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Original Article

Split Domination Number in Vertex Semi-Middle Graph

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Abstract *- Let $G(p,q)$ be a connected graph and $M_v(G)$ be its corresponding vertex semi middle graph. A dominating set $D \subseteq V[M_v(G)]$ is split dominating set $\langle V[M_v(G)] - D\rangle$ is disconnected. The minimum size of D is called the split domination number of $M_v(G)$ and is denoted by $\gamma_s[M_v(G)]$. In this paper we obtain several results on split domination number.*

Keywords **-** *Domination number, Split domination number, Vertex semi-middle graph.*

I. INTRODUCTION

Domination is an area in graph theory with an extensive research activity. We consider simple, finite, undirected, non-trivial and connected graphs for our study. In literature, the concept of graph theory terminology not presented here can be found in Harary [1]. In a graph G, a set $D \subseteq V$ is dominating set of G if every vertex in $V - D$ is adjacent to some vertex in D. The domination number of a graph G is the minimum size of D. Some studies on domination and other graph valued functions in graph theory were studied in [2, 3, 4, 5]. The vertex semi middle graph $M_v(G)$ of a graph G was studied in [19] and is defined as follows. The vertex semi-middle graph of a graph G, denoted by $M_v(G)$ is a planar graph whose vertex set is $V(G) \cup E(G) \cup R(G)$ and two vertices of $M_v(G)$ are adjacent if and only if they corresponds to two adjacent edges of G or one corresponds to a vertex and other to an edge incident with it or one corresponds to a vertex other to a region in which vertex lies on the region. Let $R' = \{r'_1, r'_2, \dots r'_m\} \subseteq V[M_v(G)]$ for the region set $\{r_1, r_2, \dots r_m\}$ of G. Let $V' = \{v'_1, v'_2, \dots v'_p\} \subseteq V[M_v(G)]$ for the vertex set $\{v_1, v_2, \dots v_p\}$ of G. Let $E' = \{e'_1, e'_2, \dots e'_q\} \subseteq V[M_v(G)]$ for the edge set $\{e_1, e_2, \dots e_q\}$ of G such that $V[M_v(G)] = V' \cup E' \cup R'$. The study of domination number of jump graph [12] motivated us to introduce split domination number in vertex semi middle graph.

II. PRELIMINARIES

Theorem 2.1. [20] For the path $P_n, \gamma[M_v(P_n)] = \gamma[L(P_n)] + 1$.

Theorem 2.2. [20] For the cycle $C_n, n \geq 4$,

$$\gamma[M_v(C_n)] = \begin{cases} \frac{n}{3} + 2 & if\ n = 3k, k \geq 2. \\ \left\lceil \frac{n}{3} + 1 \right\rceil & if\ n = 3k+1\ or\ n = 3k+2, k \geq 1. \end{cases}$$

Theorem 2.3. [20] For any graph G, $\gamma[M_v(G)] \geq \left\lceil \frac{P}{1+\Delta(G)} \right\rceil$.

III. SPLIT DOMINATION NUMBER IN VERTEX SEMI-MIDDLE GRAPH

A dominating set D of $M_v(G)$ is a split dominating set if $\langle V[M_v(G)] - D\rangle$ is disconnected(connected). The minimum cardinality of D is called split domination number of $M_v(G)$ and is denoted by $\gamma_s[M_v(G)]$. A minimum split dominating set is denoted by $\gamma_s - set$.

In the Figure 3.1, the split dominating set of $M_v(G)$ is D = {1, 3, r_1', r_2'}, $\gamma_s[M_v(G)] = 3$.

