

Original Article

# Some Special Operators on Bipolar Intuitionistic Fuzzy Ideal and Bipolar Intuitionistic Anti Fuzzy Ideal of a BP-Algebra

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**Abstract** - The concept of a bipolar intuitionistic fuzzy ideal and bipolar intuitionistic anti fuzzy ideal are a new algebraic structure of BP-algebra and to use special operators. The purpose of this study is to implement the fuzzy set theory and ideal theory of a BP-algebra. The relation between the operation of special operators  $P_{\alpha,\alpha',\beta,\beta'}$ ,  $Q_{\alpha,\alpha',\beta,\beta'}$  and  $G_{\alpha,\alpha',\beta,\beta'}$  on bipolar intuitionistic fuzzy ideal and bipolar intuitionistic anti fuzzy ideal are established.

**Keywords** - BP-algebra, fuzzy ideal, bipolar fuzzy ideal, bipolar intuitionistic fuzzy ideal, bipolar intuitionistic anti fuzzy ideal,  $P_{\alpha,\alpha',\beta,\beta'}$ ,  $Q_{\alpha,\alpha',\beta,\beta'}$  and  $G_{\alpha,\alpha',\beta,\beta'}$ .

## I. INTRODUCTION

The concept of fuzzy sets was initiated by I.A.Zadeh [14] then it has become a vigorous area of research in engineering, medical science, graph theory. S.S.Ahn [2] gave the idea of BP-algebra. Bipolar valued fuzzy sets was introduced by K.J.Lee [6] are an extension of fuzzy sets whose positive membership degree range is enlarged from the interval [0, 1] to [-1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the positive membership degree (0, 1] indicates that elements somewhat satisfies the property and the negative membership degree [-1, 0) indicates that elements somewhat satisfies the implicit counter property. The author W.R.Zhang [15] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1998. K.Chakrabarty and Biswas R.Nanda [3] investigated note on union and intersection of intuitionistic fuzzy sets. A.Rajeshkumar [13] was analyzed fuzzy groups and level subgroups. K.Gunasekaran, S.Nandakumar and S.Sivakaminathan [16] introduced the definition of bipolar intuitionistic fuzzy ideal of a BP-algebra.

## II. PRELIMINARIES

### Definition: 1

Let A and B be any two bipolar intuitionistic fuzzy set  $A = (\mu_A^P, \mu_A^N, v_A^P, v_A^N)$  and  $B = (\mu_B^P, \mu_B^N, v_B^P, v_B^N)$  in X, we define

- i)  $A \cap B = \{(x, \min(\mu_A^P(x), \mu_B^P(x)), \max(\mu_A^N(x), \mu_B^N(x)), \max(v_A^P(x), v_B^P(x)), \min(v_A^N(x), v_B^N(x))) / x \in X\}$
- ii)  $A \cup B = \{(x, \max(\mu_A^P(x), \mu_B^P(x)), \min(\mu_A^N(x), \mu_B^N(x)), \min(v_A^P(x), v_B^P(x)), \max(v_A^N(x), v_B^N(x))) / x \in X\}$
- iii)  $\bar{A} = \{(x, v_A^P(x), v_A^N(x), \mu_A^P(x), \mu_A^N(x)) / x \in X\}.$

### Definition: 2

A bipolar intuitionistic fuzzy set  $A = \{\mu_A^P, \mu_A^N, v_A^P, v_A^N / x \in X\}$  of BP-algebra X is called a bipolar intuitionistic fuzzy ideal of X if it satisfies the following conditions:

- i)  $\mu_A^P(0) \geq \mu_A^P(x)$  and  $\mu_A^N(0) \leq \mu_A^N(x)$
- ii)  $\mu_A^P(x) \geq \min\{\mu_A^P(x * y), \mu_A^P(y)\}$



- iii)  $\mu_A^N(x) \leq \max \{ \mu_A^N(x * y), \mu_A^N(y) \}$
- iv)  $\nu_A^P(0) \leq \nu_A^P(x)$  and  $\nu_A^N(0) \geq \nu_A^N(x)$
- v)  $\nu_A^P(x) \leq \max \{ \nu_A^P(x * y), \nu_A^P(y) \}$
- vi)  $\nu_A^N(x) \geq \min \{ \nu_A^N(x * y), \nu_A^N(y) \}$ , for all  $x, y \in X$ .

**Definition: 3**

A bipolar intuitionistic fuzzy set  $A = \{ \mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N / x \in X \}$  of BP-algebra  $X$  is called a bipolar intuitionistic anti fuzzy ideal of  $X$  if it satisfies the following conditions:

- i)  $\mu_A^P(0) \leq \mu_A^P(x)$  and  $\mu_A^N(0) \geq \mu_A^N(x)$
- ii)  $\mu_A^P(x) \leq \max \{ \mu_A^P(x * y), \mu_A^P(y) \}$
- iii)  $\mu_A^N(x) \geq \min \{ \mu_A^N(x * y), \mu_A^N(y) \}$
- iv)  $\nu_A^P(0) \geq \nu_A^P(x)$  and  $\nu_A^N(0) \leq \nu_A^N(x)$
- v)  $\nu_A^P(x) \geq \min \{ \nu_A^P(x * y), \nu_A^P(y) \}$
- vi)  $\nu_A^N(x) \leq \max \{ \nu_A^N(x * y), \nu_A^N(y) \}$ , for all  $x, y \in X$ .

**Definition: 4**

Let  $A$  is a bipolar intuitionistic fuzzy set of  $X$ , then

$$P_{\alpha, \alpha', \beta, \beta'}(A) = \{ (x, \max(\alpha, \mu_A^P(x)), \min(\alpha', \mu_A^N(x)), \min(\beta, \nu_A^P(x)), \max(\beta', \nu_A^N(x)) / x \in X \}, \text{ for } \alpha, \beta \in [0, 1], \alpha', \beta' \in [-1, 0] \text{ and } \alpha + \beta \leq 1, \alpha' + \beta' \geq -1.$$

**Definition: 5**

Let  $A$  is a bipolar intuitionistic fuzzy set of  $X$ , then

$$Q_{\alpha, \alpha', \beta, \beta'}(A) = \{ (x, \min(\alpha, \mu_A^P(x)), \max(\alpha', \mu_A^N(x)), \max(\beta, \nu_A^P(x)), \min(\beta', \nu_A^N(x)) / x \in X \}, \text{ for } \alpha, \beta \in [0, 1], \alpha', \beta' \in [-1, 0] \text{ and } \alpha + \beta \leq 1, \alpha' + \beta' \geq -1.$$

**Definition: 6**

Let  $A$  is a bipolar intuitionistic fuzzy set of  $X$ , then

$$G_{\alpha, \alpha', \beta, \beta'}(A) = \{ (x, \alpha \mu_A^P(x), \alpha' \mu_A^N(x), \beta \nu_A^P(x), \beta' \nu_A^N(x)) / x \in X \}, \text{ for } \alpha, \beta \in [0, 1], \alpha', \beta' \in [-1, 0] \text{ and } \alpha + \beta \leq 1, \alpha' + \beta' \geq -1.$$

### III. SPECIAL OPERATORS ON BIPOLAR INTUITIONISTIC FUZZY IDEAL

**Theorem: 1**

If  $A$  is a bipolar intuitionistic fuzzy ideal of  $X$ , then  $P_{\alpha, \alpha', \beta, \beta'}(A)$  is a bipolar intuitionistic fuzzy ideal of  $X$ .

