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Original Article

Perfectly RG Continuous Mappings and RG Irresolute Mappings in Bipolar Pythagorean Fuzzy Topological Spaces

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Abstract - *The main focus of this paper is to introduce the concept of Perfectly Regular Generalized Continuous Mappings and Regular Generalized Irresolute Mappings in Bipolar Pythagorean Fuzzy Topological spaces and study some of their basic properties. We further study some of the basic properties of Perfectly Regular Continuous mappings and Regular Generalized Irresolute Mappings interrelation with other existing Bipolar Pythagorean Fuzzy mappings in Bipolar Pythagorean Fuzzy Topological Spaces*

Keywords - *Bipolar Pythagorean Fuzzy sets, Bipolar Pythagorean Fuzzy Topology, Bipolar Pythagorean Fuzzy Regular Generalized Closed sets, Bipolar Pythagorean Fuzzy Perfectly Regular Generalized Continuous Mappings, Bipolar Pythagorean Fuzzy Regular Generalized Irresolute Mappings.*

I. INTRODUCTION

Zadeh introduced Fuzzy sets in 1965[1]. After that Atanassov [2] introduced the notion of intuitionistic fuzzy sets and Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. Yager [4] proposed another class of nonstandard fuzzy sets, called Pythagorean fuzzy sets and Murat Olgun, Mehmet Ünver, Seyhmus Yardimci[5] introduced the notion of Pythagorean fuzzy topological spaces. Zhang [6] introduced the extension of fuzzy set with bipolarity, called Bipolar value fuzzy sets. Bosc and Pivert [12] said that "Bipolarity" refers to the propensity of the human mind to reason and make decisions on the basis of positive and negative effects. Positive information states what is possible, satisfactory, permitted, desired or considered as acceptable. Negative statement corresponds to what is impossible, rejected or forbidden. Negative preferences to constraints, since they specify which values or objects have to be rejected, while positive preferences corresponds to wishes, as they specify which objects aremore desirable than others, without rejecting those that do not meet the wishes. In bipolar valued fuzzy set interval of membership value is [-1,1]. The positive membership degrees represents the possibilities of something to be happened whereas the negative membership degrees represents the impossibilities.

In this paper, we introduce, Bipolar Pythagorean Fuzzy Perfectly Regular Generalized Continuous Mappings (BPFp RG continuous mappings), Bipolar Pythagorean Fuzzy Regular Generalized Irresolute Mappings (BPFRG continuous mappings) and discussed its properties.

II. PRELIMINARIES

Definition 2.1: Let X be the non empty universe of discourse. A fuzzy set A in X, $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set A; $\mu_A(x) \in [0,1]$ is the membership of $x \in X$.

Definition 2.2: Let X be the non empty universe of discourse. An Intuitionistic fuzzy set(IFS) A in X is given by A=$\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A(x) \in [0,1]$ and $\nu_A(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$to the set A, respectively, and $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$ for each $x \in X$.

The degree of indeterminacy $I_A = 1 - (\mu_A(x) - \nu_A(x))$ for each $x \in X$.

Definition 2.3: Let X be the non empty universe of discourse. A Pythagorean fuzzy set(PFS) P in X is given by P=$\{\langle x, \mu_P(x), \nu_P(x)\rangle : x \in X\}$ where the functions $\mu_P(x) \in [0,1]$ and $\nu_P(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set P, respectively, and $0 \leqslant \mu_P^2(x) + \nu_P^2(x) \leqslant 1$ for each $x \in X$.

The degree of indeterminacy $I_P = \sqrt{1 - \mu_P^2(x) - \nu_P^2(x)}$ for each $x \in X$.

Definition 2.4: Let X be a non empty set. A Bipolar Pythagorean Fuzzy Set $A = \{(x, \mu_A^+, \mu_A^-, \nu_A^+, \nu_A^-): x \in X\}$ where $\mu_A^+: X \to [0,1]$, $\nu_A^+: X \to [0,1]$, $\mu_A^-: X \to [-1,0], \nu_A^-: X \to [-1,0]$
are the mappings such that $0 \leqslant (\mu_A^+(x))^2 + (\nu_A^+(x))^2 \leqslant 1$ and $0 \leqslant (\mu_A^-(x))^2 + (\nu_A^-(x))^2 \leqslant 1$ where
$\mu_A^+(x)$ denote the positive membership degree.
$\nu_A^+(x)$ denote the positive non membership degree.
$\mu_A^-(x)$ denote the negative membership degree.
$\nu_A^-(x)$ denote the negative non membership degree.