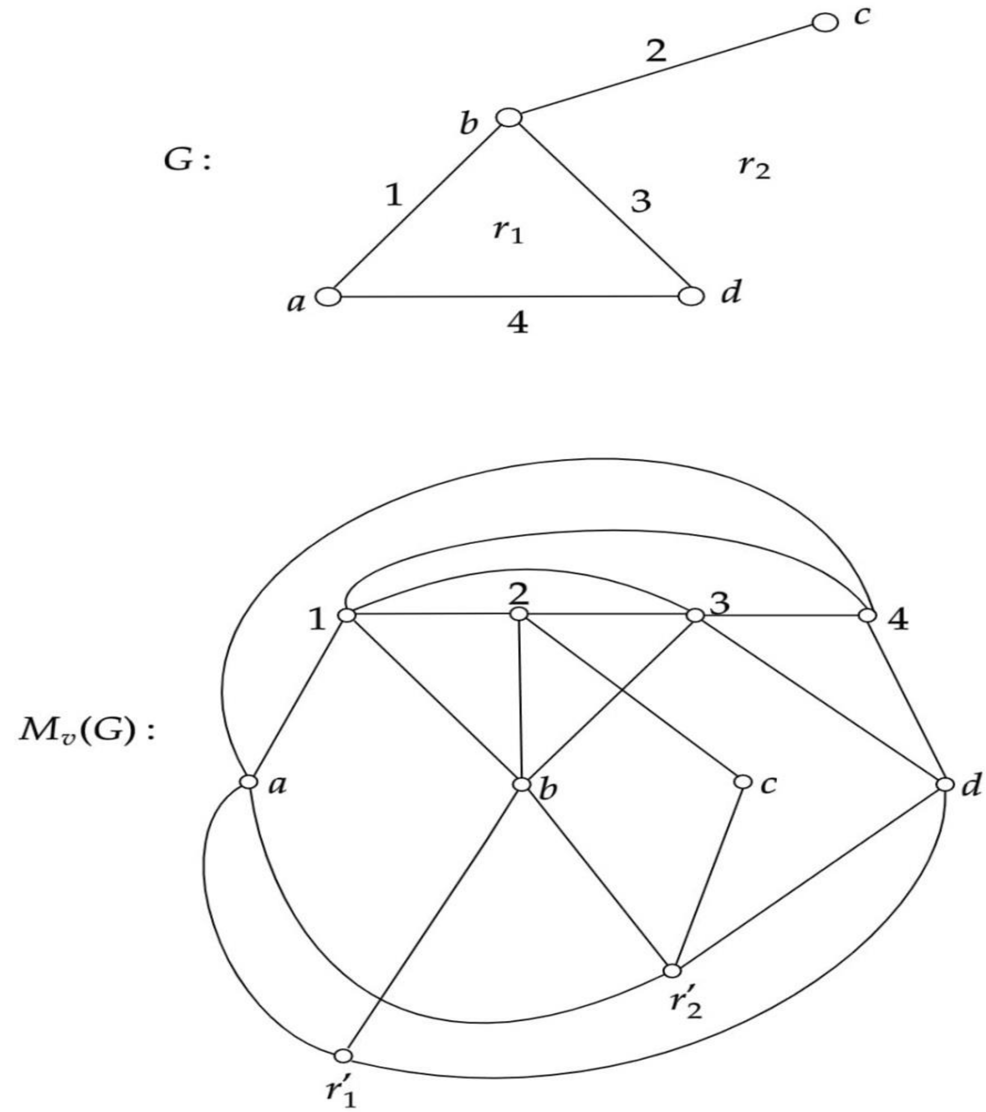

Fig. 3.1: The Graph G and its $M_v(G)$

We begin with some observations.

Observation 3.1. For every star $K_{1,n}, \gamma_s[M_v(K_{1,n})] = 2$.

Observation 3.2. For any path $P_n, \gamma_s[M_v(P_n)] = \gamma[M_v(P_n)]$.

Observation 3.3. For the cycle $C_3, \gamma_s[M_v(C_3)] = 4$.

