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Original Article

Few Closure Properties of Context-Free Multi-Strings Token Petri Net

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Abstract *- Multi-Strings Token Petri Net was introduced earlier. Context-Free Multi-Strings Token Petri Nets will be closed under union and concatenation.*

Keywords *- Multi-Strings Token Petri Net (MSTPN), Context-Free Language (CFL), Context-Free Grammar (CFG).*

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I. INTRODUCTION

The notion of Petri net has begun in Carl Adam Petri's dissertation which was presented in the year 1962 [1]. A Petri net can provide an elegant mathematical framework for modeling concurrent systems and their behavior [2]. Also, they have been used for the description and analysis of system of parallel process [3] to [6]. The more applicable version of Petri net called labeled Petri net was introduced and studied by many researchers [7] to [9]. In [10], Hack introduced Petri net languages generated by labeled transitions of Petri nets over some alphabets. Later, many researchers investigated the properties of various types of Petri net languages [11] to [16]. In [17], String-Token Petri Net was introduced and studied, where the transitions are labeled by the evaluation rules to generate the languages. Context-Free String-Token Petri Net and Parallel Context-Free String Token Petri Net can be found in [18] and [19]. Context-Free Multi-Strings Token Petri Net is introduced in [20].

Here, few closure properties of Multi-Strings Token Petri Net are discussed.

II. BASIC DEFINITIONS

Definition 2.1: Context-Free Grammar(CFG) and Context-Free Language(CFL) are defined in [21].

Definition 2.2: Evolution rules are found in [18].

Definition 2.3 A MSTPN is denoted as $N = (P, T, V, F, R(t), M_0)$ in which, P, T, V, F and R(t) are same as in [18]. $M_0: P \to x; \forall x \in V^*$ is the start marking and $T \cup P \neq \phi; T \cap P = \phi$.[20]

Definition 2.*4* Many systems' activity may be characterised in terms of the system's locations and modifications. A location or marking in an MSTPN is modified as per the underlying transition (firing) rules in order to imitate the dynamic response of the structure.

(i)When each input location(place) p of a transition t includes a collection of strings containing left side terms of the transition rules, the transition is said to be activated. For example, $t_i: A \to aAbc \,/\, B \to xDy \,/\, E \to aAx$, then input location p_j of t_i should contain strings $\{A, B, E\}$. Suppose input location p_k of t_m consists of $\{aAb, dB, aEb, abd\}$ then abd can be retained for the immediate firing of transition other than insertion rule, when insertion rule is applied, abd will be carried out to the next immediate location.

(ii) An active transition fires.

(iii) Suppose $t: X \to Y \,/\, A \to dB \,/\, D \to aXb$ and a collection of strings of input location p of t is $\{aXby, A, mDYa\}$, then t is activated as the leftmost non-terminal in each of the string is $\{aXby, A, mDYa\}$ appears in the left side expression of t. So, when t fires $\{aXby, A, mDYa\}$ will be removed from the input location of t and $\{aYby, dB, maXbYa\}$ will be deposited in the output location of t.

(iv) Identity rule like $abc \to abc$ can be used in the MSTPN as far as all strings in the corresponding location become terminals.[20]

III. CLOSURE PROPERTIES

Theorem 3.1 *The clan of Context-Free MSTPN languages is closed in respect of union.*

Proof. Let $N_1 = (P_1, T_1, V_1, F_1, R_1(t), M_1)$ be a MSTPN causes a CFL L_1 and $N_2 = (P_2, T_2, V_2, F_2, R_2(t), M_2)$ be a MSTPN causes a CFL L_2 . Now, a MSTPN $N = (P_1 \cup P_2 \cup \{P\}, T_1 \cup T_2 \cup \{t_\alpha, t_\beta\}, V_1 \cup V_2 \cup \{S, \varepsilon\}, F_1 \cup F_2 \cup \{\text{arcs from p to } t_\alpha, p \text{ to } t_\beta, t_\alpha \text{to initial location of } N_1,\ t_\beta \text{ to initial location of } N_2\}, R_1(t) \cup R_2(t) \cup \{t_\alpha: S \to \varepsilon, t_\beta: S \to \varepsilon\}, M_3)$ can be built to cause $L_1 \cup L_2$. Here, in this built process, a location p is built with the token S and connect p and N_1 by t_α, also connect p and N_2 by t_β. It is illustrated in Figure 1.

