

Original Article

Neutrosophic Contra Irresolute Beta Omega Continuous Mapping

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Abstract - In this paper, we introduce the concepts of neutrosophic contra irresolute beta omega continuous mapping and analyze the properties of this mapping. Furthermore, we have defined and studied the concept of neutrosophic almost contra beta omega continuous mapping. Moreover, we have studied neutrosophic beta omega convergence.

Keywords - Neutrosophic contra irresolute beta omega continuous mapping, Neutrosophic almost contra beta omega continuous mapping, Neutrosophic beta omega convergent, Neutrosophic beta omega connected.

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1. Introduction

Fuzzy set theory introduced by Zadeh[25] has laid the foundation for the new mathematical theories in the research of mathematics. The concept “neutrosophic set” was first given by Smarandache[9]. Neutrosophic operations and Neutrosophic topological spaces have been investigated by Salama[20]. Later, Dhavaseelan[8] introduced neutrosophic contra irresolute beta omega continuous mapping. Here, we shall introduce neutrosophic contra irresolute beta omega continuous mapping, neutrosophic almost contra beta omega continuous mapping, neutrosophic beta omega convergent, neutrosophic beta omega connected.

2. Preliminaries

Definition 2.1. [9] Let X be a non-empty fixed set. A neutrosophic set (NS) G is an object having the form $G = \{ \langle x, \mu_G(x), \sigma_G(x), \nu_G(x) \rangle : x \in X \}$ where $\mu_G(x)$, $\sigma_G(x)$ and $\nu_G(x)$ represent the degree of membership, degree of indeterminacy and the degree of nonmembership respectively of each element $x \in X$ to the set G . A neutrosophic set $G = \{ \langle x, \mu_G(x), \sigma_G(x), \nu_G(x) \rangle : x \in X \}$ can be identified as an ordered triple $\langle \mu_G, \sigma_G, \nu_G \rangle$ in $]0, 1+[$ on X .

Definition 2.2. [2] For any two sets G and H ,

1. $G \subseteq H \Leftrightarrow \mu_G(x) \leq \mu_H(x), \sigma_G(x) \leq \sigma_H(x)$ and $\nu_G(x) \geq \nu_H(x), x \in X$
2. $G \cap H = \langle x, \mu_G(x) \wedge \mu_H(x), \sigma_G(x) \wedge \sigma_H(x), \nu_G(x) \vee \nu_H(x) \rangle$
3. $G \cup H = \langle x, \mu_G(x) \vee \mu_H(x), \sigma_G(x) \vee \sigma_H(x), \nu_G(x) \wedge \nu_H(x) \rangle$
4. $G^c = \{ \langle x, \nu_G(x), 1 - \sigma_G(x), \mu_G(x) \rangle : x \in X \}$
5. $0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$
6. $1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$.

Definition 2.3. [20] A neutrosophic topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:

1. $0_N, 1_N \subseteq \tau$
2. $G_1 \cap G_2 \subseteq \tau$ for any $G_1, G_2 \subseteq \tau$
3. $\cup G_i \subseteq \tau$ where $\{G_i : i \in J\} \subseteq \tau$



Here the pair (X, τ) is a neutrosophic topological space (NTS) and any neutrosophic set in τ is known as a neutrosophic open set (N-open set) in X . A neutrosophic set G is a neutrosophic closed set (N-closed set) if and only if its complement G^c is a neutrosophic open set in X .

Definition 2.4. [16] A neutrosophic set G of a neutrosophic topological space (X, τ) is called neutrosophic beta omega closed ($N\beta\omega$ -closed) if $\beta cl_N(G) \subseteq U$ whenever $G \subseteq U$ and U is $N\omega$ -open in (X, τ) .

Definition 2.5. [6] A nonempty family \mathcal{F} of N-open sets on (X, τ) is known as N – filter

1. if $0_N \notin \mathcal{F}$.
2. If $G, H \in \mathcal{F}$ then $G \cap H \in \mathcal{F}$.
3. If $G \in \mathcal{F}$ and $G \subseteq H$ then $H \in \mathcal{F}$.

Definition 2.6. [8] A nonempty family B of N-open sets on \mathcal{F} is named as N – filter base

1. If $0_N \notin B$.
2. If $G, H \in B$ then $I \subseteq G \cap H$ for some $I \in B$.

Definition 2.7. [8] A N-filter \mathcal{F} is called N – convergent to a N-point $x_{r,s,t}$ of a NTS (X, τ) if for each N-open set G of (X, τ) containing $x_{r,s,t}$, there exists a N-set $H \in \mathcal{F}$ such that $H \subseteq G$

Definition 2.8. [8] A N-filter \mathcal{F} is called NRC-convergent to a N-point $x_{r,s,t}$ of a NTS (X, τ) if for each NR-closed set G of (X, τ) containing $x_{r,s,t}$, there exists a N-set $H \in \mathcal{F}$ such that $H \subseteq G$

Definition 2.9. [8] A space (X, τ) is called as N-connected if (X, τ) cannot be written as union of two disjoint non empty N-open sets.