**Proof:** Given  $A$  is a bipolar intuitionistic fuzzy ideal of  $X$ . Consider  $0, x, y \in A$ .

- i) Now  $\mu_{P_{\alpha, \alpha', \beta, \beta'}}(0) = \max(\alpha, \mu_A^P(0))$ 

$$\geq \max(\alpha, \mu_A^P(x))$$

$$= \mu_{P_{\alpha, \alpha', \beta, \beta'}}(x)$$

Therefore  $\mu_{P_{\alpha, \alpha', \beta, \beta'}}(0) \geq \mu_{P_{\alpha, \alpha', \beta, \beta'}}(x)$ .

Now  $\mu_{P_{\alpha, \alpha', \beta, \beta'}}^N(0) = \min(\alpha', \mu_A^N(0))$ 

$$\leq \min(\alpha', \mu_A^N(x))$$

$$= \mu_{P_{\alpha, \alpha', \beta, \beta'}}^N(x)$$

Therefore  $\mu_{P_{\alpha, \alpha', \beta, \beta'}}^N(0) \leq \mu_{P_{\alpha, \alpha', \beta, \beta'}}^N(x)$ .
- ii) Now  $\mu_{P_{\alpha, \alpha', \beta, \beta'}}(x) = \max(\alpha, \mu_A^P(x))$ 

$$\geq \max(\alpha, \min\{\mu_A^P(x * y), \mu_A^P(y)\})$$

$$= \min\{\max(\alpha, \mu_A^P(x * y)), \max(\alpha, \mu_A^P(y))\}$$

$$= \min\{\mu_{P_{\alpha, \alpha', \beta, \beta'}}^P(x * y), \mu_{P_{\alpha, \alpha', \beta, \beta'}}^P(y)\}$$

Therefore  $\mu_{P_{\alpha, \alpha', \beta, \beta'}}(x) \geq \min\{\mu_{P_{\alpha, \alpha', \beta, \beta'}}^P(x * y), \mu_{P_{\alpha, \alpha', \beta, \beta'}}^P(y)\}$ .
- iii) Now  $\mu_{P_{\alpha, \alpha', \beta, \beta'}}^N(x) = \min(\alpha', \mu_A^N(x))$ 

$$\leq \min(\alpha', \max\{\mu_A^N(x * y), \mu_A^N(y)\})$$

$$= \max \{ \min (\alpha', \mu_A^N(x * y)), \min (\alpha', \mu_A^N(y)) \} \\ = \max \{ \mu_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(x * y), \mu_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(y) \}$$

Therefore  $\mu_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(x) \leq \max \{ \mu_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(x * y), \mu_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(y) \}$ .

iv) Now  $v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(0) = \min (\beta, v_A^P(0))$

$$\leq \min (\beta, v_A^P(x)) \\ = v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(x)$$

Therefore  $v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(0) \leq v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(x)$ .

Now  $v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(0) = \max (\beta', v_A^N(0))$

$$\geq \max (\beta', v_A^N(x)) \\ = v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(x)$$

Therefore  $v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(0) \geq v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(x)$ .

v) Now  $v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(x) = \min (\beta, v_A^P(x))$

$$\leq \min (\beta, \max \{ v_A^P(x * y), v_A^P(y) \}) \\ = \max \{ \min (\beta, v_A^P(x * y)), \min (\beta, v_A^P(y)) \} \\ = \max \{ v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(x * y), v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(y) \}$$

Therefore  $v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(x) \leq \max \{ v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(x * y), v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(y) \}$ .

vi) Now  $v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(x) = \max (\beta', v_A^N(x))$

$$\geq \max (\beta', \min \{ v_A^N(x * y), v_A^N(y) \}) \\ = \min \{ \max (\beta', v_A^N(x * y)), \max (\beta', v_A^N(y)) \} \\ = \min \{ v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(x * y), v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(y) \}$$

Therefore  $v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(x) \geq \min \{ v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(x * y), v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(y) \}$ .

Therefore  $P_{\alpha, \alpha', \beta, \beta'}(A)$  is a bipolar intuitionistic fuzzy ideal of X.

### Theorem: 2

If A and B are bipolar intuitionistic fuzzy ideal of X, then  $P_{\alpha, \alpha', \beta, \beta'}(A \cap B) = P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)$  is also a bipolar intuitionistic fuzzy ideal of X, and for every  $\alpha, \beta \in [0, 1]$ ,  $\alpha', \beta' \in [-1, 0]$  and  $\alpha + \beta \leq 1$ ,  $\alpha' + \beta' \geq -1$ .

**Proof:** Let A and B are bipolar intuitionistic fuzzy ideal of X. Consider 0, x, y  $\in A \cap B$  then 0, x, y  $\in A$  and 0, x, y  $\in B$ .

i) Now  $\mu_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^P(0) = \max (\alpha, \mu_{A \cap B}^P(0))$

$$= \max (\alpha, \min \{ \mu_A^P(0), \mu_B^P(0) \}) \\ \geq \max (\alpha, \min \{ \mu_A^P(x), \mu_B^P(x) \}) \\ = \min \{ \max (\alpha, \mu_A^P(x)), \max (\alpha, \mu_B^P(x)) \} \\ = \min \{ \mu_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(x), \mu_{P_{\alpha, \alpha', \beta, \beta'}(B)}^P(x) \} \\ = \mu_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^P(x)$$

Therefore  $\mu_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^P(0) \geq \mu_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^P(x)$ .

Now  $\mu_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^N(0) = \min (\alpha', \mu_{A \cap B}^N(0))$

$$= \min (\alpha', \max \{ \mu_A^N(0), \mu_B^N(0) \}) \\ \leq \min (\alpha', \max \{ \mu_A^N(x), \mu_B^N(x) \}) \\ = \max \{ \min (\alpha', \mu_A^N(x)), \min (\alpha', \mu_B^N(x)) \} \\ = \max \{ \mu_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(x), \mu_{P_{\alpha, \alpha', \beta, \beta'}(B)}^N(x) \} \\ = \mu_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^N(x)$$

Therefore  $\mu_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^N(0) \leq \mu_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^N(x)$ .

ii) Now  $\mu_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^P(x) = \max (\alpha, \mu_{A \cap B}^P(x))$

$$= \max (\alpha, \min \{ \mu_A^P(x), \mu_B^P(x) \})$$

$$\geq \max (\alpha, \min \{ \min \{ \mu_A^P(x * y), \mu_A^P(y) \}, \min \{ \mu_B^P(x * y), \mu_B^P(y) \} \})$$

$$\begin{aligned}
 &= \max(\alpha, \min\{\min\{\mu_A^P(x * y), \mu_B^P(x * y)\}, \min\{\mu_A^P(y), \mu_B^P(y)\}\}) \\
 &= \min\{\max(\alpha, \min\{\mu_A^P(x * y), \mu_B^P(x * y)\}), \max(\alpha, \min\{\mu_A^P(y), \mu_B^P(y)\})\} \\
 &= \min\{\min\{\max(\alpha, \mu_A^P(x * y)), \max(\alpha, \mu_B^P(x * y))\}, \min\{\max(\alpha, \mu_A^P(y)), \max(\alpha, \mu_B^P(y))\}\} \\
 &= \min\{\min\{\mu_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(x * y), \mu_{P_{\alpha, \alpha', \beta, \beta'}(B)}^P(x * y)\}, \min\{\mu_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(y), \mu_{P_{\alpha, \alpha', \beta, \beta'}(B)}^P(y)\}\} \\
 &= \min\{\mu_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^P(x * y), \mu_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^P(y)\}
 \end{aligned}$$