IV. MAIN RESULTS

Theorem 4.1. For the cycle C_n , $n \geq 4$,

$$\gamma_s[M_v(C_n)] = \begin{cases} \frac{n}{3} + 2 & if\ n = 3k, k \geq 2. \\ \left\lceil \frac{n}{3} + 2 \right\rceil & if\ n = 3k + 1\ or\ n = 3k + 2, k \geq 1. \end{cases}$$

Proof. Consider $G = C_n$ and $V(C_n) = \{v_1, v_2, \ldots \ldots v_n\}$ for n ≥ 4 . Assume D denote the dominating set of $M_v(C_n)$, defined as follows.

$$D = \begin{cases} r'_1, r'_2, e'_1, e'_4, \ldots \ldots \ldots e'_{n-2} & if\ n = 3k, k \geq 2. \\ v'_1, r'_1, e'_3, e'_6, \ldots \ldots \ldots e'_{n-1} & if\ n = 3k + 1, k \geq 1. \\ v'_1, r'_1, e'_3, e'_6, \ldots \ldots \ldots e'_{n-2} & if\ n = 3k + 2, k \geq 1. \end{cases}$$

Clearly, D itself is a $\gamma_s - set$ for n = 3k . Now $D' = D \cup \{r'_2\}$ is a set such that $V[M_v(C_n)] - D'$ is disconnected for n = 3k +1 or n = 3k + 2 . Thus

$$\gamma_s[M_v(C_n)] = \begin{cases} \frac{n}{3} + 2 & if\ n = 3k, k \geq 2. \\ \left\lceil \frac{n}{3} + 2 \right\rceil & if\ n = 3k + 1\ or\ n = 3k + 2, k \geq 1. \end{cases}$$

Theorem 4.2. For every graph G , $\gamma_s[M_v(G)] \geq \gamma[M_v(G)]$.

Proof. By definition,$V[M_v(G)] = V' \cup E' \cup R'$. Consider the dominating set $D = \{u'_i / u'_i \in V[M_v(G)]\}$.
Here we have to consider four cases.

Case 1. Let $G = P_n$. With the Theorem 2.1, $\gamma[M_v(P_n)] = \gamma[L(P_n)] + 1$ and $\langle V[M_v(G)] - D \rangle$ itself is a disconnected graph. By Observation 3.2, $\gamma[M_v(P_n)] = \gamma_s[M_v(P_n)]$. Hence $\gamma[M_v(P_n)] \geq \gamma_s[M_v(P_n)] = \gamma[L(P_n)] + 1$. It follows.

Case 2. Let G be a tree. It is obvious that, $\gamma_s[M_v(T)] \geq \gamma[M_v(T)]$

Case 3. Now we consider the cycle C_n. The following is found in Theorem 2.2 and Theorem 4.1, $\gamma_s[M_v(C_n)] \geq \gamma[M_v(C_n)]$.

Case 4. Let any graph be G . By the Theorem 2.1, Observation 3.2 and Theorem 4.1, we can say that $\gamma_s[M_v(G)] \geq \gamma[M_v(G)]$.
From the above cases, we can say that $\gamma_s[M_v(G)] \geq \gamma[M_v(G)]$.

Theorem 4.3. $\gamma_s[M_v(G)] \geq \left\lceil \frac{P}{1+\Delta(G)} \right\rceil$ for every graph $G(p, q)$.

Proof.
From Theorem 2.3,

$$\gamma[M_v(G)] \geq \left\lceil \frac{P}{1 + \Delta(G)} \right\rceil \ldots \ldots \ldots \ldots (1)$$

By Theorem 4.2,

$$\gamma_s[M_v(G)] \geq \gamma[M_v(G)] \ldots \ldots \ldots \ldots (2)$$

We have from equation (1) and equation (2),

$$\gamma_s[M_v(G)] \geq \left\lceil \frac{P}{1+\Delta(G)} \right\rceil \ldots\ldots\ldots\ldots (3)$$

Theorem 4.4. Let $G(p,q)$ be a graph, $\gamma_s[M_v(G)] \geq \left\lceil \frac{diam(G)+1}{3} \right\rceil$.

