volume 68 Issue 3, 90-93, March 2022

Binary Contra Continuous and Binary Contra Semi Continuous Functions in Binary Topological Spaces

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Received Date: 14 February 2022
Revised Date: 02 April 2022
Accepted Date: 09 April 2022

Abstract *- The authors [6] introduced the concept of binary topology between two sets and investigate its basic properties where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. in this paper we introduce and study binary contra continuous functions and binary contra semi continuous functions in binary topological spaces.*

Keywords *- Binary topology, Binary contra continuous function, Binary contra semi continuous function.*

I. INTRODUCTION

Levine[5] introduced semi open and semi continuous functions in topological spaces.The authors [6] introduced the concept of binary topology and discussed some of its basic properties.In1980, Jain introduced totally continuous functions. in 1995, T.M. Nour introduced the concept of totally semi-continuous functions as a generalization of totally continuous functions. Dontchev introduced the notion of contra continuity in topological spaces in 1996. A new weaker form of this class of functions is called contra semi-continuous function which is introduced and investigated by Dontchev and Noiri. The purpose of this paper is to introduce binary contra continuous functions and binary contra semi continuous functions in binary topological spaces. Section 2 deals with basic concepts. Binary contra continuous functions and binary contra semi continuous functions in binary topological spaces are discussed in section 3. Throughout the paper, $\wp(X)$ denotes the power set of X.

II. PRELIMINARIES

Let X and Y be any two nonempty sets. A binary topology [6] from X to Y is a binary structure $M \subseteq \wp(X) \times \wp(Y)$ that satisfies the axioms namely (i) $(\varnothing,\varnothing)$ and $(X,Y) \in M$,(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in M$ whenever $(A_1, B_1) \in M$ and $(A_2, B_2) \in M$, and (iii) If $\{(A_\alpha,B_\alpha):\alpha\in\Delta\}$ is a family of members of M , then $\left(\bigcup_{\alpha\in\Delta} A_\alpha, \bigcup_{\alpha\in\Delta} B_\alpha\right) \in M$. If M is a binary topology from X to Y then the triplet (X, Y, M) is called a binary topological space and the members of M are called the binary open subsets of the binary topological space (X,Y, M). The elements of $X\times Y$ are called the binary points of the binary topological space (X,Y, M). If Y=X then M is called a binary topology on X in which case we write (X, M) as a binary topological space. The examples of binary topological spaces are given in [6].

Definition 2.1.[6]Let X and Y be any two nonempty sets and let (A,B) and $(C,D) \in \wp(X)\times\wp(Y)$. We say that $(A,B)\subseteq(C,D)$ if $A \subseteq C$ and $B \subseteq D$.

Definition 2.2.[6] Let (X,Y, M) be a binary topological space and $A\subseteq X$, $B \subseteq Y$. Then (A,B) is called binary closed in (X,Y, M) if $(X\backslash A, Y\backslash B) \in M$.

Definition 2.3. Let X and Y be any two nonempty sets and let (A,B) and $(C,D) \in \wp(X)\times\wp(Y)$. We say that $(A,B)\not\subset (C,D)$ if one of the following holds :

(i)$A\subseteq C$ and $B\not\subset D$ (ii) $A \not\subset C$ and $B \subseteq D$ (iii) $A \not\subset C$ and $B\not\subset D$.

Definition 2.4.[6] Let (X,Y,M) be a binary topological space and let (Z,τ) be a topological space. Let $f: Z\rightarrow X\times Y$ be a function. Then f is called binary continuous if $f^{-1}(A, B)$ is open in Z for every binary open set (A,B) in (X,Y,M).

Definition 2.5.[5] A subset A of a topological space X is said to be semi open if there exists an open set U such that $U \subseteq A \subseteq ClU$ or equivalently $A \subseteq Cl(IntA)$.

Definition 2.6.[1] The complement of a semi open set is called semi closed.

Definition 2.7.[5] Let $f: X \to X*$ be a function where X and X∗ are topological spaces. Then f is said to be semi continuous if $f^{-1}(U)$ is semi open in X for every open set U in X∗.

