International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373 / https://doi.org/10.14445/22315373/IJMTT-V68I4P517

Volume 68 Issue 4, 107-110, April 2022

Original Article

The Aspect of Ceteris Paribus in Dynamical System

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Received Date: 19 March 2022
Revised Date: 23 April 2022
Accepted Date: 28 April 2022

Abstract *- In this paper, we introduce a new chaotic dynamical system that describes the interactions of particular kinds of populations in the circumstances of well-known postulates ceteris paribus in population dynamics. To accomplish this, we propose a mathematical model which consists of three-dimensional autonomous ordinary differential equations modelled: a population of mentally worried individuals, a population of physically stressed individuals, and a population of joyful persons. Initially, the model is non-chaotic and then we add a periodic forcing term to this and detect the chaotic behaviour under the condition of ceteris paribus.*

Keywords *- Ceteris Paribus, Chaos, Lyapunov Exponents, Stress, Periodic Forcing term.*

MSC 2010 No.: *34H10, 34D06, 34D20, 34C60, 92B05*

I. INTRODUCTION

A manageable mathematical description of psychological processes within the human body is useful in the field biomedical sciences and clinical research. These mathematical explanation helps to enhance the knowledge to the medical professionals or doctors. It can be use as research tool to illustrate, how psychological processes adapt different circumstances of the surrounding, in the human body. R. Silva; C.M. Layne and A. Steinberg; D.M. Chambliss and R.K. Schutt; G.B. David and J.O. Andrew; and M.J.C. Crump et al. ([20], [5]-[6],[8] and[15]), described human psychological behaviour by using the Law of ceteris paribus, which states that "the effect of one element on the body system when other factors remain unchanged". The Law ceteris paribus might be considered a crucial tool in the process of developing models for complex occurrences in biomedical sciences given in A.J. Culyer [1]. P. Nijkamp [19] observed the complex behaviour of dynamical systems by using the condition of ceteris paribus. In the field of medicine and health, the condition of ceteris paribus is studied by B. Guillonneau,; L.H. Thu et al.; Lilienfeld and M. Abraham; S. Weisgrau; and Z. Marek et al. ([3], [13]-[14], [24]-[25]). The condition of ceteris Paribus is widely used in the field of economics given in [4] and [21]. M. Kuzba et al. [17], explained the concept of machine learning with CAS under the circumstances of ceteris paribus.

From the last three decades, many researchers observed the chaos in dynamical systems as a result of ceteris paribus. Scientists, mathematicians, and engineers have spent a great deal of time studying chaos as a fascinating complex phenomenon in the last four decades. In various technological processes, such as information and computer science, power system protection, biological system analysis, flow dynamics and liquid mixing, encryption and communication, and so on, chaos has recently been discovered to be very beneficial and has significant potential explained by G. Chen and Dong X.; J. Lu.; and S. Chen and D. Lu. ([9]-[10], [12], [7]).

Recently, chaos theory has been applied to social and psychological issues. In this context, M. Battaglini [16] observed chaos in the dynamics of social problems due to ceteris paribus condition. The natural progression of these pioneering studies motivates researchers to delve deeper into the complexities of human psychology.

In this research work, we construct a dynamical model consisting of a system of ordinary differential equations which characterizes the three kinds of populations based on the facts studied by Chen and Lu [11]. The most important assumption we make about the model is that it is not a steady state system. This means that we calculate all over results in a time-dependent manner. To this aim, we divide the population into three categories namely: mentally stressed persons; physically stressed persons, and happy persons. Further, for a particular choice of parameter and initial conditions, we observe that the model is non-chaotic, and then by using the forcing term with the population of mentally stressed peoples, we observe that model exhibits the chaotic behaviour.

