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Original Article

The Semi-Image Neighbouhood Block Graph Crossing Number and Forbidden Subgraphs

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Abstract - *Let $G = (V,E)$ be a simple connected undirected graph with vertex set V and edge set E. The advent of graph theory has played a prominent role in wide variety of engineering applications and optimizes its use in many applications. We characterize graphs whose semi-image neighbourhood block graphs are never has crossing number k (k=1 or 2). Also we prove that the semi-image neighbouhood block graph of a graph which are planar and outerplanar in terms of forbidden subgraphs.*

***Keywords* -** *Semi-Iimage Neighbourhood, Block, Crossing Number, Forbidden, Minimally Nonouterplanar.*

I. INTRODUCTION

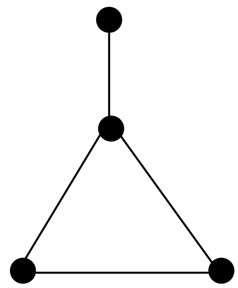
All graphs considered here are finite, undirected and without loops or multiple lines. We use the terminology of [2]. The open-neighbourhoodN(u) of a point u in V(G) is the set of points adjacent to u. E(G)].∈N(u) = [v/uv In 1975, Kulli [4] introduced the idea of a minimally nonouterplanar graph. The inner point i(G) of a planar graph G is the minimum possible number of points not belonging to the boundary of the exterior region in any embedding of G in the plane. Obviously G is outerplanar if and only if i(G)=0. A graph G is minimallynonouterplanari(G)=1 and G is n-minimally (n≥2) nonouterplanar if i(G)=n. A graph is planar if it can be≥nonouterplanar if i(G)=1, and G is n-minimally (n drawn on the plane in such a way that no two of its lines intersect. If B = {u1, u2, …, ur , r ≥2} is a block of a graph G, then we say that point u1 and block B are incident with each other, as are u2 and B and so on. If two blocks B1 and B2 of G are incident with a common cutpoint, then they are adjacent blocks. If B = { e1, e2, ...es , s≥1}is a block of a graph G, then we say that line e1 and block B are incident with each other, as are e2 and B and so on. This idea was introduced by Kulli in [5]. The points, lines and blocks of a graph are called its members.

For each point vi of G, we take a new point ui and the resulting set of points is denoted by V1(G).

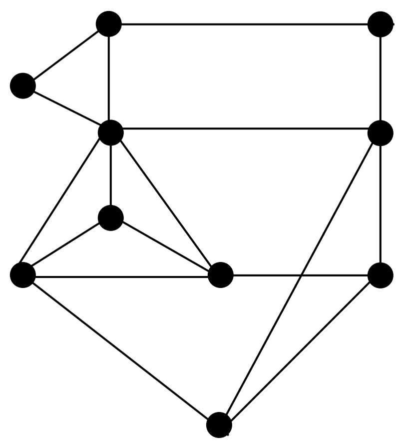
The semi-image neighborhood block graph $NB_I(G)$ of a graph G. The semi-image neighbourhoood block graph $NB_I(G)$ of a graph G is defined as the graph having point set $V(G)\cup V'(G)\cup b(G)$, two points of $V(G)$ and $V'(G)$ are adjacent if they are adjacent in G, two points $V_i$and V'_j are adjacent if $i=j$, with two points of $V(G)$ are incident with $b(G)$.This concept was introduced by Kulli and Niranjan [17]. Many other graph valued functions in graph theory were studied, [e.g in [1],[3],[6]-[16] [18]-[21] and [23]-[37]].

Theimage neighbourhood graph $N_I(G)$ of a graph G is defined as the graph having point set $V(G)\cup V'(G)$, two points of $V(G)$ and $V'(G)$ are adjacent if they are adjacent in $V(G)$, two points V_i and V'_j are adjacent if $i=j$. This concept was introduced by M.H.Muddebihal, Usha.P and Milind S.C. [22].A graph, the semi-image neighbourhood block $NB_I(G)$ and image neighbourhood graph $N_I(G)$ are shown in Figure 1.In 1975,

G:



$NB_I(G)$:



$N_I(G)$:

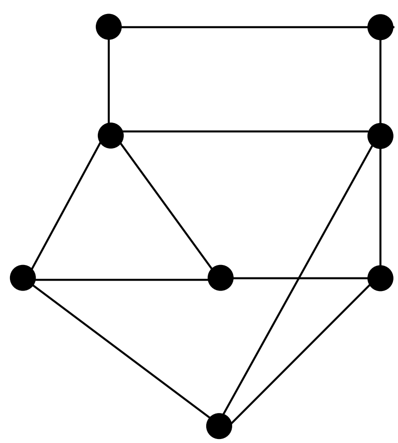


Fig. 1

II. PRELIMINARIES

We use the following result to prove our main results.