Definition2.8. A subset A of a topological space X is said to be clopen if it is both open and closed.

Definition2.9.[2] Let $f: X \to X*$ be a function where X and X∗ are topological spaces. Then f is said to be contra continuous if $f^{-1}(U)$ is closed in X for every open set U in X∗.

Definition2.10.[3] Let $f: X \to X*$ be a function where X and X∗ are topological spaces. Then f is said to be contra semi continuous if $f^{-1}(U)$ is semi closed in X for every open set U in X∗.

Definition 2.11.[7] Let (Z,τ) be a topological space and (X,Y,M) be a binary topological space. Then the map $f: Z \to X \times Y$ is called totally binary continuous if $f^{-1}(A,B)$ is clopen in Z for every binary open set (A,B) in (X,Y,M).

Definition 2.12.[7] Let (Z,τ) be a topological space and (X,Y,M) be a binary topological space. Then the map $f: Z \to X \times Y$ is called totally binary semi continuous if $f^{-1}(A,B)$ is semi clopen in Z for every binary open set (A,B) in (X,Y,M).

Definition 2.13.[7] Let (Z,τ) be a topological space and (X,Y,M) be a binary topological space. Then the map $f: Z \to X \times Y$ is called strongly binary continuous if $f^{-1}(A,B)$ is clopen in Z for every binary set (A,B) in (X,Y,M).

Definition 2.14.[7] Let (Z,τ) be a topological space and (X,Y,M) be a binary topological space. Then the map $f: Z \to X \times Y$ is called strongly binary semi continuous if $f^{-1}(A,B)$ is semi clopen in Z for every binary set (A,B) in (X,Y,M).

Now, we define binary contra continuous and binary contra semi continuous functions in binary topological spaces by using closed sets and semi closed sets in topological spaces. Further we establish the relationship between these functions with varies types of binary continuous functions.

III. BINARY CONTRA CONTINUOUS FUNCTIONS

In this section, we introduce binary contra continuous and binary contra semi continuous functions and study some of their basic properties.

Definition 3.1. Let (Z,τ) be a topological space and (X,Y,M) be a binary topological space. Then the map $f: Z \to X \times Y$ is called binary contra continuous if $f^{-1}(A,B)$ is closed in Z for every binary open set (A,B) in (X,Y,M).

Definition 3.2. Let (Z,τ) be a topological space and (X,Y, M) be a binary topological space. Then the map $f: Z \to X \times Y$ is called binary contra semi continuous if $f^{-1}(A,B)$ is semi closed in Z for every binary open set (A,B) in (X,Y, M).

Example 3.3. Consider $Z=\{a,b,c\}$, $X=\{x_1,x_2\}$ and $Y=\{y_1,y_2\}$. Let $\tau=\{\emptyset,Z,\{b,c\}\}$ and $M=\{(\emptyset,\emptyset),(X,Y),(\{x_1\},\{y_1\})\}$. Clearly τ is a topology on Z and M is a binary topology from X to Y. Define $f: Z \to X \times Y$ by $f(a)=(x_1,y_1)$, $f(b)=(x_2,y_2)$ and $f(c)=(x_1,y_2)$. The closed sets in Z are $\emptyset$, Z, $\{a\}$.We shall find the inverse image of every binary open sets in (X,Y,M).

Now, $f^{-1}(\emptyset,\emptyset)=\{z\in Z: f(z)\in(\emptyset,\emptyset)\}=\emptyset$, $f^{-1}(X,Y)=\{a,b,c\}=Z$ and $f^{-1}(\{x_1\},\{y_1\})=\{z\in Z: f(z)\in(\{x_1\},\{y_1\})\}=\{a\}$. This shows that the inverse image of every binary open set in (X,Y,M) is closed in Z. Hence f is binary contra continuous.

The proof of the following Proposition is straight forward.

Proposition 3.4. Every binary contra continuous function is binary contra semi continuous.