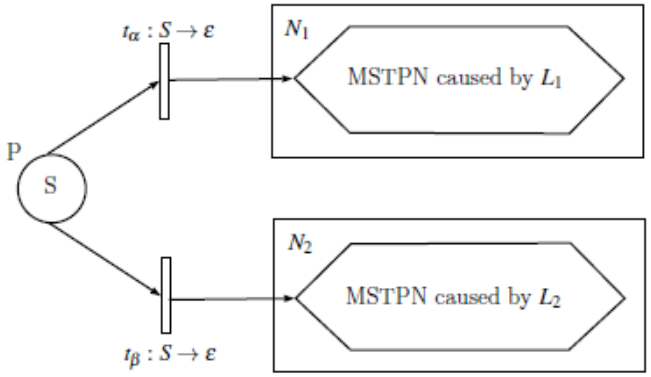

Figure 1. MSTPN caused by $L_1 \cup L_2$

The similar approach is applicable to any number of CFL. i.e., if $L_1, L_2, L_3, \ldots, L_n$ are CFL's, then $L_1 \cup L_2 \cup L_3 \cup \ldots \cup L_n$ is also a CFL, MSTPN for $L_1 \cup L_2 \cup L_3 \cup \ldots \cup L_n$ can be built in the same way to that of MSTPN N in Figure 1.

Thus, it is proved that, the clan of Context-Free MSTPN Languages is closed in respect of union.

Example 3.1 *A MSTPN causing the CFL L_1 is shown in Figure 2, where $L_1 = \{a^n b^n / n \geq 1\} \cup \{wcw^R / w \in (a, b)^*\} \cup \{a^i b^j / j > i\}$.*

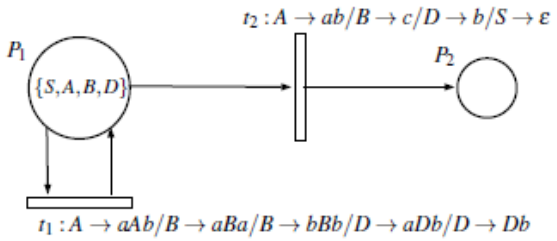

Fig. 2

A MSTPN causing Context-Free Language L_2 is illustrated in Figure 3, where $L_2 = \{\frac{w}{w} \in T^+$ consists of equal number of a′s and b′s$\}$.

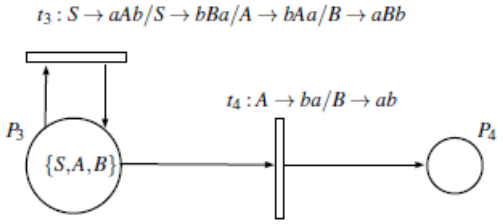

Fig. 3

From Figure 4, it can be seen that a MSTPN N_3 causes only $L_1 \cup L_2$.

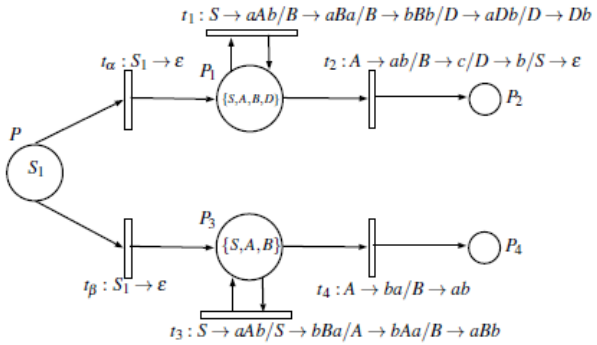

Fig. 4

Thus, it is shown that $L(N_3) = L(N_1) \cup L(N_2)$. Now, it has been verified that Context-Free Language $L(N_3)$ which is caused by MSTPN N_1 and N_2 causes $L_1 \cup L_2$.