Theorem A.[17]. A nonseperablenonouterplanar graph G is minimally nonouterplanar if and only if it fails to contain a subgraphhomeomorphic to one of the graphs of Figure2.

Theorem B [17]. The semi-image neighbourhood block graph $NB_I(G)$ of a graph G is planar if and only if G is outerplanar.

Theorem C [17]. The semi-image neighbourhood block graph $NB_I(G)$ of a graph G is outerplanar if and only if every component of G is a path.

III. MAIN RESULTS

Lemma 1. *If G is K_4 or $K_{2,3}$, then $Cr[NB_I(G)]=3$.*

Proof. Suppose *G* is K_4 or $K_{2,3}$. Then it is easy to see that the graphs $NB_I(K_4)$ and $NB_I(K_{2,3})$ are nonplanar and drawing in Figure 3 shows that its crossing number is at most 3. One may also observe that $NB_I(K_4)$ and $NB_I(K_{2,3})$ cannot be drawn without having a K_4 or $K_{2,3}$ line crossed. However, Figure 4 shows that the deletion of such a point leaves a nonplanar graph (it contains a homeomorphic of $K_{2,3}$). Therefore the crossing number must equal to 3.

We prove that for any graph *G*, semi-image neighbourhood block graph $NB_I(G)$ never has crossing number $k(k=1 \text{ or } 2)$.

Theorem 1. *The semi-image neighborhood block graph $NB_I(G)$ of a graph G never has crossing number $k(k=1 \text{ or } 2)$.*

Proof. We consider the following two cases.

Case 1. Suppose $\Delta(G)\leq 2$. Then *G* is either a path or a cycle. We now consider the following subcases of this case.

Subcase1.1. Assume *G* is a path. Then by Theorem B and Theorem A, $NB_I(G)$ is planar and has crossing number zero.

Subcase 1.2. Assume *G* is a cycle. Then by Theorem A, $NB_I(G)$ is planar. Hence $Cr[NB_I(G)]=0$.

Case 2. Suppose $\Delta(G)\geq 3$. Then we have the following subcases.

Subcase 2.1. Assume *G* is tree. Then by Theorem A, $Cr[NB_I(G)]=0$.

Subcase 2.2. Assume *G* is not a tree. Again we consider subcases of subcase 2.2.

Subcase 2.2.1. Suppose every block of *G* is a cycle or *G* has a block which contains at least two cycles such that $i(G)=0$. Then by Theorem A, $NB_I(G)$ has crossing number zero.

Subcase 2.2.2. Suppose every block of *G* is a cycle or *G* has a block, which contains at least two cycles. Assume there exists a block *B* with $i(B)\geq 1$. Then by Lemma 1, $NB_I(B)$ has at least 3 crossings. Hence $Cr[NB_I(G)]\geq 7$.

We have exhausted all possibilities and in each case we found that $NB_I(G)$ of a graph *G* never has crossing number $k(k=1 \text{ or } 2)$.

We now present characterizations of graphs whose semi-image neighbourhood block graphs are planar and outerplanar in terms of forbidden subgraphs.

Theorem 2. *The semi-image neighbourhood block graph $NB_I(G)$ of a graph G is planar if and only if G has no subgraphhomeomorphic to $K_{2,3}$ or K_4.*

Proof. Let $NB_I(G)$ is planar. By Theorem B, $i(G)=0$. Suppose *G* has subgraphhomeomorphic to $K_{2,3}$ or K_4. Then $NB_I(K_{2,3})$ or $NB_I(K_4)$ is nonplanar by Lemma 1, a contradiction. Hence *G*ahs no subgarphhomeomorphic to $K_{2,3}$ or K_4.