Proof. Let $V(G) = \{v_1, v_2, v_3, \ldots\ldots v_n\}$ be the vertex set then ∃ u, v ∈ V (G) and d(u, v) forms a diametral path in G. Evidently, d(u, v) = diam(G). Consider D be the γ – set of $M_v(G)$. If $\langle V[M_v(G)] - D \rangle$ is disconnected then $\langle D \rangle$ itself forms the $\gamma_s - set$ of $M_v(G)$. Otherwise, ∃ r_j' ∈ V $[M_v(G)]$ − D having maximum degree, 2 ≤ j ≤ m such that $\langle V[M_v(G)] - D \cup \{r_j'\} \rangle$ consists more than one component. Consequently, D ∪ $\{r_j'\}$ forms a $\gamma_s - set$ of $M_v(G)$. Therefore, the diametral path contains at most $\gamma_s[M_v(G)] - 1$ edges joining the neighbourhood of the vertices of D ∪ $\{r_j'\}$. Therefore, $2\gamma_s[M_v(G)] + \gamma_s[M_v(G)] - 1 \geq$ diam(G) . Which gives $\gamma_s[M_v(G)] \geq \left\lceil \frac{diam(G)+1}{3} \right\rceil$.

Theorem 4.5. For every graph G , $\gamma_s[M_v(G)] \leq q$ for p ≥ 4 .

Proof. Consider $G(p,q)$ be any graph. Let D be a γ–set of $M_v(G)$. If $\langle V[M_v(G)] - D \rangle$ is disconnected then $\langle D \rangle$ itself is a $\gamma_s - set$ of $M_v(G)$.) . Otherwise, ∃ r_j' ∈ V$[M_v(G)]$−D having maximum degree, 2 ≤ j ≤ m such that $\langle V[M_v(G)] - (D \cup r_j') \rangle$ is disconnected. Obviously, D ∪ r_j' forms a $\gamma_s - set$ of $M_v(G)$. Thus, $\gamma_s[M_v(G)] \leq q$.

Theorem 4.6. For every tree $T(p,q)$, $\gamma_s[M_v(T)] \leq \alpha_1(T)$.

Proof. Let $E_1 = \{e_1, e_2, \ldots\ldots e_k, 1 \leq k \leq q\}$ be the minimum set of edges in G , so that $|E_1| = \alpha_1(T)$. Consider the dominating set D of $M_v(T)$. By the Theorem 4.2, $\gamma_s[M_v(T)] = \gamma[M_v(T)]$ and $\langle V[M_v(T)] - D \rangle$ is disconnected. As a result, D itself forms the $\gamma_s - set$ of $M_v(T)$. Thereafter, |D| ≤ $|E_1|$. Hence $\gamma_s[M_v(T)] \leq \alpha_1(T)$.

Theorem 4.7. For every graph $G(p,q)$, $\gamma_s[M_v(G)] \leq diam(G) + \alpha_0(G)$.

Proof. Let $\alpha_0(G)$ be the vertex covering number of G. Let $V(G) = \{v_1, v_2, \ldots\ldots v_n\}$ then ∃ v_i, v_j ∈ V (G) such that K = d(v_i, v_j) = $\{v_1, v_2, \ldots\ldots v_k\}$ forms a diametral path in G. Consider D be a γ − set in $M_v(G)$. $\langle V[M_v(G)] - D \rangle$ is disconnected, then $\langle D \rangle$ itself is a $\gamma_s - set$ of $M_v(G)$.

Hence

$$|D| \leq |K \cup A|.$$
$$|D| \leq |K| \cup |A|.$$
$$\gamma_s[M_v(G)] \leq diam(G) + \alpha_0(G).$$

Otherwise, ∃ r_j' ∈ V $[M_v(G)]$ − D having maximum degree, 2 ≤ j ≤ m such that $\langle V[M_v(G)] - (D \cup r_j') \rangle$ consists of many components. Evidently, a $\gamma_s - set$ of $M_v(G)$ is generated by D ∪ $\{r_j'\}$. Since D ∪ $\{r_j'\}$ includes diametral path, we have

$$|D \cup \{r_j'\}| \leq |K \cup A| .$$
$$|D \cup \{r_j'\}| \leq |K| \cup |A| .$$
$$\gamma_s[M_v(G)] \leq diam(G) + \alpha_0(G).$$

V. CONCLUSION

In this paper we established split domination results on vertex semi-middle graph. Many bounds on domination number of vertex semi-middle graph are obtained.

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