Definition2.5:Let$A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle: x \in X\}$ and$B = \{\langle x, \mu_B^+(x), \nu_B^+(x), \mu_B^-(x), \nu_B^-(x)\rangle: x \in X\}$ be two Bipolar Pythagorean Fuzzy sets over X. Then
(i) The Bipolar Pythagorean fuzzy Complement of A is defined by $A^c = \{\langle x, \nu_A^+(x), \mu_A^+(x), \nu_A^-(x), \mu_A^-(x)\rangle: x \in X\}$,
(ii) The Bipolar Pythagorean fuzzy intersection of A and B is defined by

$$A \cap B = \{\langle x, \min\{\mu_A^+(x), \mu_B^+(x)\}, \max\{\nu_A^+(x), \nu_B^+(x)\}, \max\{\mu_A^-(x), \mu_B^-(x)\}, \min\{\nu_A^-(x), \nu_B^-(x)\}\rangle: x \in X\}$$

(iii) The Bipolar Pythagorean fuzzy union of A and B is defined by

$$A \cup B = \{\langle x, \max\{\mu_A^+(x), \mu_B^+(x)\}, \min\{\nu_A^+(x), \nu_B^+(x)\}, \min\{\mu_A^-(x), \mu_B^-(x)\}, \max\{\nu_A^-(x), \nu_B^-(x)\}\rangle: x \in X\}$$

(iv) A is a Bipolar Pythagorean subset of B and write $A \subseteq B$ if $\mu_A^+(x) \leqslant \mu_B^+(x), \nu_A^+(x) \geqslant \nu_B^+(x), \mu_A^-(x) \geqslant \mu_B^-(x), \nu_A^-(x) \leqslant \nu_B^-(x)\rangle$ *for each* $x \in X$
(v) $0_X = \{\langle x, 0,1,0,-1\rangle: x \in X\}$and
$1_X = \{\langle x, 1,0,-1,0\rangle: x \in X\}$.

Definition 2.6: Bipolar Pythagorean Fuzzy Topological Spaces: Let $X \neq \emptyset$ be a set and τ_p be a family of Bipolar Pythagorean fuzzy subsets of X. If
T_1 $0_X, 1_X \in \tau_p$.
T_2 For any $P_1, P_2 \in \tau_p$, we have $P_1 \cap P_2 \in \tau_p$.
T_3 $\cup P_i \in \tau_p$ for an arbitrary family $\{P_i : i \in J\} \subseteq \tau_p$.

Then τ_p is called Bipolar Pythagorean Fuzzy Topology on X and the pair (X, τ_p) is said to be Bipolar Pythagorean Fuzzy Topological space. Each member of τ_p is called Bipolar Pythagorean fuzzy open set(BPFOS). The complement of a Bipolar Pythagorean Fuzzy open set is called a Bipolar Pythagorean fuzzy Closed set(BPFCS).

Definition 2.7: Let (X, τ_p) be a BPFTS and
$P = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle: x \in X\}$ be a BPFS over X. Then the Bipolar Pythagorean Fuzzy Interior, Bipolar Pythagorean Fuzzy Closure of P are defined by:
a) $\mathcal{B}_{\mathrm{PF}}int(P)$ =∪{G / G is a BPFOS in (X, τ_p) and G ⊆ P}.
b) $\mathcal{B}_{\mathrm{PF}}cl(P)$ =∩{K / K is a BPFCS in (X, τ_p) and P⊆ K}.
It is clear that
a) $\mathcal{B}_{\mathrm{PF}}int(P)$ is the biggest Bipolar Pythagorean Fuzzy Open set contained in P.
b) $\mathcal{B}_{\mathrm{PF}}\ cl(P)$ is the smallest Bipolar Pythagorean Fuzzy Closed set containing P.

Definition 2.8. If BPFS $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle: x \in X\}$ in a BPTS (X, τ_p) is said to be
(a) Bipolar Pythagorean Fuzzy Semi closed set (BPFSCS) if $\mathcal{B}_{\mathrm{PF}}int(\mathcal{B}_{\mathrm{PF}}cl(A)) \subseteq A$.
(b) Bipolar Pythagorean Fuzzy Semi open set (BPFSOS) if $A \subseteq \mathcal{B}_{\mathrm{PF}}cl(\mathcal{B}_{\mathrm{PF}}int(A))$.
(c) Bipolar Pythagorean Fuzzy Preclosed set (BPFPCS) if $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{B}_{\mathrm{PF}}int(A)) \subseteq A$.
(d) Bipolar Pythagorean Fuzzy Preopen set (BPFPOS) if $A \subseteq \mathcal{B}_{\mathrm{PF}}int(\mathcal{B}_{\mathrm{PF}}cl(A))$.
(e) Bipolar Pythagorean Fuzzy α closed set (BPFαCS) if $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{B}_{\mathrm{PF}}int(cl(A)) \subseteq A$.
(f) Bipolar Pythagorean Fuzzy α open set (BPFαOS) if $A \subseteq \mathcal{B}_{\mathrm{PF}}int(\mathcal{B}_{\mathrm{PF}}cl(int(A))$.
(g) Bipolar Pythagorean Fuzzy γ closed set (BPFγCS) if $A \subseteq \mathcal{B}_{\mathrm{PF}}int(\mathcal{B}_{\mathrm{PF}}cl(A) \cup \mathcal{B}_{\mathrm{PF}}cl(\mathcal{B}_{\mathrm{PF}}int(A))$.
(h) Bipolar Pythagorean Fuzzy γ open set (BPFγOS) if
$\mathcal{B}_{\mathrm{PF}}cl(\mathcal{B}_{\mathrm{PF}}int(A) \cup in\mathcal{B}_{\mathrm{PF}}t(\mathcal{B}_{\mathrm{PF}}cl(A)) \subseteq A$.
(i) Bipolar Pythagorean Fuzzy regular closed set (BPFRCS) if $A = \mathcal{B}_{\mathrm{PF}}cl(\mathcal{B}_{\mathrm{PF}}int(A))$.
(j) Bipolar Pythagorean Fuzzy regular open set (BPFROS) if $A = \mathcal{B}_{\mathrm{PF}}int(\mathcal{B}_{\mathrm{PF}}cl(A))$.
(k) A Bipolar Pythagorean Fuzzy set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Generalized closed set (BPFGCS), if $\mathcal{B}_{\mathrm{PF}}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is BPFOS in (X, τ_p).
(l) A Bipolar Pythagorean Fuzzy set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Generalized open set (BPFGCS), if A^c is a BPFGCS in (X, τ_p).
(m) A Bipolar Pythagorean Fuzzy set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Regular Generalized closed set (BPFGCS), if $\mathcal{B}_{\mathrm{PF}}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is BPFROS in (X, τ_p).