Therefore

$$\begin{aligned}
 \mu_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^P(x) &\geq \min\{\mu_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^P(x * y), \mu_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^P(y)\}. \\
 \text{iii) Now } \mu_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^N(x) &= \min(\alpha', \mu_{A \cap B}^N(x)) \\
 &= \min(\alpha', \max\{\mu_A^N(x), \mu_B^N(x)\}) \\
 &\leq \min(\alpha', \max\{\max\{\mu_A^N(x * y), \mu_A^N(y)\}, \max\{\mu_B^N(x * y), \mu_B^N(y)\}\}) \\
 &= \min(\alpha', \max\{\max\{\mu_A^N(x * y), \mu_B^N(x * y)\}, \max\{\mu_A^N(y), \mu_B^N(y)\}\}) \\
 &= \max\{\min(\alpha', \max\{\mu_A^N(x * y), \mu_B^N(x * y)\}), \min(\alpha', \max\{\mu_A^N(y), \mu_B^N(y)\})\} \\
 &= \max\{\max\{\min(\alpha', \mu_A^N(x * y)), \min(\alpha', \mu_B^N(x * y))\}, \max\{\min(\alpha', \mu_A^N(y)), \min(\alpha', \mu_B^N(y))\}\} \\
 &= \max\{\max\{\mu_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(x * y), \mu_{P_{\alpha, \alpha', \beta, \beta'}(B)}^N(x * y)\}, \max\{\mu_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(y), \mu_{P_{\alpha, \alpha', \beta, \beta'}(B)}^N(y)\}\} \\
 &= \max\{\mu_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^N(x * y), \mu_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^N(y)\}
 \end{aligned}$$

Therefore

$$\begin{aligned}
 \mu_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^N(x) &\leq \max\{\mu_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^N(x * y), \mu_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^N(y)\}. \\
 \text{iv) Now } v_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^P(0) &= \min(\beta, v_{A \cap B}^P(0)) \\
 &= \min(\beta, \max(v_A^P(0), v_B^P(0))) \\
 &\leq \min(\beta, \max(v_A^P(x), v_B^P(x))) \\
 &= \max\{\min(\beta, v_A^P(x)), \min(\beta, v_B^P(x))\} \\
 &= \max\{v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(x), v_{P_{\alpha, \alpha', \beta, \beta'}(B)}^P(x)\} \\
 &= v_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^P(x)
 \end{aligned}$$

Therefore  $v_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^P(0) \leq v_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^P(x)$ .

$$\begin{aligned}
 \text{Now } v_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^N(0) &= \max(\beta', v_{A \cap B}^N(0)) \\
 &= \max(\beta', \min(v_A^N(0), v_B^N(0))) \\
 &\geq \max(\beta', \min(v_A^N(x), v_B^N(x))) \\
 &= \min\{\max(\beta', v_A^N(x)), \max(\beta', v_B^N(x))\} \\
 &= \min\{v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^N(x), v_{P_{\alpha, \alpha', \beta, \beta'}(B)}^N(x)\} \\
 &= v_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^N(x)
 \end{aligned}$$

Therefore  $v_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^N(0) \geq v_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^N(x)$ .

$$\begin{aligned}
 \text{v) Now } v_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^P(x) &= \min(\beta, v_{A \cap B}^P(x)) \\
 &= \min(\beta, \max\{v_A^P(x), v_B^P(x)\}) \\
 &\leq \min(\beta, \max\{\max\{v_A^P(x * y), v_A^P(y)\}, \max\{v_B^P(x * y), v_B^P(y)\}\}) \\
 &= \min(\beta, \max\{\max\{v_A^P(x * y), v_B^P(x * y)\}, \max\{v_A^P(y), v_B^P(y)\}\}) \\
 &= \max\{\min(\beta, \max\{v_A^P(x * y), v_B^P(x * y)\}), \min(\beta, \max\{v_A^P(y), v_B^P(y)\})\} \\
 &= \max\{\max\{\min(\beta, v_A^P(x * y)), \min(\beta, v_B^P(x * y))\}, \max\{\min(\beta, v_A^P(y)), \min(\beta, v_B^P(y))\}\} \\
 &= \max\{\max\{v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(x * y), v_{P_{\alpha, \alpha', \beta, \beta'}(B)}^P(x * y)\}, \max\{v_{P_{\alpha, \alpha', \beta, \beta'}(A)}^P(y), v_{P_{\alpha, \alpha', \beta, \beta'}(B)}^P(y)\}\} \\
 &= \max\{v_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^P(x * y), v_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^P(y)\}
 \end{aligned}$$

Therefore

$$v_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^P(x) \leq \max\{v_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^P(x * y), v_{P_{\alpha, \alpha', \beta, \beta'}(A) \cap P_{\alpha, \alpha', \beta, \beta'}(B)}^P(y)\}.$$

$$\begin{aligned}
 \text{vi) Now } v_{P_{\alpha, \alpha', \beta, \beta'}(A \cap B)}^N(x) &= \max(\beta', v_{A \cap B}^N(x)) \\
 &= \max(\beta', \min(v_A^N(x), v_B^N(x))) \\
 &\geq \max(\beta', \min(\min\{v_A^N(x * y), v_A^N(y)\}, \min\{v_B^N(x * y), v_B^N(y)\})) \\
 &= \max(\beta', \min(\min\{v_A^N(x * y), v_B^N(x * y)\}, \min\{v_A^N(y), v_B^N(y)\}))
 \end{aligned}$$

$$\begin{aligned}
 &= \min\{\max(\beta', \min\{\nu_A^N(x * y), \nu_B^N(x * y)\}), \max(\beta', \min\{\nu_A^N(y), \nu_B^N(y)\})\} \\
 &= \min\{\min\{\max(\beta', \nu_A^N(x * y)), \max(\beta', \nu_B^N(x * y))\}, \min\{\max(\beta', \nu_A^N(y)), \max(\beta', \nu_B^N(y))\}\} \\
 &= \min\{\min\{\nu_{P_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y)), \nu_{P_{\alpha,\alpha',\beta,\beta'}(B)}^N(x * y)\}, \min\{\nu_{P_{\alpha,\alpha',\beta,\beta'}(A)}^N(y), \nu_{P_{\alpha,\alpha',\beta,\beta'}(B)}^N(y)\}\} \\
 &= \min\{\nu_{P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)}}^N(x * y), \nu_{P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)}}^N(y)\}
 \end{aligned}$$

Therefore

$$\nu_{P_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(x) \geq \min\{\nu_{P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)}}^N(x * y), \nu_{P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)}}^N(y)\}.$$

Therefore  $P_{\alpha,\alpha',\beta,\beta'}(A \cap B) = P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)$  is a bipolar intuitionistic fuzzy ideal of X.

### Theorem: 3

If A is a bipolar intuitionistic fuzzy ideal of X, then  $Q_{\alpha,\alpha',\beta,\beta'}(A)$  is a bipolar intuitionistic fuzzy ideal of X.