Proof. Let (A,B) be a binary open set in (X,Y,M). Since f is binary contra continuous, we have $f^{-1}(A, B)$ is closed in Z. We know that every closed set is semi closed. Hence $f^{-1}(A,B)$ is semi closed in Z. Thus f is binary contra semi continuous.

The converse of Proposition 3.4 need not be true which is shown in the following example.

Example 3.5. Consider Z={a,b,c}, X={x_1,x_2} and Y={y_1,y_2}. Let τ ={∅, Z ,{b},{a,b}} and M = {(∅,∅),(X,Y),({x_1},{y_1})}.Clearly τ is a topology on Z and M is a binary topology from X toY. The closed sets in Z are ∅,Z,{c},{a,c}.The semi open sets in Z are ∅,Z,{b},{a,b},{b,c}. Hence, the semi closed sets in Z are ∅, Z,{a,c},{c},{a}. Define f:Z→ X×Y by f(a)=(x_1,y_1) and f(b) = (x_2,y_2) = f(c).We shall find the inverse image of every binary open sets in (X,Y,M). Now, f^{-1}(∅,∅)={z∈Z: f(z)∈(∅,∅)}=∅, f^{-1}(X,Y) = {a,b,c}= Z and f^{-1}({ x_1},{y_1}) ={a}.This shows that f is binary contra semi continuous, since f $^{-1}$({ x_1},{y_1})= {a} which is semi closed in Z. But f is not binary contra continuous, since f $^{-1}$({ x_1},{y_1})= {a} which is not closed in Z.

It is evident that every totally binary continuous function is binary contra continuous. But the converse need not be true as can be seen from the following example.

Example 3.6. Consider Z={a,b,c}, X={x_1 ,x_2} and Y={y_1 ,y_2}. Let τ ={∅, Z ,{a},{b},{a,b}} and

M ={(∅,∅), (X,Y), ({x_1},{y_2})}. Clearly τ is a topology on Z and M is a binary topology from X to Y. The closed sets in Z are ∅, Z,{c},{a,c},{b,c}. Define f: Z→ X×Y by f(a) = (x_1,y_1) and f(b)= (x_1,y_2) = f(c). Clearly f is binary contra continuous but not totally binary continuous.

for, f^{-1}(∅,∅) ={z∈Z:f(z)∈(∅,∅)}= ∅, f^{-1}(X,Y) = {a,b,c}=Z and f^{-1}({ x_1},{y_2})={b,c} which is closed in Z but not open in Z.

It is obvious that every totally binary semi continuous function is binary contra semi continuous. But the converse need not be true which is shown in the following example.

Example 3.7. Consider Z={a,b,c}, X={x_1,x_2} and Y={y_1,y_2}. Let τ={∅,Z,{b,c}} and M ={(∅, ∅),(X,Y),({x_1},{y_1})}. Clearly τ is a topology on Z and M is a binary topology from X to Y. Define f: Z→ X×Y by f(a) =(x_1,y_1) , f(b)= (x_2,y_2) and f(c)= (x_1,y_2). The closed sets in Z are ∅, Z, {a}. The semi open sets in Z are ∅, Z,{b,c}. Hence the semi closed sets in Z are ∅, Z,{a}.We shall find the inverse image of every binary open sets in (X,Y,M). Now, f^{-1}(∅,∅) ={z∈Z:f(z)∈ (∅,∅)}= ∅, f^{-1}(X,Y) ={a,b,c}= Z and f^{-1}({x_1},{y_1}) = {z∈Z:f(z)∈ ({x_1},{y_1})}={a}.This shows that the inverse image of every binary open set in (X,Y,M) is semi closed in Z. Hence f is binary contra semi continuous. But{a} is not semi open in Z. Therefore,{a} is not semi clopen in Z. Thus f is not totally binary semi continuous.

We observe that every strongly binary continuous function is binary contra continuous. But the converse need not be true. This can be seen from the following example.