(n) A Bipolar Pythagorean Fuzzy set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Regular Generalized Open set (BPFGOS), if $\mathcal{B}_{\text{PF}}int(A) \supseteq U$ whenever $A \supseteq U$ and U is BPFRCS in (X, τ_p).

Definition 2.9: A Bipolar Pythagorean Fuzzy Set A of a Bipolar Pythagorean Fuzzy Topological Space (X, τ_p) is called Bipolar Pythagorean Regular Generalized closed (BPFRGCS in short), if $\mathcal{B}_{\text{PF}}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is BPF regular Open in (X, τ_p).

Definition 2.10: A Bipolar Pythagorean Fuzzy Set A of a Bipolar Pythagorean Fuzzy Topological Space (X, τ_p) is called Bipolar Pythagorean Regular Generalized Open (BPFRGOS in short), if $\mathcal{B}_{\text{PF}}int(A) \supseteq U$ whenever $A \supseteq U$ and U is BPFR-closed in (X, τ_p).

Definition 2.11: Let $\mathcal{H}$ be a mapping from an BPFTS in (X, τ_p) into a BPFTS (Y, σ_p). Then $\mathcal{H}$ is said to be Bipolar Pythagorean Fuzzy Continuous mapping if $\mathcal{H}^{-1}(A) \in BPFO(X)$ for every $A \in (Y, \sigma_p)$.

Definition 2.12: A mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ is said to be
(i) BPF semi continuous mapping if $\mathcal{H}^{-1}(A) \in$ BPFSO(X) for every $A \in (Y, \sigma_p)$.
(ii) BPFα continuous mapping if $\mathcal{H}^{-1}(A) \in$ BPFαO(X) for every $A \in (Y, \sigma_p)$.
(iii) BPF Pre continuous mapping if $\mathcal{H}^{-1}(A) \in$ BPFPO(X) for every $A \in (Y, \sigma_p)$.
(iv) BPFγ continuous mapping if $\mathcal{H}^{-1}(A) \in$ BPFγO(X) for every $A \in (Y, \sigma_p)$.

Definition 2.13: A mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ is said to be BPF Generalized Continuous mapping (BPFG continuous mapping) if $\mathcal{H}^{-1}(A) \in$ BPFGC(X) for every BPFCS A in (Y, σ_p).

Definition 2.14: A mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ is said to be BPF α Generalized Continuous mapping (BPFαG continuous mapping) if $\mathcal{H}^{-1}(A) \in$ BPFαC(X) for every BPFCS A in (Y, σ_p).

Definition 2.15: A BPFTS (X, τ_p) is said to be a $BPFR_cT_{\frac{1}{2}}$ space (Bipolar Pythagorean Fuzzy Regular $_cT_{\frac{1}{2}}$ space) if every BPFRGCS in (X, τ_p) is a BPFCS in (X, τ_p).

Definition 2.16: A BPFTS (X, τ_p) is said to be a $BPFR_{\mathcal{R}}T_{\frac{1}{2}}$ space (Bipolar Pythagorean Fuzzy Regular Generalized $_{\mathcal{R}}T_{\frac{1}{2}}$ space) if every BPFRGCS in (X, τ_p) is a BPFGCS in (X, τ_p).

Definition 2.17: A BPFTS (X, τ_p) is said to be a $BPFR_\alpha T_{\frac{1}{2}}$ space (Bipolar Pythagorean Fuzzy Regular Generalized $_\alpha T_{\frac{1}{2}}$ space) if every BPFRGCS in (X, τ_p) is a BPFαCS in (X, τ_p).

Definition 2.18: A mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy irresolute mapping (BPF irresolute mapping) if $\mathcal{H}^{-1}(A) \in$ BPFCS(X) for every BPFCS A in (Y, σ_p).

Definition 2.19: A mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Generalized irresolute mapping (BPFG irresolute mapping) if $\mathcal{H}^{-1}(A) \in$ BPFGCS(X) for every BPFGCS A in (Y, σ_p).