II. LYAPUNOV EXPONENTS

The chaotic behaviour of dynamical systems is quantified by Lyapunov exponents. We may correlate Lyapunov exponents with the rate of expansion and contraction of paths in state space with more than one dimension, so a measure of sensitivity to small changes in initial circumstances can be used to describe chaotic behaviour in a dynamical System. A. Lyapunov exponent can be determined using the time step *(t)* and the initial separation *(d_0)* as follows:

$$\lambda = \lim_{n\to\infty}\left(\frac{1}{n\Delta t}\sum_{i=1}^{n} ln\left|\frac{d_i}{d_0)}\right|\right). \qquad (1)$$

At step i, d_i is the distance between neighbouring points given in Otto and Day [22], Sandri [18] and Andrews and Nakita [2]. A Lyapunov exponent exists for each dimension of a dynamical system, which can be arranged in descending order ($\lambda_1 \geq \lambda_2 \geq \lambda_3 \ldots$) to give λ_1, the maximal Lyapunov exponent (MLE). Only the MLE will be explored in this paper, because chaos can be recognised by the criteria "MLE > 0 implies chaotic behaviour". Abernethy and Gooding [23] observed that MLE will dominate the divergence between neighbouring orbits for big enough n, and hence Equation (1) can be understood as an expression for the MLE.

III. MATHEMATICAL MODEL

Mathematically, chaos has been studied by many researchers but few of them worked upon chaos under the circumstances of ceteris paribus. In this section, we concentrate upon the Law of ceteris paribus and design a model, which structured by the three compartments. To achieve this, we categories the population in three classes and correspond to each class, we set a variable which represents the population namely: m, mentally stressed persons; p, physically stressed persons, and h, happy persons with following assumptions:

A1. Population of happy peoples decreases when they interact with the population of mentally stressed peoples.
A2. Population of happy people decreases.
A3. Population of Physically stressed people increase linearly.

The model, which is based on these assumptions, consists of three ordinary differential equations that classify the change in population, given as follows:

$$\frac{dm}{dt} = m^2 - ah^2 - p,$$
$$\frac{dp}{dt} = -amh + b,$$
$$\frac{dh}{dt} = mp - ch, \qquad (2)$$

with initial conditions *m(0) = m0, p(0) = p0, h(0) = h0.*
For System (2), we notice that with parameters $a = 5, b = 27, c = 2,$ and initial condition $m(0) = 1, p(0) = 1, h(0) = 10$, the Lyapunov exponents are $(-0.00013862, -2.69489, -7.43205)$. Therefore, the System (2) non-chaotic, shown in Fig. and
Fig shown the corresponding Lyapunov exponents spectra of System (2).

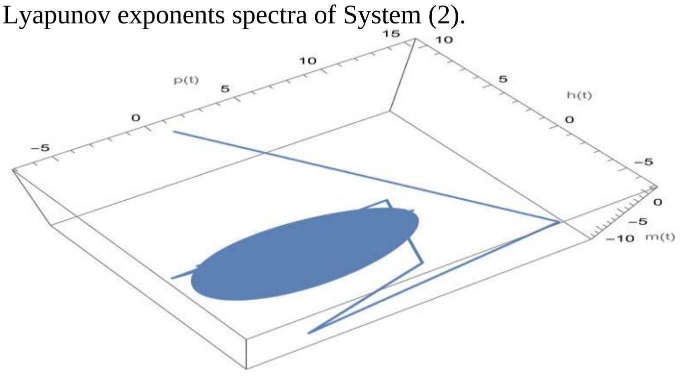

Fig. 1 Phase portraits of System 2

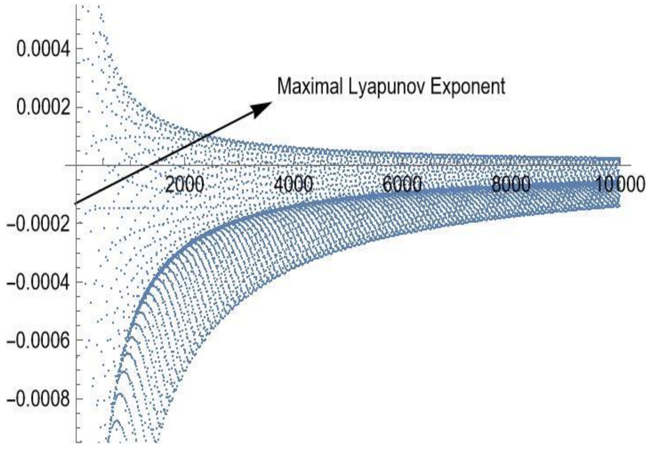

Fig. 2 Spectra of Lyapunov Exponents of System 2

Further we modified this model by using a periodic forcing term defined as "$k\ sin(2\ \pi t)$" and the condition of ceteris paribus, then System (2)**Error! Reference source not found.** redesign as