**Proof:** Given A is a bipolar intuitionistic fuzzy ideal of X. Consider 0, x, y  $\in A$ .

$$\begin{aligned}
 i) \quad \text{Now } \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(0) &= \min(\alpha, \mu_A^P(0)) \\
 &\geq \min(\alpha, \mu_A^P(x)) \\
 &= \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x)
 \end{aligned}$$

Therefore  $\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(0) \geq \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x)$ .

$$\begin{aligned}
 \text{Now } \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(0) &= \max(\alpha', \mu_A^N(0)) \\
 &\leq \max(\alpha', \mu_A^N(x)) \\
 &= \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x)
 \end{aligned}$$

Therefore  $\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(0) \leq \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x)$ .

$$\begin{aligned}
 ii) \quad \text{Now } \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x) &= \min(\alpha, \mu_A^P(x)) \\
 &\geq \min(\alpha, \min\{\mu_A^P(x * y), \mu_A^P(y)\}) \\
 &= \min\{\min(\alpha, \mu_A^P(x * y)), \min(\alpha, \mu_A^P(y))\} \\
 &= \min\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x * y), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(y)\}
 \end{aligned}$$

Therefore  $\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x) \geq \min\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x * y), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(y)\}$ .

$$\begin{aligned}
 iii) \quad \text{Now } \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x) &= \max(\alpha', \mu_A^N(x)) \\
 &\leq \max(\alpha', \max\{\mu_A^N(x * y), \mu_A^N(y)\}) \\
 &= \max\{\max(\alpha', \mu_A^N(x * y)), \max(\alpha', \mu_A^N(y))\} \\
 &= \max\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(y)\}
 \end{aligned}$$

Therefore  $\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x) \leq \max\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(y)\}$ .

$$\begin{aligned}
 iv) \quad \text{Now } \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(0) &= \max(\beta, \nu_A^P(0)) \\
 &\leq \max(\beta, \nu_A^P(x)) \\
 &= \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x)
 \end{aligned}$$

Therefore  $\nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(0) \leq \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x)$ .

$$\begin{aligned}
 \text{Now } \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(0) &= \min(\beta', \nu_A^N(0)) \\
 &\geq \min(\beta', \nu_A^N(x)) \\
 &= \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x)
 \end{aligned}$$

Therefore  $\nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(0) \geq \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x)$ .

$$\begin{aligned}
 v) \quad \text{Now } \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x) &= \max(\beta, \nu_A^P(x)) \\
 &\leq \max(\beta, \max\{\nu_A^P(x * y), \nu_A^P(y)\}) \\
 &= \max\{\max(\beta, \nu_A^P(x * y)), \max(\beta, \nu_A^P(y))\} \\
 &= \max\{\nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x * y), \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(y)\}
 \end{aligned}$$

Therefore  $\nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x) \leq \max\{\nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x * y), \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(y)\}$ .

$$vi) \quad \text{Now } \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x) = \min(\beta', \nu_A^N(x))$$

$$\begin{aligned} &\geq \min(\beta', \min\{\nu_A^N(x * y), \nu_A^N(y)\}) \\ &= \min\{\min(\beta', \nu_A^N(x * y)), \min(\beta', \nu_A^N(y))\} \\ &= \min\{\nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y), \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(y)\} \end{aligned}$$

Therefore  $\nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x) \geq \min\{\nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y), \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(y)\}$ .

Therefore  $Q_{\alpha,\alpha',\beta,\beta'}(A)$  is a bipolar intuitionistic fuzzy ideal of  $X$ .

**Theorem: 4**

If  $A$  and  $B$  are bipolar intuitionistic fuzzy ideal of  $X$ , then  $Q_{\alpha,\alpha',\beta,\beta'}(A \cap B) = Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)$  is also a bipolar intuitionistic fuzzy ideal of  $X$ , and for every  $\alpha, \beta \in [0, 1]$ ,  $\alpha', \beta' \in [-1, 0]$  and  $\alpha + \beta \leq 1$ ,  $\alpha' + \beta' \geq -1$ .

**Proof:** Let  $A$  and  $B$  are bipolar intuitionistic fuzzy ideal of  $X$ . Consider  $0, x, y \in A \cap B$  then  $0, x, y \in A$  and  $0, x, y \in B$ .

$$\begin{aligned} i) \quad \text{Now } \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(0) &= \min(\alpha, \mu_{A \cap B}^P(0)) \\ &= \min(\alpha, \min\{\mu_A^P(0), \mu_B^P(0)\}) \\ &\geq \min(\alpha, \min\{\mu_A^P(x), \mu_B^P(x)\}) \\ &= \min\{\min(\alpha, \mu_A^P(x)), \min(\alpha, \mu_B^P(x))\} \\ &= \min\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(x)\} \\ &= \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(x) \end{aligned}$$

Therefore  $\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(0) \geq \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(x)$ .

$$\begin{aligned} \text{Now } \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(0) &= \max(\alpha', \mu_{A \cap B}^N(0)) \\ &= \max(\alpha', \max\{\mu_A^N(0), \mu_B^N(0)\}) \\ &\leq \max(\alpha', \max\{\mu_A^N(x), \mu_B^N(x)\}) \\ &= \max\{\max(\alpha', \mu_A^N(x)), \max(\alpha', \mu_B^N(x))\} \\ &= \max\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(x)\} \\ &= \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(x) \end{aligned}$$

Therefore  $\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(0) \leq \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(x)$ .

$$\begin{aligned} ii) \quad \text{Now } \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(x) &= \min(\alpha, \mu_{A \cap B}^P(x)) \\ &= \min(\alpha, \min\{\mu_A^P(x), \mu_B^P(x)\}) \\ &\geq \min(\alpha, \min\{\min\{\mu_A^P(x * y), \mu_A^P(y)\}, \min\{\mu_B^P(x * y), \mu_B^P(y)\}\}) \\ &= \min(\alpha, \min\{\min\{\mu_A^P(x * y), \mu_B^P(x * y)\}, \min\{\mu_A^P(y), \mu_B^P(y)\}\}) \\ &= \min\{\min(\alpha, \min\{\mu_A^P(x * y), \mu_B^P(x * y)\}), \min(\alpha, \min\{\mu_A^P(y), \mu_B^P(y)\})\} \\ &= \min\{\min\{\min(\alpha, \mu_A^P(x * y)), \min(\alpha, \mu_B^P(x * y))\}, \min\{\min(\alpha, \mu_A^P(y)), \min(\alpha, \mu_B^P(y))\}\} \\ &= \min\{\min\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x * y), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(x * y)\}, \min\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(y), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(y)\}\} \\ &= \min\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(x * y), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(y)\} \end{aligned}$$

Therefore

$$\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(x) \geq \min\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(x * y), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(y)\}.$$

$$\begin{aligned} iii) \quad \text{Now } \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(x) &= \max(\alpha', \mu_{A \cap B}^N(x)) \\ &= \max(\alpha', \max\{\mu_A^N(x), \mu_B^N(x)\}) \\ &\leq \max(\alpha', \max\{\max\{\mu_A^N(x * y), \mu_A^N(y)\}, \max\{\mu_B^N(x * y), \mu_B^N(y)\}\}) \\ &= \max(\alpha', \max\{\max\{\mu_A^N(x * y), \mu_B^N(x * y)\}, \max\{\mu_A^N(y), \mu_B^N(y)\}\}) \\ &= \max\{\max(\alpha', \max\{\mu_A^N(x * y), \mu_B^N(x * y)\}), \max(\alpha', \max\{\mu_A^N(y), \mu_B^N(y)\})\} \\ &= \max\{\max\{\max(\alpha', \mu_A^N(x * y)), \max(\alpha', \mu_B^N(x * y))\}, \max\{\max(\alpha', \mu_A^N(y)), \max(\alpha', \mu_B^N(y))\}\} \\ &= \max\{\max\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(x * y)\}, \max\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(y), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(y)\}\} \\ &= \max\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(x * y), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(y)\} \end{aligned}$$

Therefore

$$\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(x) \leq \max\{\mu_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(x * y), \mu_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(y)\}.$$

$$\begin{aligned} iv) \quad \text{Now } \nu_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(0) &= \max(\beta, \nu_{A \cap B}^P(0)) \\ &= \max(\beta, \max(\nu_A^P(0), \nu_B^P(0))) \end{aligned}$$

$$\begin{aligned}
 &\leq \max(\beta, \max(v_A^P(x), v_B^P(x))) \\
 &= \max\{\max(\beta, v_A^P(x)), \max(\beta, v_B^P(x))\} \\
 &= \max\{v_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x), v_{Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(x)\} \\
 &= v_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(x)
 \end{aligned}$$