III. BIPOLAR PYTHAGOREAN FUZZY PERFECTLY REGULAR GENERALIZED CONTINUOUS MAPPINGS (BPFpRG CONTINUOUS MAPPINGS)

In this section we have introduced Bipolar Pythagorean Fuzzy Perfectly Regular Generalized continuous mappings and studied some of its properties

Definition 3.1: A map $\mathcal{H}: X, \tau_p) \to (Y, \sigma_p)$ is said to be BPF Perfectly Regular Generalized continuous mapping if $\mathcal{H}^{-1}(A)$ is BPF clopen in (X, τ_p) for every BPFRCS A in (Y, σ_p).

Theorem 3.2: Let $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ be map. Then the following are equivalent.
(a) $\mathcal{H}$ is BPF perfectly Regular Generalized continuous.
(b) The inverse image of BPFRGOS in (Y, σ_p) is BPF clopen in (X, τ_p).
(c) The inverse image of BPFRGCS in (Y, σ_p) is BPF clopen in (X, τ_p).

Proof: (i) $\mathcal{H}$ is BPF perfectly Regular Generalized continuous The inverse image of BPFRGOS in (Y, σ_p) is BPF clopen in (X, τ_p), from the definition.
(ii) (b) (c) : Let G be any BPFRGCS in (Y, σ_p). Then G^c is BPFRGOS in (Y, σ_p). Hence by assumption $\mathcal{H}^{-1}(G^c)$ is BPF clopen in (X, τ_p).
(iii) (c) (a):Let H be any BPFRGOS in (Y, σ_p). Then H^c is BPFRGCS in (Y, σ_p). As by (c) $\mathcal{H}^{-1}(H^c)$ is BPF clopen in (X, τ_p) which implies that $\mathcal{H}^{-1}(H)$ is BPF clopen in (X, τ_p). Hence $\mathcal{H}$ is BPF Perfectly Regular Generalized Continuous Mapping (BPFpRG continuous mapping).

Proposition 3.3: Every BPFpRG continuous mapping is a BPF continuous mapping but not conversely in general.
Proof: Let $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFpRG continuous mapping. Let A be a BPFCS in (Y, σ_p). Since every BPFCS is a BPFRGCS, A is a BPFRGCS in (Y, σ_p). Since $\mathcal{H}$ is a BPFpRG continuous mapping, $\mathcal{H}^{-1}(A)$ is a BPF Clopen in (X, τ_p). Thus $\mathcal{H}^{-1}(A)$ is a BPFCS in (X, τ_p). Hence $\mathcal{H}$ is a BPF continuous mapping (BPFpRG continuous mapping).

***Example 3.4*:** Let X={a,b} and Y={u,v} and T_1=(x, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)), T_2=(y, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)). Then $\tau_p=\{0_p, T_1, 1_p\}$ and $\sigma_p=\{0_p, T_2, 1_p\}$ be a BPFTs on X and Y respectively. Define a mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ by $\mathcal{H}(a) = u$ and $\mathcal{H}(b) = v$. The BPFS A = ⟨y, (0.8, 0.6), (0.3, 0.5), (-0.9, -0.7), (-0.4, -0.5)) is a BPFCS in (Y, σ_p). Then$\mathcal{H}^{-1}(A)$ is a BPFCS in (X, τ_p). Therefore $\mathcal{H}$ is a BPF continuous mapping, but not a BPFpRG continuous mapping. Since for a BPFRGCS A = ⟨y, (0.8, 0.6), (0.3, 0.5), (-0.9, -0.7), (-0.4, -0.5)) in (Y, σ_p), $\mathcal{H}^{-1}(A)$ is not a BPF clopen in (X, τ_p), as $\mathcal{B}_{\text{PF}}\, cl(\mathcal{H}^{-1}(A)) = T_1^c = \mathcal{H}^{-1}(A)$ but $\mathcal{B}_{\text{PF}} int(\mathcal{H}^{-1}(A)) = T_1 \neq \mathcal{H}^{-1}(A)$.

Proposition 3.5: Every BPFpRG continuous mapping is a BPFG continuous mapping but not conversely in general.

Proof: Let $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFpRG continuous mapping. Let A be a BPFCS in Y. Since every BPFCS is a BPFRGCS, A is a BPFRGCS in (Y, σ_p). Since $\mathcal{H}$ is a BPFpRG continuous mapping, $\mathcal{H}^{-1}(A)$ is a BPF Clopen in (X, τ_p). Thus $\mathcal{H}^{-1}(A)$ is a BPFCS in (X, τ_p). Since every BPFCS is BPFGCS , $\mathcal{H}^{-1}(A)$ is a BPFGCS in (X, τ_p). Hence $\mathcal{H}$ is a BPFG continuous mapping.