Conversely, *G* has no subgarphhomeomorphic to $K_{2,3}$ or K_4. Then clearly *G* is outerplanar. By Theorem B, the semi-image neighbourhood block graph $NB_I(G)$ is planar. This completes the proof.

Theorem 3. *The semi-image neighbourhood block graph $NB_I(G)$ is outerplanar if and only if G has no subgraphhomeomorphic to $K_{1,3}$ or C_3.*

Proof. Let $NB_I(G)$ is outerplanar. Then by Theorem C, every component of *G* is a path. Hence *G* has no subgraphhomeomorphic to $K_{1,3}$ or C_3.

Conversely, suppose *G* has no subgraphhomeomorphic to $K_{1,3}$or C_3. Then $\Delta(G)\leq 2$. Thus *G* must be a path. This completes the proof.

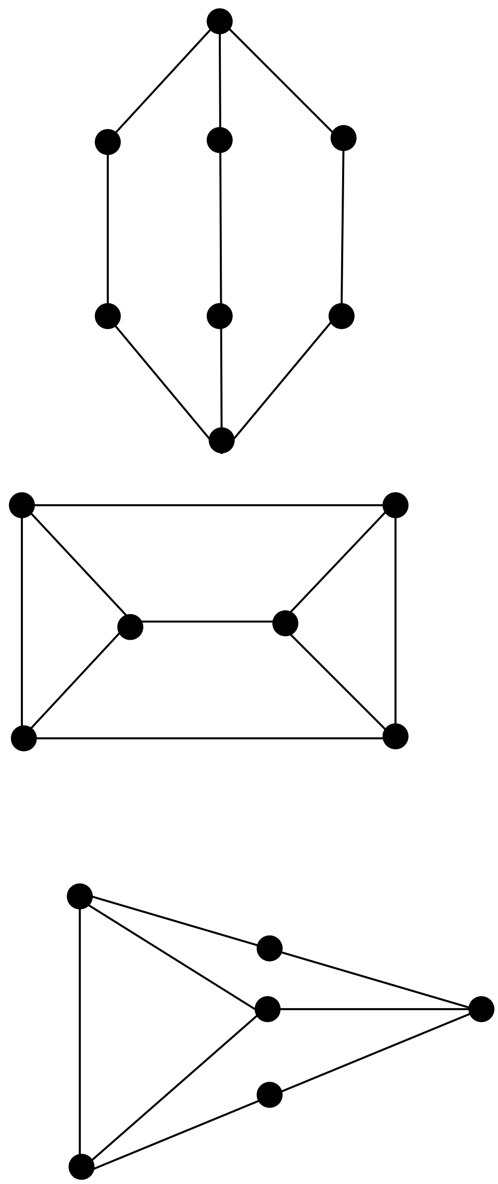
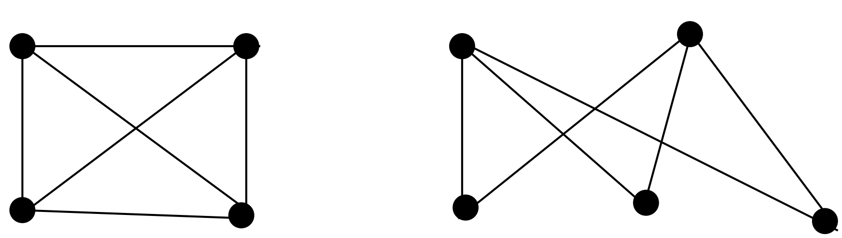