Therefore  $v_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(0) \leq v_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(x)$ .

$$\begin{aligned}
 \text{Now } v_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(0) &= \min(\beta', v_{A \cap B}^N(0)) \\
 &= \min(\beta', \min(v_A^N(0), v_B^N(0))) \\
 &\geq \min(\beta', \min(v_A^N(x), v_B^N(x))) \\
 &= \min\{\min(\beta', v_A^N(x)), \min(\beta', v_B^N(x))\} \\
 &= \min\{v_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x), v_{Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(x)\} \\
 &= v_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(x)
 \end{aligned}$$

Therefore  $v_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(0) \geq v_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(x)$ .

$$\begin{aligned}
 \text{v) Now } v_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(x) &= \max(\beta, v_{A \cap B}^P(x)) \\
 &= \max(\beta, \max\{v_A^P(x), v_B^P(x)\}) \\
 &\leq \max(\beta, \max\{\max\{v_A^P(x * y), v_A^P(y)\}, \max\{v_B^P(x * y), v_B^P(y)\}\}) \\
 &= \max(\beta, \max\{\max\{v_A^P(x * y), v_B^P(x * y)\}, \max\{v_A^P(y), v_B^P(y)\}\}) \\
 &= \max\{\max(\beta, \max\{v_A^P(x * y), v_B^P(x * y)\}), \max(\beta, \max\{v_A^P(y), v_B^P(y)\})\} \\
 &= \max\{\max(\beta, v_A^P(x * y)), \max(\beta, v_B^P(x * y))\}, \max\{\max(\beta, v_A^P(y)), \max(\beta, v_B^P(y))\} \\
 &= \max\{\max\{v_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(x * y), v_{Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(x * y)\}, \max\{v_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^P(y), v_{Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(y)\}\} \\
 &= \max\{v_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(x * y), v_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(y)\}
 \end{aligned}$$

Therefore

$$v_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(x) \leq \max\{v_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(x * y), v_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^P(y)\}.$$

$$\begin{aligned}
 \text{vi) Now } v_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(x) &= \min(\beta', v_{A \cap B}^N(x)) \\
 &= \min(\beta', \min(v_A^N(x), v_B^N(x))) \\
 &\geq \min(\beta', \min(\min\{v_A^N(x * y), v_A^N(y)\}, \min\{v_B^N(x * y), v_B^N(y)\})) \\
 &= \min(\beta', \min(\min\{v_A^N(x * y), v_B^N(x * y)\}, \min\{v_A^N(y), v_B^N(y)\})) \\
 &= \min\{\min(\beta', \min\{v_A^N(x * y), v_B^N(x * y)\}), \min(\beta', \min\{v_A^N(y), v_B^N(y)\})\} \\
 &= \min\{\min(\min(\beta', v_A^N(x * y)), \min(\beta', v_B^N(x * y))), \min(\min(\beta', v_A^N(y)), \min(\beta', v_B^N(y)))\} \\
 &= \min\{\min\{v_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y), v_{Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(x * y)\}, \min\{v_{Q_{\alpha,\alpha',\beta,\beta'}(A)}^N(y), v_{Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(y)\}\} \\
 &= \min\{v_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(x * y), v_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(y)\}
 \end{aligned}$$

Therefore

$$v_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(x) \geq \min\{v_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(x * y), v_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}^N(y)\}.$$

Therefore  $Q_{\alpha,\alpha',\beta,\beta'}(A \cap B) = Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)$  is also a bipolar intuitionistic fuzzy ideal of X.

### Theorem: 5

If A is a bipolar intuitionistic fuzzy ideal of X, then  $G_{\alpha,\alpha',\beta,\beta'}(A)$  is also a bipolar intuitionistic fuzzy ideal of X.

**Proof:** Given A is a bipolar intuitionistic fuzzy ideal of X. Consider 0, x, y  $\in$  A.

$$\begin{aligned}
 \text{i) Now } \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(0) &= \alpha \mu_A^P(0) \\
 &\geq \alpha \mu_A^P(x) \\
 &= \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x)
 \end{aligned}$$

Therefore  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(0) \geq \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x)$ .

$$\begin{aligned}
 \text{Now } \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(0) &= \alpha' \mu_A^N(0) \\
 &\leq \alpha' \mu_A^N(x) \\
 &= \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x)
 \end{aligned}$$

Therefore  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(0) \leq \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x)$ .

- ii) Now  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x) = \alpha \mu_A^P(x)$
- $$\begin{aligned} &\geq \alpha \min \{ \mu_A^P(x * y), \mu_A^P(y) \} \\ &= \min \{ \alpha \mu_A^P(x * y), \alpha \mu_A^P(y) \} \\ &= \min \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x * y), \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(y) \} \end{aligned}$$
- Therefore  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x) \geq \min \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x * y), \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(y) \}$ .
- iii) Now  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x) = \alpha' \mu_A^N(x)$
- $$\begin{aligned} &\leq \alpha' \max \{ \mu_A^N(x * y), \mu_A^N(y) \} \\ &= \max \{ \alpha' \mu_A^N(x * y), \alpha' \mu_A^N(y) \} \\ &= \max \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y), \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(y) \} \end{aligned}$$
- Therefore  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x) \leq \max \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y), \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(y) \}$ .
- iv) Now  $v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(0) = \beta v_A^P(0)$
- $$\begin{aligned} &\leq \beta v_A^P(x) \\ &= v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x) \end{aligned}$$
- Therefore  $v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(0) \leq v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x)$ .
- Now  $v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(0) = \beta' v_A^N(0)$
- $$\begin{aligned} &\geq \beta' v_A^N(x) \\ &= v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x) \end{aligned}$$
- Therefore  $v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(0) \geq v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x)$ .
- v) Now  $v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x) = \beta v_A^P(x)$
- $$\begin{aligned} &\leq \beta \max \{ v_A^P(x * y), v_A^P(y) \} \\ &= \max \{ \beta v_A^P(x * y), \beta v_A^P(y) \} \\ &= \max \{ v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x * y), v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(y) \} \end{aligned}$$
- Therefore  $v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x) \leq \max \{ v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x * y), v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(y) \}$ .
- vi) Now  $v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x) = \beta' v_A^N(x)$
- $$\begin{aligned} &\geq \beta' \min \{ v_A^N(x * y), v_A^N(y) \} \\ &= \min \{ \beta' v_A^N(x * y), \beta' v_A^N(y) \} \\ &= \min \{ v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y), v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(y) \} \end{aligned}$$
- Therefore  $v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x) \geq \min \{ v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y), v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(y) \}$ .
- Therefore  $G_{\alpha,\alpha',\beta,\beta'}(A)$  is a bipolar intuitionistic fuzzy ideal of X.

### Theorem: 6

If A and B are bipolar intuitionistic fuzzy ideal of X, then  $G_{\alpha,\alpha',\beta,\beta'}(A \cap B) = G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)$  is also a bipolar intuitionistic fuzzy ideal of X, and for every  $\alpha, \beta \in [0, 1]$ ,  $\alpha', \beta' \in [-1, 0]$  and  $\alpha + \beta \leq 1$ ,  $\alpha' + \beta' \geq -1$ .