Example 3.6: Let X={a,b}, Y={u,v} and T_1=(x, (0.5, 0.4), (0.6, 0.5), (-0.4, -0.3), (-0.5, -0.4)), T_2=(y, (0.7, 0.8), (0.3, 0.3), (-0.8, -0.8), (-0.3, -0.4)). Then $\tau_p=\{0_p, T_1, 1_p\}$ and $\sigma_p=\{0_p, T_2, 1_p\}$ be a BPFTs on X and Y respectively. Define a mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ by $\mathcal{H}(a) = u$ and $\mathcal{H}(b) = v$. The BPFS A = ⟨y, (0.2, 0.3), (0.8, 0.9), (-0.2, -0.2), (-0.9, -0.8)) is a BPFCS in (Y, σ_p). Then$\mathcal{H}^{-1}(A)$ is a BPFCS in (X, τ_p). Therefore $\mathcal{H}$ is a BPFG continuous mapping, but not a BPFpRG continuous mapping. Since for a BPFRGCS A = ⟨y, (0.2, 0.3), (0.8, 0.9), (-0.2, -0.2), (-0.9, -0.8)) in (Y, σ_p), and $\mathcal{H}^{-1}(A)$ is not a BPF clopen in (X, τ_p), as $\mathcal{B}_{\text{PF}} cl(\mathcal{H}^{-1}(A)) = T_1^c \neq \mathcal{H}^{-1}(A)$ but $\mathcal{B}_{\text{PF}} int(\mathcal{H}^{-1}(A)) = 0_p \neq \mathcal{H}^{-1}(A)$.

Proposotion 3.7: Every BPFpRG continuous mapping is a BPFα continuous mapping but not conversely in general.

Proof: Let $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFpRG continuous mapping. Let A be a BPFCS in Y. Since every BPFCS is a BPFRGCS, A is a BPFRGCS in (Y, σ_p). Since $\mathcal{H}$ is a BPFpRG continuous mapping, $\mathcal{H}^{-1}(A)$ is a BPF Clopen in (X, τ_p). Thus $\mathcal{H}^{-1}(A)$ is a BPFCS in (X, τ_p). Since every BPFCS is BPFαCS , $\mathcal{H}^{-1}(A)$ is a BPFαCS in (X, τ_p). Hence $\mathcal{H}$ is a BPFα continuous mapping.

Example 3.8: Let X={a,b}, Y={u,v} and T_1=(x, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)), T_2=(y, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)). Then $\tau_p=\{0_p, T_1, 1_p\}$ and $\sigma_p=\{0_p, T_2, 1_p\}$ be a BPFTs on X and Y respectively. Define a mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ by $\mathcal{H}(a) = u$ and $\mathcal{H}(b) = v$. The BPFS A = ⟨y, (0.8, 0.6), (0.3, 0.5), (-0.9, -0.7), (-0.4, -0.5)) is a BPFαCS in (Y, σ_p). Then$\mathcal{H}^{-1}(A)$ is a BPFCS in (X, τ_p). Therefore $\mathcal{H}$ is a BPF continuous mapping, but not a BPFpRG continuous mapping. Since for a BPFαCS A = ⟨y, (0.8, 0.6), (0.3, 0.5), (-0.9, -0.7), (-0.4, -0.5)) in (Y, σ_p), $\mathcal{H}^{-1}(A)$ is not a BPF clopen in (X, τ_p), as $\mathcal{B}_{\text{PF}} cl(\mathcal{H}^{-1}(A)) = T_1^c = \mathcal{H}^{-1}(A)$ but $\mathcal{B}_{\text{PF}} int(\mathcal{H}^{-1}(A)) = T_1 \neq \mathcal{H}^{-1}(A)$.

Proposition 3.9: Ever BPFpRG continuous mapping is a BPFR continuous mapping but not conversely in general.

Proof: Let $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFpRG continuous mapping. Let A be a BPFCS in Y. Since every BPFCS is a BPFRGCS, A is a BPFRGCS in (Y, σ_p). Since $\mathcal{H}$ is a BPFpRG continuous mapping, $\mathcal{H}^{-1}(A)$ is a BPF Clopen in (X, τ_p).

Thus $\mathcal{H}^{-1}(A)$ is a BPFRCS in (X,τ_p). Since every BPFRCS is BPFRGCS , $\mathcal{H}^{-1}(A)$ is a BPFRCS in (X,τ_p). Hence $\mathcal{H}$ is a BPFR continuous mapping.

Example 3.10: Let X={a,b}, Y={u,v} and T_1=(x, (0.2, 0.2), (0.8, 0.8), (-0.2, -0.1), (-0.8, -0.7)), T_2=(y, (0.6, 0.4), (0.7, 0.5), (-0.6, -0.4), (-0.7, -0.6)). Then $\tau_p=\{0_p, T_1, 1_p\}$ and $\sigma_p=\{0_p, T_2, 1_p\}$ be a BPFTs on X and Y respectively. Define a mapping $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ by $\mathcal{H}(a)=u$ and $\mathcal{H}(b)=v$. Let us consider the BPFRCS A in (Y,σ_p). Then The BPFS A= (y, (0.7, 0.5), (0.6, 0.4), (-0.7, -0.6), (-0.6, -0.4)) Then $\mathcal{H}^{-1}(A)$ is a BPFRCS in (X,τ_p). Therefore $\mathcal{H}$ is a BPFRG continuous mapping, but not a BPFpRG continuous mapping. Since for a BPFRGCS A= (y, (0.7, 0.5), (0.6, 0.4), (-0.7, -0.6), (-0.6, -0.4)) in (Y,σ_p), and $\mathcal{H}^{-1}(A)$ is not a BPF clopen in (X,τ_p), as $\mathcal{B}_{PF}cl(\mathcal{H}^{-1}(A))=T_1^c\neq\mathcal{H}^{-1}(A)$ but $\mathcal{B}_{PF}int(\mathcal{H}^{-1}(A))=T_1\neq\mathcal{H}^{-1}(A)$.