Theorem 3.2 *The clan of Context-Free MSTPN languages is closed in respect of concatenation.*

Proof. Let $N_1 = (P_1, T_1, V_1, F_1, R_1(t), M_1)$ be a MSTPN causes a CFL L_1 and $N_2 = (P_2, T_2, V_2, F_2, R_2(t), M_2)$ be a MSTPN causes a CFL L_2 . Now, a MSTPN $N = (P_1 \cup P_2, T_1 \cup T_2 \cup \{t_\alpha\}, V_1 \cup V_2 \cup \{\varepsilon\}, F_1 \cup F_2 \cup$ {a location with terminal strings of N_1 to t_α, t_α to the initial location of $N_2\}, R_1(t) \cup R_2(t) \cup \{t_\alpha : \varepsilon \to \varepsilon\}, M_4)$ can be built to cause $L_2.L_1$. It is illustrated in Figure 5.

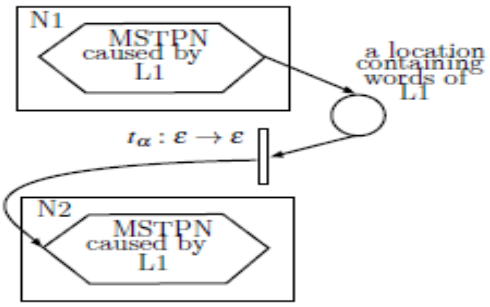

Fig. 5 MSTPN causing $L_2.L_1$

It can be seen that from the location with terminal strings of L_1, connect this location with the initial location (a location with tokens) of N_2 by a new transition $t_\alpha: \varepsilon \to \varepsilon$. When t_α fires, along with εL_1, initial tokens of N_2 will be deposited on the initial location of N_2 using leftmost insertion. After all the sequence of firings of transitions of N_2, $L_2.L_1$ will be obtained on the location with only terminal strings of N_2.

Similarly, $L_1.L_2$ is obtained by taking N_2 first and then N_1. Also, it can be seen that $L_1.L_2 \neq L_2.L_1$.

The similar approach is applicable to any number of CFL. i.e., if $L_1, L_2, \dots, L_n$ are CFL, MSTPN for $L_1.L_2.\dots.L_n$ can be built in the same way to that of MSTPN N in Figure 5.

Thus, it is proved that, the clan of Context-Free MSTPN Languages is closed in respect of concatenation.

Example 3.2 *Consider a MSTPN causing the Context-Free Language L_1 as given in Example 3.1. Also, Consider a MSTPN causing the Context-Free Language L_2 as given in Example 3.1.*

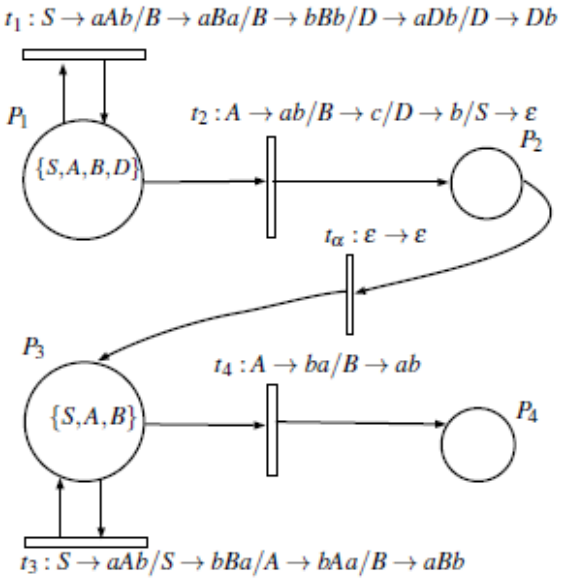

Fig. 6 MSTPN causing $L_2.L_1$

Now, it can be seen that, from Figure 6, Context-Free Languages which are caused by MSTPN's L_1 and L_2 is closed in respect of concatenation. That is, MSTPN N_4 causes $L_2.L_1$. In Figure 6, $t_\alpha: \varepsilon \to \varepsilon$ is taken as a transition between P_2 and P_3.

IV. CONCLUSION

Thus, every Context-Free Multi-Strings Token Petri Net is closed in respect of union and concatenation.

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