$$\frac{dm}{dt} = m^2 - ah^2 - p + ksin(2\pi t),$$
$$\frac{dp}{dt} = -amh + b,$$
$$\frac{dh}{dt} = mp - ch, \tag{3}$$

with initial conditions $m(0) = m0,\ p(0) = p0,\ h(0) = h0.$

IV. EXISTENCE OF CHAOS

In this section, we show that the System (*3*) exhibits the chaotic behaviour. For this purpose, we need to calculate the Lyapunov exponents of System (*3*). We set the value of parameter *"k = 20"*. For this particular value of k the Lyapunov Exponents of System (*3*) found as $(0.195341, -1.49738, -9.09906)$ and simulation to find these exponents is carried with the help of software Mathematica. As a result, System (*3*) appears to be a chaotic dynamical system and its phase portrait is shown in Fig. Fig represents the Lyapunov spectra of System (*3*), which shows a positive value of MLE.

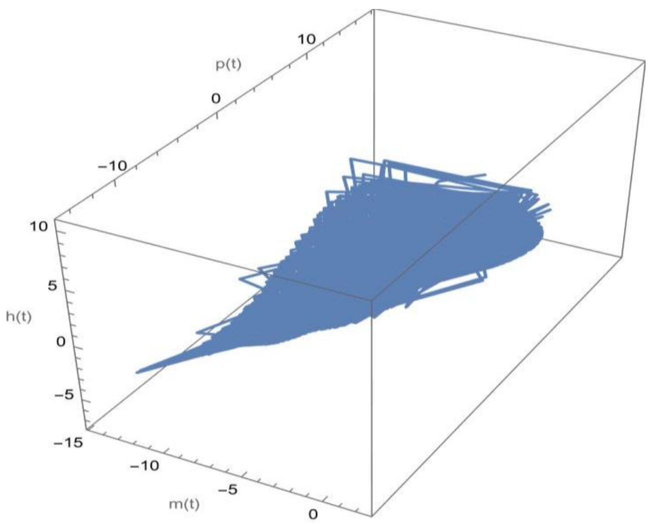

Fig. 3 Evolution of Chaos in modified System 3

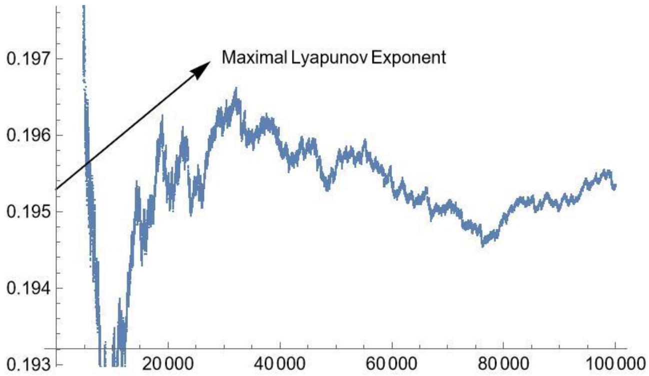

Fig. 4 Lyapunov spectra of System 3

V. CONCLUSION

The status of our emotional, psychological, and social well-being is referred to as mental health. The way we deal with stress, how we relate to people, and the decisions we make in our daily life are all influenced by our mental health. So, we need to maintain good mental health and leading a more satisfying and joyful life. For this we need a disordered mind. The purpose of this study is to introduce the chaos in the mind effectively and relieve stress to the chaotic mind in order to live a happy life.

ACKNOWLEDGEMENT

I would like to express my deep gratitude to the Centre for Fundamental Research in Space Dynamics and Celestial Mechanics for providing all the research facilities for the completion of this research work.

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