Theorem 3.11: A mapping $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ is a BPFpRG continuous mapping Iff the inverse image of each BPFRGOS is a BPF clopen in (X,τ_p).

Proof: Necessity: Let $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ be a (Y,σ_p) BPFpRG continuous mapping. Let A be a BPFRGOS in (Y,σ_p). Since $\mathcal{H}$ is a BPFpRG continuous mapping, $\mathcal{H}^{-1}(A^c)$ is BPF clopen in (X,τ_p). As $\mathcal{H}^{-1}(A^c)=(\mathcal{H}^{-1}(A))^c$, we have $\mathcal{H}^{-1}(A)$ is a BPF clopen in (X,τ_p).

Sufficiency: Let B be a BPFRGCS in (Y,σ_p). Then B^c is a BPFRGOS in (Y,σ_p). By hypothesis, $(\mathcal{H}^{-1}(B^c))$ is BPF clopen in (X,τ_p) $(\mathcal{H}^{-1}(B))^c$ is BPF clopen in (X,τ_p), as $\mathcal{H}^{-1}(B^c)=(\mathcal{H}^{-1}(B))^c$. Therefore $\mathcal{H}$ is a BPFpRG continuous mapping.

Theorem 3.12: Let $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ be a BPF continuous mapping and $\mathcal{R}:(Y,\sigma_p)\to(Z,\zeta_p)$ is a BPFpRG continuous mapping, then $\mathcal{R}o\mathcal{H}:(X,\tau_p)\to(Z,\zeta_p)$ is a BPFpRG continuous mapping.

Proof: Let A be a BPFRGCS in (Z,ζ_p). Since $\mathcal{R}$ is a BPFpRG continuous mapping, $\mathcal{R}^{-1}(A)$ is a BPF clopen in (Y,σ_p). Since $\mathcal{H}$ is a BPF continuous mapping, then $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))$ is a BPFCS in (X,τ_p), and also BPFOS in (X,τ_p). Hence $\mathcal{R}o\mathcal{H}$ is a BPFpRG continuous mapping.

Theorem 3.13: The composition of two BPFpRG continuous mapping is a BPFpRG continuous mapping in general.

Proof: Let $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ and $\mathcal{R}:(Y,\sigma_p)\to(Z,\zeta_p)$ be any two BPFpRG continuous mappings. Let A be a BPFRGCS in (Z,ζ_p). By hypothesis, $\mathcal{R}^{-1}(A)$ is BPF clopen in (Y,σ_p) and hence it is BPFCS in (Y,σ_p). Since every BPFCS is BPFRGCS, $\mathcal{R}^{-1}(A)$ is a BPFRGCS in (Y,σ_p) and $\mathcal{H}$ is a BPFpRG continuous mapping, $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))=(\mathcal{R}o\mathcal{H})^{-1}(A)$ is BPF clopen in (X,τ_p). Hence $\mathcal{R}o\mathcal{H}$ is BPFRG continuous mapping.

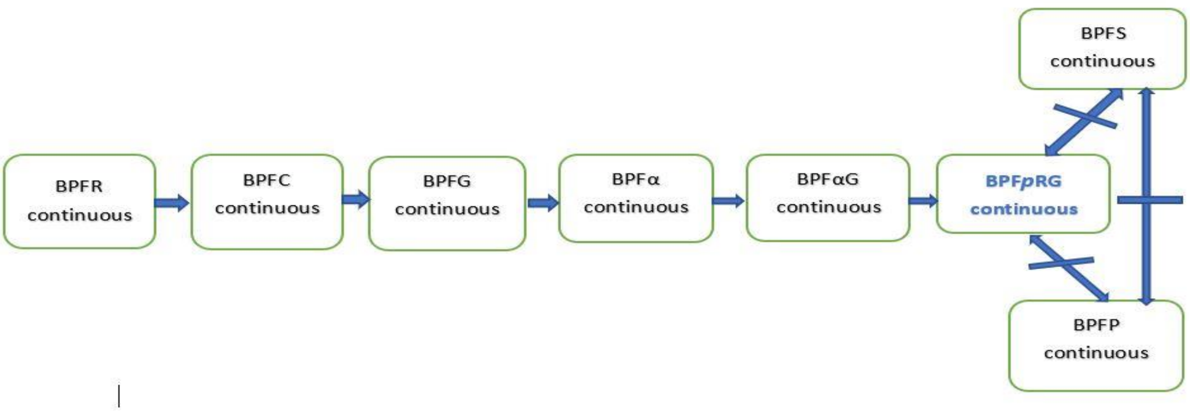

Fig. 1 Relation between BPF*p*RG continuous mappings with other BPFs

IV. BIPOLAR PYTHAGOREAN FUZZY REGULAR GENERALIZED IRRESOLUTE MAPPINGS

In this section we have introduced Bipolar Pythagorean Fuzzy Regular Generalized Irresolute Mappings and studied some of its properties.

Definition 4.1: A mapping $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Regular Generalized irresolute mapping (BPFRG irresolute) if $\mathcal{H}^{-1}(A)$ is a BPFRGCS in (X,τ_p) for every BPFRGCS A in (Y,σ_p).