**Proof:** Let A and B are bipolar intuitionistic fuzzy ideal of X. Consider 0, x, y  $\in A \cap B$  then 0, x, y  $\in A$  and 0, x, y  $\in B$ .

i) Now  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(0) = \alpha \mu_{A \cap B}^P(0)$

$$\begin{aligned} &= \alpha \min \{ \mu_A^P(0), \mu_B^P(0) \} \\ &\geq \alpha \min \{ \mu_A^P(x), \mu_B^P(x) \} \\ &= \min \{ \alpha \mu_A^P(x), \alpha \mu_B^P(x) \} \\ &= \min \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x), \mu_{G_{\alpha,\alpha',\beta,\beta'}(B)}^P(x) \} \\ &= \mu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(x) \end{aligned}$$

Therefore  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(0) \geq \mu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^P(x)$

Now  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(0) = \alpha' \mu_{A \cap B}^N(0)$

$$\begin{aligned} &= \alpha' \max \{ \mu_A^N(0), \mu_B^N(0) \} \\ &\leq \alpha' \max \{ \mu_A^N(x), \mu_B^N(x) \} \end{aligned}$$

$$\begin{aligned}
 &= \max \{\alpha' \mu_A^N(x), \alpha' \mu_B^N(x)\} \\
 &= \max \{\mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x), \mu_{G_{\alpha,\alpha',\beta,\beta'}(B)}^N(x)\} \\
 &= \mu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^N(x)
 \end{aligned}$$

Therefore  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(0) \leq \mu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^N(x)$ .

ii) Now  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(x) = \alpha \mu_{A \cap B}^P(x)$

$$\begin{aligned}
 &= \alpha \min \{ \mu_A^P(x), \mu_B^P(x) \} \\
 &\geq \alpha \min \{ \min \{ \mu_A^P(x * y), \mu_B^P(y) \}, \min \{ \mu_B^P(x * y), \mu_B^P(y) \} \} \\
 &= \alpha \min \{ \min \{ \mu_A^P(x * y), \mu_B^P(x * y) \}, \min \{ \mu_A^P(y), \mu_B^P(y) \} \} \\
 &= \min \{ \min \{ \alpha \mu_A^P(x * y), \alpha \mu_B^P(x * y) \}, \min \{ \alpha \mu_A^P(y), \alpha \mu_B^P(y) \} \} \\
 &= \min \{ \min \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x * y), \mu_{G_{\alpha,\alpha',\beta,\beta'}(B)}^P(x * y) \}, \min \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(y), \mu_{G_{\alpha,\alpha',\beta,\beta'}(B)}^P(y) \} \} \\
 &= \min \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^P(x * y), \mu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^P(y) \}
 \end{aligned}$$

Therefore

$$\mu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(x) \geq \min \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^P(x * y), \mu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^P(y) \}.$$

iii) Now  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(x) = \alpha' \mu_{A \cap B}^N(x)$

$$\begin{aligned}
 &= \alpha' \max \{ \mu_A^N(x), \mu_B^N(x) \} \\
 &\leq \alpha' \max \{ \max \{ \mu_A^N(x * y), \mu_A^N(y) \}, \max \{ \mu_B^N(x * y), \mu_B^N(y) \} \} \\
 &= \alpha' \max \{ \max \{ \mu_A^N(x * y), \mu_B^N(x * y) \}, \max \{ \mu_A^N(y), \mu_B^N(y) \} \} \\
 &= \max \{ \max \{ \alpha' \mu_A^N(x * y), \alpha' \mu_B^N(x * y) \}, \max \{ \alpha' \mu_A^N(y), \alpha' \mu_B^N(y) \} \} \\
 &= \max \{ \max \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y), \mu_{G_{\alpha,\alpha',\beta,\beta'}(B)}^N(x * y) \}, \max \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(y), \mu_{G_{\alpha,\alpha',\beta,\beta'}(B)}^N(y) \} \} \\
 &= \max \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^N(x * y), \mu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^N(y) \}
 \end{aligned}$$

Therefore

$$\mu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(x) \leq \max \{ \mu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^N(x * y), \mu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^N(y) \}.$$

iv) Now  $\nu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(0) = \beta \nu_{A \cap B}^P(0)$

$$\begin{aligned}
 &= \beta \max \{ \nu_A^P(0), \nu_B^P(0) \} \\
 &\leq \beta \max \{ \nu_A^P(x), \nu_B^P(x) \} \\
 &= \max \{ \beta \nu_A^P(x), \beta \nu_B^P(x) \} \\
 &= \max \{ \nu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x), \nu_{G_{\alpha,\alpha',\beta,\beta'}(B)}^P(x) \} \\
 &= \nu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^P(x)
 \end{aligned}$$

Therefore  $\nu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(0) \leq \nu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^P(x)$ .

Now  $\nu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(0) = \beta' \nu_{A \cap B}^N(0)$

$$\begin{aligned}
 &= \beta' \min \{ \nu_A^N(0), \nu_B^N(0) \} \\
 &\geq \beta' \min \{ \nu_A^N(x), \nu_B^N(x) \} \\
 &= \min \{ \beta' \nu_A^N(x), \beta' \nu_B^N(x) \} \\
 &= \min \{ \nu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x), \nu_{G_{\alpha,\alpha',\beta,\beta'}(B)}^N(x) \} \\
 &= \nu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^N(x)
 \end{aligned}$$

Therefore  $\nu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(0) \geq \nu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^N(x)$ .

v) Now  $\nu_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(x) = \beta \nu_{A \cap B}^P(x)$

$$\begin{aligned}
 &= \beta \max \{ \nu_A^P(x), \nu_B^P(x) \} \\
 &\leq \beta \max \{ \max \{ \nu_A^P(x * y), \nu_A^P(y) \}, \max \{ \nu_B^P(x * y), \nu_B^P(y) \} \} \\
 &= \beta \max \{ \max \{ \nu_A^P(x * y), \nu_B^P(x * y) \}, \max \{ \nu_A^P(y), \nu_B^P(y) \} \} \\
 &= \max \{ \max \{ \beta \nu_A^P(x * y), \beta \nu_B^P(x * y) \}, \max \{ \beta \nu_A^P(y), \beta \nu_B^P(y) \} \} \\
 &= \max \{ \max \{ \nu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(x * y), \nu_{G_{\alpha,\alpha',\beta,\beta'}(B)}^P(x * y) \}, \max \{ \nu_{G_{\alpha,\alpha',\beta,\beta'}(A)}^P(y), \nu_{G_{\alpha,\alpha',\beta,\beta'}(B)}^P(y) \} \} \\
 &= \max \{ \nu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^P(x * y), \nu_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^P(y) \}
 \end{aligned}$$

Therefore

$$\begin{aligned}
 v_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^P(x) &\leq \max \{ v_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^P(x * y), v_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}^N(y) \}. \\
 \text{vi) Now } v_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(x) &= \beta' v_{A \cap B}^N(x) \\
 &= \beta' \min \{ v_A^N(x), v_B^N(x) \} \\
 &\geq \beta' \min \{ \min \{ v_A^N(x * y), v_A^N(y) \}, \min \{ v_B^N(x * y), v_B^N(y) \} \} \\
 &= \beta' \min \{ \min \{ v_A^N(x * y), v_B^N(x * y) \}, \min \{ v_A^N(y), v_B^N(y) \} \} \\
 &= \min \{ \min \{ \beta' v_A^N(x * y), \beta' v_B^N(x * y) \}, \min \{ \beta' v_A^N(y), \beta' v_B^N(y) \} \} \\
 &= \min \{ \min \{ v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(x * y), v_{G_{\alpha,\alpha',\beta,\beta'}(B)}^N(x * y) \}, \min \{ v_{G_{\alpha,\alpha',\beta,\beta'}(A)}^N(y), v_{G_{\alpha,\alpha',\beta,\beta'}(B)}^N(y) \} \} \\
 &= \min \{ v_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}}^N(x * y), v_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}}^N(y) \}
 \end{aligned}$$

Therefore

$$v_{G_{\alpha,\alpha',\beta,\beta'}(A \cap B)}^N(x) \geq \min \{ v_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}}^N(x * y), v_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}}^N(y) \}.$$

Therefore  $G_{\alpha,\alpha',\beta,\beta'}(A \cap B) = G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)$  is also a bipolar intuitionistic fuzzy ideal of X.