Example 3.8 Consider Z={a,b,c}, X={x_1 ,x_2} and Y={y_1 ,y_2}. Let τ ={∅, Z ,{a},{b},{a,b}} and M ={(∅,∅),(X,Y),({x_1},{y_2})}. Clearly τ is a topology on Z and M is a binary topology from X to Y. The closed sets in Z are ∅, Z ,{c},{a,c},{b,c}. Hence the clopen sets in Z are ∅, Z . Define f:Z→X×Y by f(a)=(x_1,y_1) and f(b)= (x_1,y_2)= f(c).Then f is not strongly binary continuous. We shall find the inverse image of every binary open sets in (X,Y,M). f^{-1}(∅,∅) ={z∈Z :f(z)∈ (∅,∅)}=∅, f^{-1}(∅,{y_1})= ∅,f^{-1} (∅,{y_2})= ∅, f^{-1} (∅,Y) = ∅, f^{-1}({x_1},∅) = ∅, f^{-1}({x_1}{y_1}) ={a}, f^{-1}({x_1 },{y_2 }) ={b,c}, f^{-1} ({x_1},Y) = Z, f^{-1}({x_2},∅) = ∅, f^{-1} ({x_2}{y_1}) = ∅, f^{-1}({x_2},{y_2}= ∅, f^{-1}({x_2},Y) = ∅, f^{-1} (X,∅) = ∅, f^{-1} (X,{y_1})={a}, f^{-1} (X,{y_2}={b,c}, f^{-1} (X,Y) = Z. This gives that f is not strongly binary continuous, since {b,c} and {a}are not clopen in Z . But the inverse image of every binary open set in (X,Y,M) is closed in Z. Hence f is binary contra continuous.

Also, we observe that every strongly binary semi continuous function is binary contra semi I continuous. But the converse need not be true as can be seen from the following example.

Example 3.9 Consider Z={a,b,c}, X={x_1 ,x_2} and Y={y_1 ,y_2}. Let τ ={∅, Z ,{a},{b},{a,b}} and M ={(∅,∅),(X,Y),({x_1},{y_2})}. Clearly τ is a topology on Z and M is a binary topology from X to Y. The closed sets in Z are ∅, Z ,{c},{a,c},{b ,c}. Hence the clopen sets in Z are ∅, Z . Now the semi open sets in Z are ∅,Z,{a},{b},{a,b},{a,c},{b,c}. Hence the semi closed sets in Z are ∅,Z,{a},{b},{c},{a,c}and{b,c}. Thus the semi clopen sets in Z are ∅, Z,{a},{b},{a,c},{b,c}. Define f: Z→ X×Y by f(a) =(x_1,y_1), f(b)=(x_1,y_2) and f(c)=(x_2,y_1). We shall find the inverse image of every binary open sets in (X,Y,M). Now, f^{-1}(∅,∅) ={z∈Z: f(z)∈(∅,∅)} = ∅, f^{-1} (X,Y) = Z. f^{-1}({x_1 },{y_2 }) ={b,c}.This shows that f is binary contra semi continuous. Now, f^{-1}(∅,{y_1}) = ∅, f^{-1} (∅, {y_2}) = ∅, f^{-1} (∅,Y) = ∅, f^{-1}({x_1}, ∅) = ∅, f^{-1}({x_1},{y_1}) ={a}, f^{-1}({x_1 },{y_2}) = {b, c}, f^{-1}({x_1},Y) = Z, f^{-1}({x_2},∅) = ∅, f^{-1}({x_2},{y_1})= {c}, f^{-1}({x_2},{y_2})= ∅, f^{-1}({x_2}, Y)= {c}, f^{-1} (X , ∅) = ∅, f^{-1} (X,{y_1})={a}, f^{-1} (X,{y_2}={b,c}, f^{-1} (X,Y)=Z. This gives that f is not strongly binary semi continuous, since f^{-1} ({x_2},{y_1})={c} which is not semi clopen in Z.

From the above discussion with [7], we have the following,

$$
\begin{array}{ccccc}
\text{strongly binary continuous} & \Rightarrow & \text{totally binary continuous} & \Rightarrow & \text{binary contra continuous.} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\text{strongly binary semi continuous} & \Rightarrow & \text{totally binary semi continuous} & \Rightarrow & \text{binary contra semi continuous .}
\end{array}
$$

IV. CONCLUSION

Binary contra continuous, binary contra semi continuous functions are introduced and their relation between varies types of binary continuous functions are discussed.