Example 4.2: Let X={a,b} and Y={u,v} and T_1=(x, (0.6, 0.7), (0.6, 0.6), (-0.5, -0.6), (-0.5, -0.5)), T_2=(y, (0.5, 0.4), (0.5, 0.4), (-0.6, -0.5), (-0.6, -0.5)). Then $\tau_p=\{0_p,T_1,1_p\}$ and $\sigma_p=\{0_p,T_2,1_p\}$ be a BPFTs on X and Y respectively. Define a mapping $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ by $\mathcal{H}(a)=u$ and $\mathcal{H}(b)=v$. The BPFS A = (y, (0.2, 0), (0.6, 0.7), (-0.3, -0.1), (-0.7, -0.6)) is said to be BPFRG irresolute mapping (BPFRG irresolute), since $\mathcal{H}^{-1}(A)$ = (x, (0.2, 0), (0.6, 0.7), (-0.3, -0.1), (-0.7, -0.6)) is a BPFRGCS in (X,τ_p) for every BPFRGCS A in (Y,σ_p).

Theorem 4.3: If $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ is a BPFRG irresolute, then $\mathcal{H}$ is BPFRG continuous mapping but not conversely.

Proof: Let $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ be a BPFG irresolute mapping in (X,τ_p). Let A be any BPFCS in (Y,σ_p), Since every BPFCS is a BPFRGCS, A is a BPFRGCS in (Y,σ_p). By hypothesis, $\mathcal{H}^{-1}(A)$ is a BPFRGCS in (X,τ_p). Hence $\mathcal{H}$ is a BPFRG continuous mapping.

Example 4.4: Let X={a,b}, Y={u,v} and T_1=(x, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)), T_2=(y, (0.4, 0.6), (0.5, 0.6), (-0.5, -0.7), (-0.6, -0.7)). Then $\tau_p=\{0_p,T_1,1_p\}$ and $\sigma_p=\{0_p,T_2,1_p\}$ be a BPFTs on X and Y respectively. Define a mapping $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ by $\mathcal{H}(a)=u$ and $\mathcal{H}(b)=v$. The BPFS A = (y, (0.5, 0.6), (0.4, 0.6), (-0.6, -0.7), (-0.5, -0.7)) is not BPFRGCS in (Y,σ_p), since $\mathcal{B}_{\text{PF}}cl(A)=T_2^c\nsubseteq U$, whenever $A\subseteq U$ but $\mathcal{H}^{-1}(A)$ is BPFRGCS in (X,τ_p), as $\mathcal{B}_{\text{PF}}cl(\mathcal{H}^{-1}(A))=T_1^c\subseteq 1_p$, whenever $\mathcal{H}^{-1}(A)\subseteq 1_p$. Therefore $\mathcal{H}$ is not BPFRG irresolute mapping.

Theorem 4.5: If $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ and $\mathcal{R}:(Y,\sigma_p)\to(Z,\zeta_p)$ are two BPFRG irresolute mappings, then $\mathcal{R}o\mathcal{H}:(X,\tau_p)\to(Z,\zeta_p)$ is a BPFRG irresolute mapping.

Proof: Let A be BPFRGCS in (Z,ζ_p). Then by hypothesis, $\mathcal{R}^{-1}(A)$ is a BPFRGCS in (Y,σ_p). Since $\mathcal{R}$ is a BPFRG irresolute mapping, $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))$ is a BPFRGCS in (X,τ_p). then, $(\mathcal{R}o\mathcal{H})^{-1}(A)$ is a BPFRGCS in (X,τ_p). Therefore, $\mathcal{R}o\mathcal{H}$ is a BPFRG irresolute mapping.

Example 4.6: Let X={a,b}, Y={u,v} and Z={p,q} and T_1=(x, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)), T_2=(y, (0.4, 0.6), (0.5, 0.6), (-0.5, -0.7), (-0.6, -0.7)), T_3=(z, (0.5, 0.7), (0.5, 0.7), (-0.6, -0.8), (-0.7, -0.8)). Then $\tau_p=\{0_p,T_1,1_p\}$, $\sigma_p=\{0_p,T_2,1_p\}$ and $\zeta_p=\{0_p,T_3,1_p\}$ be a BPFTs on X, Y and Z respectively. Define the mapping $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ by $\mathcal{H}(a)=u$ and $\mathcal{H}(b)=v$ and $\mathcal{R}:(Y,\sigma_p)\to(Z,\zeta_p)$ by $\mathcal{R}(u)=p$ and $\mathcal{H}(v)=q$. The BPFS A = (z, (0.4, 0.3), (0.5, 0.7), (-0.5, -0.8), (-0.7, -0.8)) is a BPFRGCS in (Z,ζ_p), since $\mathcal{B}_{\text{PF}}cl(A)=T_3^c\subseteq T_3$, whenever $A\subseteq\{T_3,1_p\}$ and $\mathcal{R}^{-1}(A)$ is a BPFRGCS in (Y,σ_p), since $\mathcal{B}_{\text{PF}}cl(\mathcal{R}^{-1}(A))=1_p\subseteq U$, whenever $\mathcal{R}^{-1}(A)\subseteq U$ and $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))$ is a BPFRGCS in (X,τ_p), since $\mathcal{B}_{\text{PF}}cl\left(\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))\right)=1_p\subseteq U$, whenever $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))\subseteq U$. Therefore $\mathcal{R}o\mathcal{H}$ is BPFRG irresolute mapping.