### Theorem: 7

If A is a bipolar intuitionistic fuzzy ideal of X, then  $\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})} = Q_{\beta,\beta',\alpha,\alpha'}(A)$  is also a bipolar intuitionistic fuzzy ideal of X.

**Proof:** Given A is a bipolar intuitionistic fuzzy ideal of X. Consider 0, x, y  $\in A$ .

$$\begin{aligned}
 \text{i) Now } \mu_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^P(0) &= v_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^P(0) \\
 &= \min \{ \beta, v_{\bar{A}}^P(0) \} \\
 &= \min \{ \beta, \mu_A^P(0) \} \\
 &\geq \min \{ \beta, \mu_A^P(x) \} \\
 &= \mu_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^P(x)
 \end{aligned}$$

$$\text{Therefore } \mu_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^P(0) \geq \mu_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^P(x).$$

$$\begin{aligned}
 \text{Now } \mu_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^N(0) &= v_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^N(0) \\
 &= \max \{ \beta', v_{\bar{A}}^N(0) \} \\
 &= \max \{ \beta', \mu_A^N(0) \} \\
 &\leq \max \{ \beta', \mu_A^N(x) \} \\
 &= \mu_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^N(x)
 \end{aligned}$$

$$\text{Therefore } \mu_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^N(0) \leq \mu_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^N(x).$$

$$\begin{aligned}
 \text{ii) Now } \mu_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^P(x) &= v_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^P(x) \\
 &= \min \{ \beta, v_{\bar{A}}^P(x) \} \\
 &= \min \{ \beta, \mu_A^P(x) \} \\
 &\geq \min \{ \beta, \min \{ \mu_A^P(x * y), \mu_A^P(y) \} \} \\
 &= \min \{ \min \{ \beta, \mu_A^P(x * y) \}, \min \{ \beta, \mu_A^P(y) \} \} \\
 &= \min \{ \mu_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^P(x * y), \mu_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^P(y) \}
 \end{aligned}$$

$$\text{Therefore } \mu_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^P(x) \geq \min \{ \mu_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^P(x * y), \mu_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^P(y) \}.$$

$$\begin{aligned}
 \text{iii) Now } \mu_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^N(x) &= v_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^N(x) \\
 &= \max \{ \beta', v_{\bar{A}}^N(x) \} \\
 &= \max \{ \beta', \mu_A^N(x) \} \\
 &\leq \max \{ \beta', \max \{ \mu_A^N(x * y), \mu_A^N(y) \} \} \\
 &= \max \{ \max \{ \beta', \mu_A^N(x * y) \}, \max \{ \beta', \mu_A^N(y) \} \} \\
 &= \max \{ \mu_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^N(x * y), \mu_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^N(y) \}
 \end{aligned}$$

$$\text{Therefore } \mu_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^N(x) \leq \max \{ \mu_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^N(x * y), \mu_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^N(y) \}.$$

$$\begin{aligned}
 \text{iv) Now } v_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^P(0) &= \mu_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}^P(0) \\
 &= \max \{ \alpha, \mu_{\bar{A}}^P(0) \}
 \end{aligned}$$

$$\begin{aligned}
 &= \max \{ \alpha, v_A^P(0) \} \\
 &\leq \max \{ \alpha, v_A^P(x) \} \\
 &= v_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^P(x)
 \end{aligned}$$

Therefore  $v_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^P(0) \leq v_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^P(x)$ .

$$\begin{aligned}
 \text{Now } v_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^N(0) &= \mu_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^N(0) \\
 &= \min \{ \alpha', \mu_A^N(0) \} \\
 &= \min \{ \alpha', v_A^N(0) \} \\
 &\geq \min \{ \alpha', v_A^N(x) \} \\
 &= v_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^N(x)
 \end{aligned}$$

Therefore  $v_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^N(0) \geq v_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^N(x)$ .

$$\begin{aligned}
 \text{v) Now } v_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^P(x) &= \mu_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^P(x) \\
 &= \max \{ \alpha, \mu_A^P(x) \} \\
 &= \max \{ \alpha, v_A^P(x) \} \\
 &\leq \max \{ \alpha, \max \{ v_A^P(x * y), v_A^P(y) \} \} \\
 &= \max \{ \max \{ \alpha, v_A^P(x * y) \}, \max \{ \alpha, v_A^P(y) \} \} \\
 &= \max \{ v_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^P(x * y), v_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^P(y) \}
 \end{aligned}$$

Therefore  $v_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^P(x) \leq \max \{ v_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^P(x * y), v_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^P(y) \}$ .

$$\begin{aligned}
 \text{vi) Now } v_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^N(x) &= \mu_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^N(x) \\
 &= \min \{ \alpha', \mu_A^N(x) \} \\
 &= \min \{ \alpha', v_A^N(x) \} \\
 &\geq \min \{ \alpha', \min \{ v_A^N(x * y), v_A^N(y) \} \} \\
 &= \min \{ \min \{ \alpha', v_A^N(x * y) \}, \min \{ \alpha', v_A^N(y) \} \} \\
 &= \min \{ v_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^N(x * y), v_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^N(y) \}
 \end{aligned}$$

Therefore  $v_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^N(x) \geq \min \{ v_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^N(x * y), v_{Q_{\beta,\beta',\alpha,\alpha'}(A)}^N(y) \}$ .

Therefore  $P_{\alpha,\alpha',\beta,\beta'}(\bar{A}) = Q_{\beta,\beta',\alpha,\alpha'}(A)$  is a bipolar intuitionistic fuzzy ideal of X.

### Theorem: 8

If A is a bipolar intuitionistic fuzzy ideal of X, then  $\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})} = G_{\beta,\beta',\alpha,\alpha'}(A)$  is also a bipolar intuitionistic fuzzy ideal of X.

**Proof:** Given A is a bipolar intuitionistic fuzzy ideal of X. Consider 0, x, y  $\in$  A.

$$\begin{aligned}
 \text{i) Now } \mu_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^P(0) &= v_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^P(0) \\
 &= \beta v_A^P(0) \\
 &= \beta \mu_A^P(0) \\
 &\geq \beta \mu_A^P(x) \\
 &= \mu_{G_{\beta,\beta',\alpha,\alpha'}(A)}^P(x)
 \end{aligned}$$

Therefore  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^P(0) \geq \mu_{G_{\beta,\beta',\alpha,\alpha'}(A)}^P(x)$ .

$$\begin{aligned}
 \text{Now } \mu_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^N(0) &= v_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^N(0) \\
 &= \beta' v_A^N(0) \\
 &= \beta' \mu_A^N(0) \\
 &\leq \beta' \mu_A^N(x) \\
 &= \mu_{G_{\beta,\beta',\alpha,\alpha'}(A)}^N(x)
 \end{aligned}$$

Therefore  $\mu_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^N(0) \leq \mu_{G_{\beta,\beta',\alpha,\alpha'}(A)}^N(x)$ .

$$\begin{aligned}
 \text{ii) Now } \mu_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^P(x) &= v_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}^P(x) \\
 &= \beta v_A^P(x)
 \end{aligned}$$

$$\begin{aligned}
 &= \beta \mu_A^P(x) \\
 &\geq \beta \min \{ \mu_A^P(x * y), \mu_A^P(y) \} \\
 &= \min \{ \beta \mu_A^P(x * y), \beta \mu_A^P(y) \} \\
 &= \min \{ \mu_{G_{\beta, \beta', \alpha, \alpha'}(A)}^P(x * y), \mu_{G_{\beta, \beta', \alpha, \alpha'}(A)}^P(y) \}
 \end{aligned}$$