Fig. 2



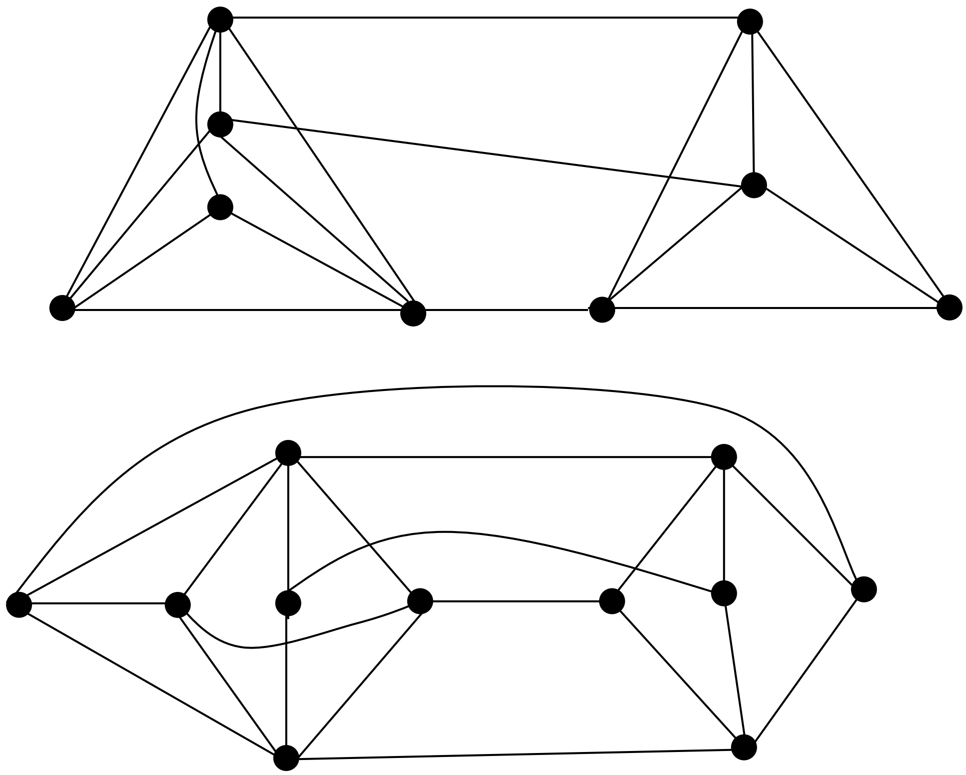
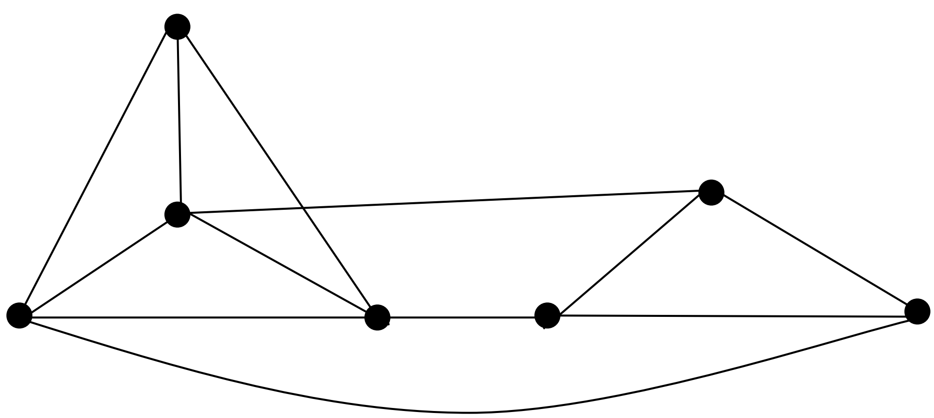


Fig. 3



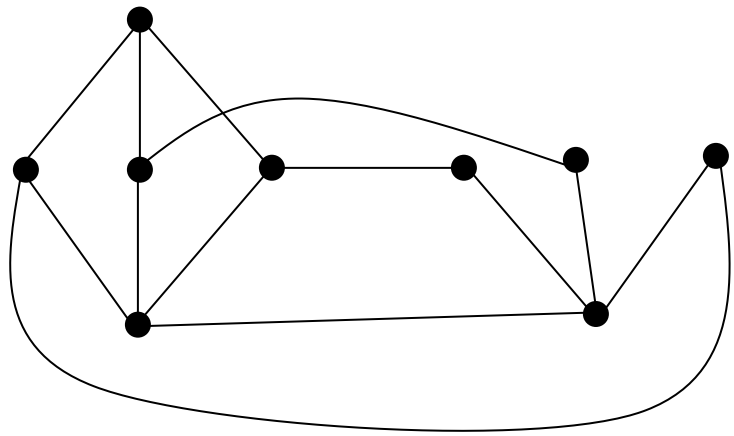


Fig. 4

IV. CONCLUSION

We present here whose semi-image neighbourhood block graphs are never has crossing number *k (k=1 or 2)*. We further to find a characterizations of graphs whose semi-image neighbourhood block graphs are planar and outerplanar in terms of forbidden subgraphs.

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