Theorem 4.7: If $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ and $\mathcal{R}:(Y,\sigma_p)\to(Z,\zeta_p)$ are two BPFRG irresolute mappings, then $\mathcal{R}o\mathcal{H}:(X,\tau_p)\to(Z,\zeta_p)$ is a BPFRG continuous mapping.

Proof: Let A be BPFCS in (Z,ζ_p). Then by hypothesis, $\mathcal{R}^{-1}(A)$ is a BPFRGCS in (Y,σ_p). Since $\mathcal{H}$ is a BPFRG irresolute mapping, $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))$ is a BPFRGCS in (X,τ_p). then, $(\mathcal{R}o\mathcal{H})^{-1}(A)$ is a BPFRGCS in (X,τ_p). Therefore, $\mathcal{R}o\mathcal{H}$ is a BPFRG continuous mapping.

Theorem 4.8: If $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ is a BPFRG irresolute mapping in a $BPFR_cT_{\frac{1}{2}}$ space in (X,τ_p), Then $\mathcal{H}$ is a BPF continuous mapping.

Proof: Let A be a BPFCS in (Y,σ_p). Then A is a BPFRG irresolute mapping in (Y,σ_p). Since $\mathcal{H}$ is a BPFRG irresolute, $\mathcal{H}^{-1}(A)$ is a BPFRGCS in (X,τ_p). Since X is a $BPFR_cT_{\frac{1}{2}}$ space, $\mathcal{H}^{-1}(A)$ is a BPFCS in (X,τ_p). Hence $\mathcal{H}$ is a BPF continuous mapping.

Theorem 4.9: If $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ is a BPFRG irresolute mapping in a $BPFR_{\mathcal{R}}T_{\frac{1}{2}}$ space in (X,τ_p), Then $\mathcal{H}$ is a BPFG irresolute mapping.

Proof: Let A be a BPFGCS in (Y, σ_p). Then A is a BPFRGCS in (Y, σ_p). Therefore $\mathcal{H}^{-1}(A)$ is a BPFRGCS in (x, τ_p), by hypothesis. Since(X, τ_p) is a $BPFR_{\mathcal{R}}T_{\frac{1}{2}}$ space , $\mathcal{H}^{-1}(A)$ is a BPFGCS in (Y, σ_p). Hence $\mathcal{H}$ is a BPFG irresolute mapping.

Theorem 4.10: Let$\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping from a BPFTS (X, τ_p) into a BPFTS (Y, σ_p). Then the following conditions are equivalent if (X, τ_p) and (Y, σ_p) are $BPFR_cT_{\frac{1}{2}}$ spaces:

(i) $\mathcal{H}$ is a BPFRG irresolute mapping.
(ii) $\mathcal{H}^{-1}(B)$ is a BPFRGOS in (X, τ_p) for each BPFRGOS in (Y, σ_p).
(iii) $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(B) \subseteq \mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B))$ for each BPFS B of (Y, σ_p).

Proof: (i)⇒(ii) : Obviously true.
(ii)⇒(iii) : Let B be any BPFS in (Y, σ_p). Clearly $B \subseteq \mathcal{B}_{\mathrm{PF}}cl(B)$. Then $\mathcal{H}^{-1}(B) \subseteq \mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B))$. Since $cl(B)$ is a BPFCS in (Y, σ_p), $\mathcal{B}_{\mathrm{PF}}cl(B)$ is a BPFRGCS in (Y, σ_p). Therefore, $\mathcal{H}^{-1}(c\mathcal{B}_{\mathrm{PF}}l(B))$ is a BPFRGCS in (X, τ_p), by hypothesis. Since (X, τ_p) is a $BPFR_cT_{\frac{1}{2}}$space, $\mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B))$ is a BPFCS in (X, τ_p). Hence $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(B) \subseteq c\mathcal{B}_{\mathrm{PF}}l(\mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B))) = \mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B))$.That is $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(B)) \subseteq \mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B))$.
(iii)⇒(i) : Let B be a BPFRGCS in (Y, σ_p). Since (Y, σ_p) is a $BPFR_cT_{\frac{1}{2}}$space, B is a BPFCS in (Y, σ_p) and $\mathcal{B}_{\mathrm{PF}}cl(B) = B$. Hence $\mathcal{H}^{-1}(B) = \mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B)) \supseteq \mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(B))$. Therefore, $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(B)) = \mathcal{H}^{-1}(B)$. This implies $\mathcal{H}^{-1}(B)$ is a BPFCS in (X, τ_p) and hence it is a BPFRGCS in (X, τ_p). Thus $\mathcal{H}$ is a BPFRG irresolute mapping.

V. CONCLUSION

We defined and studied a new concept of Perfectly Regular Generalized continuous mappings and Regular Generalized Irresolute Mappings in Bipolar Pythagorean Fuzzy Topological Spaces. The relationship between BPFPRG continuous mappings and other BPFs were proved. In the future, we intend to extent our research work in the applications of these mappings in decision making problems.

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