Therefore  $\mu_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^P(x) \geq \min \{ \mu_{G_{\beta, \beta', \alpha, \alpha'}(A)}^P(x * y), \mu_{G_{\beta, \beta', \alpha, \alpha'}(A)}^P(y) \}$ .

iii) Now  $\mu_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^N(x) = v_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^N(x)$

$$\begin{aligned}
 &= \beta' v_{\bar{A}}^N(x) \\
 &= \beta' \mu_{\bar{A}}^N(x) \\
 &\leq \beta' \max \{ \mu_A^N(x * y), \mu_A^N(y) \} \\
 &= \max \{ \beta' \mu_A^N(x * y), \beta' \mu_A^N(y) \} \\
 &= \max \{ \mu_{G_{\beta, \beta', \alpha, \alpha'}(A)}^N(x * y), \mu_{G_{\beta, \beta', \alpha, \alpha'}(A)}^N(y) \}
 \end{aligned}$$

Therefore  $\mu_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^N(x) \leq \max \{ \mu_{G_{\beta, \beta', \alpha, \alpha'}(A)}^N(x * y), \mu_{G_{\beta, \beta', \alpha, \alpha'}(A)}^N(y) \}$ .

iv) Now  $v_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^P(0) = \mu_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^P(0)$

$$\begin{aligned}
 &= \alpha \mu_{\bar{A}}^P(0) \\
 &= \alpha v_{\bar{A}}^P(0) \\
 &\leq \alpha v_A^P(x) \\
 &= v_{G_{\beta, \beta', \alpha, \alpha'}(A)}^P(x)
 \end{aligned}$$

Therefore  $v_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^P(0) \leq v_{G_{\beta, \beta', \alpha, \alpha'}(A)}^P(x)$ .

Now  $v_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^N(0) = \mu_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^N(0)$

$$\begin{aligned}
 &= \alpha' \mu_{\bar{A}}^N(0) \\
 &= \alpha' v_{\bar{A}}^N(0) \\
 &\geq \alpha' v_A^N(x) \\
 &= v_{G_{\beta, \beta', \alpha, \alpha'}(A)}^N(x)
 \end{aligned}$$

Therefore  $v_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^N(0) \geq v_{G_{\beta, \beta', \alpha, \alpha'}(A)}^N(x)$ .

v) Now  $v_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^P(x) = \mu_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^P(x)$

$$\begin{aligned}
 &= \alpha \mu_{\bar{A}}^P(x) \\
 &= \alpha v_{\bar{A}}^P(x) \\
 &\leq \alpha \max \{ v_A^P(x * y), v_A^P(y) \} \\
 &= \max \{ \alpha v_A^P(x * y), \alpha v_A^P(y) \} \\
 &= \max \{ v_{G_{\beta, \beta', \alpha, \alpha'}(A)}^P(x * y), v_{G_{\beta, \beta', \alpha, \alpha'}(A)}^P(y) \}
 \end{aligned}$$

Therefore  $v_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^P(x) \leq \max \{ v_{G_{\beta, \beta', \alpha, \alpha'}(A)}^P(x * y), v_{G_{\beta, \beta', \alpha, \alpha'}(A)}^P(y) \}$ .

vi) Now  $v_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^N(x) = \mu_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^N(x)$

$$\begin{aligned}
 &= \alpha' \mu_{\bar{A}}^N(x) \\
 &= \alpha' v_{\bar{A}}^N(x) \\
 &\geq \alpha' \min \{ v_A^N(x * y), v_A^N(y) \} \\
 &= \min \{ \alpha' v_A^N(x * y), \alpha' v_A^N(y) \} \\
 &= \min \{ v_{G_{\beta, \beta', \alpha, \alpha'}(A)}^N(x * y), v_{G_{\beta, \beta', \alpha, \alpha'}(A)}^N(y) \}
 \end{aligned}$$

Therefore  $v_{G_{\alpha, \alpha', \beta, \beta'}(\bar{A})}^N(x) \geq \min \{ v_{G_{\beta, \beta', \alpha, \alpha'}(A)}^N(x * y), v_{G_{\beta, \beta', \alpha, \alpha'}(A)}^N(y) \}$ .

Therefore  $G_{\alpha, \alpha', \beta, \beta'}(\bar{A}) = G_{\beta, \beta', \alpha, \alpha'}(A)$  is a bipolar intuitionistic fuzzy ideal of X.

**Theorem: 9**

If A is a bipolar intuitionistic anti fuzzy ideal of X, then  $P_{\alpha,\alpha',\beta,\beta'}(A)$  is a bipolar intuitionistic anti fuzzy ideal of X.

**Theorem: 10**

If A and B are bipolar intuitionistic anti fuzzy ideal of X, then  $P_{\alpha,\alpha',\beta,\beta'}(A \cap B) = P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)$  is also a bipolar intuitionistic anti fuzzy ideal of X, and for every  $\alpha, \beta \in [0, 1]$ ,  $\alpha', \beta' \in [-1, 0]$  and  $\alpha + \beta \leq 1$ ,  $\alpha' + \beta' \geq -1$ .

**Theorem: 11**

If A is a bipolar intuitionistic anti fuzzy ideal of X, then  $Q_{\alpha,\alpha',\beta,\beta'}(A)$  is a bipolar intuitionistic anti fuzzy ideal of X.

**Theorem: 12**

If A and B are bipolar intuitionistic anti fuzzy ideal of X, then  $Q_{\alpha,\alpha',\beta,\beta'}(A \cap B) = Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)$  is also a bipolar intuitionistic anti fuzzy ideal of X, and for every  $\alpha, \beta \in [0, 1]$ ,  $\alpha', \beta' \in [-1, 0]$  and  $\alpha + \beta \leq 1$ ,  $\alpha' + \beta' \geq -1$ .

**Theorem: 13**

If A is a bipolar intuitionistic anti fuzzy ideal of X, then  $G_{\alpha,\alpha',\beta,\beta'}(A)$  is also a bipolar intuitionistic anti fuzzy ideal of X.

**Theorem: 14**

If A and B are bipolar intuitionistic anti fuzzy ideal of X, then  $G_{\alpha,\alpha',\beta,\beta'}(A \cap B) = G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)$  is also a bipolar intuitionistic anti fuzzy ideal of X, and for every  $\alpha, \beta \in [0, 1]$ ,  $\alpha', \beta' \in [-1, 0]$  and  $\alpha + \beta \leq 1$ ,  $\alpha' + \beta' \geq -1$ .

**Theorem: 15**

If A is a bipolar intuitionistic anti fuzzy ideal of X, then  $\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})} = Q_{\beta,\beta',\alpha,\alpha'}(A)$  is also a bipolar intuitionistic anti fuzzy ideal of X.

**Theorem: 16**

If A is a bipolar intuitionistic anti fuzzy ideal of X, then  $\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})} = G_{\beta,\beta',\alpha,\alpha'}(A)$  is also a bipolar intuitionistic anti fuzzy ideal of X.

**IV. CONCLUSION**

In this paper, the main idea of a bipolar intuitionistic fuzzy ideal and bipolar intuitionistic anti fuzzy ideal are a new algebraic structure of BP-algebra and it is used through the special operators. The aim of this study is implemented. The relation between the operation of special operators  $P_{\alpha,\alpha',\beta,\beta'}$ ,  $Q_{\alpha,\alpha',\beta,\beta'}$  and  $G_{\alpha,\alpha',\beta,\beta'}$  on bipolar intuitionistic fuzzy ideal and bipolar intuitionistic anti fuzzy ideal are discussed. We believe that our ideas can also be applied for other algebraic system.

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