REFERENCES

[1] S.G Crossley and S.K Hildebrand , Semi Closure ,Texas J.Sci. 22(1971) 99-112
[2] J.Dontchevi, Contra Continuous Functions and Strongly S-Closed Spaces, Internat.J.Math.and Math. Sci., 19(2) (1996) 303-310.
[3] J.Dontchev and T.Noiri, Contra Semi Continuous Functions, Math. Pannonica.,10 (1999) 159-168.
[4] Engelking, Generel Topology, Polish Scientificpublishers,Warszawa. (1977).
[5] Norman Levine, Semi-Open Sets and Semi-Continuity in Topological Spaces, Amer. Math.Monthly.70 (1963) 36-41.
[6] S. Nithyanantha Jothi and P.Thangavelu, Topology Between Two Sets, Journal of Mathematical Sciences & Computer Applications, 3 (2011), 95-107.
[7] S. Nithyanantha Jothi, Binary Semi-Continuous Functions, International Journal of Mathematics Trends and Technology. 49(2) (2017) 521-155
[8] S. Nithyanantha Jothi and P.Thangavelu , On Binary Topological Spaces, Pacific–Asian Journal of Mathematics, 5(2) (2011)133-138.
[9] S. Nithyanantha Jothi and P.Thangavelu ,On Binary Continuity and Binary Separation Axi Oms, Ultra Scientist . 24(1) (2012) 121-126
[10] S. Nithyanantha Jothi and P.Thangavelu, Binary-$T_{1/2}$ Spaces, Acta Cienca. 3 (2015),241-247.
[11] S. Nithyanantha Jothi and P.Thangavelu ,Generalized Binary Closed Sets in Binary Topological Spaces, Ultra Scientist of Physical Sciences, 26(1) (2014) 25-30.
[12] S. Nithyanantha Jothi, Generalized Binary Regular Closed Sets , IRA- International Journal of Applied Sciences ISSN 2455-4499, 4 (2016) 259-263.
[13] S. Nithyanantha Jothi, Binary Semi Open Sets in Binary Topologicalspaces, International Journal of Mathematical Archive, 7(9) (2016)73-76.
[14] T. Nieminen On Ultra pseudo compact and Related Spaces, Ann.Acad.Sci.Fenn.Ser.A.I.Math. 3(1977)185-205.
[15] Njastad.O. On Some Classes of Nearly Open Sets,Paccific.J.M.,15(1965) 961-970.
[16] Noiri.T., Supercontinuity and Some Strong Forms of Continuity, Indian J.Pure Appl.Math.,15(1984) 241-250.
[17] T.M.Nour,Totally Semi-Continuous Functions, Indian J.Pure Appl.Math.,26(7) (1995) 675-678
[18] Noll.D.,Baire Spaces and Graph Theorems,, Proc.Amer.Math.Soc.,18 (1986) 141-151.
[19] Pettis.B.J.,Closed Graph and Open Mapping Theorem in Certain Topologically Complete Spaces, Bull London Math.Soc.,6 (1974) 37-41.
[20] Popa.V and Noiri,T., on β -Continuous Functions, Real Anal.Exchange,18 (1992/1993) 544-548.
[21] Ptak.V Completeness and Open Mapping Theorem, Bull. Soc.Math.France,86 (1958) 41-74.
[22] Singal.M.K Singal.A.R , Almost-Continuous Mapping, Yokohama Math .J.,16 (1968) 63-73.
[23] Stone.A.H.,Absolute FG Spaces,,Proc.Amer.Math.Soc.,80 (1980) 515-520.
[24] Stone.A.H.,Hereditarily Compac T Spaces, Amer. J. Math.., 82 (1960) 900-916
[25] Thompson .T., S-Closed Spaces, Proc.Amer.Math.Soc.,60 (1976)335-338.