3. Neutrosophic Contra Irresolute Beta Omega Continuous Mapping

Definition 3.1. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called Neutrosophic Contra Irresolute Beta Omega Continuous (contra-irresolute- $N\beta\omega$ -continuous) if $f^{-1}(U)$ is $N\beta\omega$ -closed in (X, τ) for each $N\beta\omega$ -open set U in (Y, σ) .

Example 3.1. Let $X = [0, 1]$, $\tau = \{0_N, H(x), 1_N\}$, $\sigma = \{0_N, G^c(x), H^c(x), 1_N\}$, $(X, \tau) = (Y, \sigma) = \{0_N, G(x), H(x), G^c(x), H^c(x), 1_N\}$. Where

$$G(x) = \begin{cases} \langle x, x, 1 - x \rangle, & \text{if } 0 \leq x \leq \frac{1}{2} \\ \langle 1 - x, 1 - x, x \rangle, & \text{if } \frac{1}{2} \leq x \leq 1 \end{cases}$$

$$H(x) = \begin{cases} \langle x, x, 1 - x \rangle, & \text{if } 0 \leq x \leq \frac{1}{2} \\ \langle \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \rangle, & \text{if } \frac{1}{2} \leq x \leq 1 \end{cases}$$

Then τ and σ are NTs. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a$ where $a \in [0,1]$. Then f is contra-irresolute- $N\beta\omega$ -continuous mapping.

Proposition 3.1. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra-irresolute- $N\beta\omega$ -continuous map and $g : (Y, \sigma) \rightarrow (Z, \phi)$ is a contra- $N\beta\omega$ -continuous map then $g \circ f : (X, \tau) \rightarrow (Z, \phi)$ is $N\beta\omega$ -continuous.

Proof. Let U be N-open set in (Z, ϕ) . Since g is contra- $N\beta\omega$ -continuous, $g^{-1}(U)$ is $N\beta\omega$ -closed in (Y, σ) . Since f is contra-irresolute- $N\beta\omega$ -continuous, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is $N\beta\omega$ -open in (X, τ) . Thus $g \circ f$ is $N\beta\omega$ -continuous.

Proposition 3.2. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $N\beta\omega$ -irresolute map and $g : (Y, \sigma) \rightarrow (Z, \phi)$ is a contra- $N\beta\omega$ -continuous map then $g \circ f : (X, \tau) \rightarrow (Z, \phi)$ is contra- $N\beta\omega$ -continuous.

Proof. Let U be N-open set in (Z, ϕ) . Since g is contra- $N\beta\omega$ -continuous, $g^{-1}(U)$ is $N\beta\omega$ -closed in (Y, σ) . Since f is $N\beta\omega$ -irresolute, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is $N\beta\omega$ -closed in (X, τ) . Thus $g \circ f$ is N-contra- $N\beta\omega$ -continuous.

4. Neutrosophic Almost Contra Beta Omega Continuous Mapping

Definition 4.1. Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called as neutrosophic almost contra beta omega continuous (almost contra-N $\beta\omega$ -continuous) if inverse image of each NR-open set in (Y, σ) is N $\beta\omega$ -closed set in (X, τ) .

Theorem 4.1. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function and $g : (X, \tau) \rightarrow (X, \tau) \times (Y, \sigma)$ be the graph function defined by $g(P) = (P, f(P))$ for each $P \in (X, \tau)$. If g is almost contra-N $\beta\omega$ -continuous function, then f is almost contra-N $\beta\omega$ -continuous function.

Proof. Let G be a NR-closed set in (Y, σ) . Accordingly, $(X, \tau) \times G$ is a NR-closed set in $(X, \tau) \times (Y, \sigma)$. since g is almost contra N $\beta\omega$ continuous, so that $f^{-1}(G) = g^{-1}((X, \tau) \times G)$ is a N $\beta\omega$ -open in (X, τ) . Thus f is almost contra-N $\beta\omega$ -continuous.

Definition 4.2. A N-filter \mathcal{F} is called a N $\beta\omega$ -convergent to a N-point $x_{r,s,t}$ of a neutrosophic topological space (X, τ) , if for each N $\beta\omega$ -open set G of (X, τ) containing $x_{r,s,t}$ there exists a N-set $H \in \mathcal{F}$ so as $H \subseteq G$.

Example 4.3. Let $X = \{a, b, c\}$ and $\tau = \{0_N, G, 1_N\}$ where $G = \langle \xi, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.8}, \frac{b}{0.8}, \frac{c}{0.8}) \rangle$; $H = \langle \xi, (\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.1}), (\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.1}), (\frac{a}{0.2}, \frac{b}{0.1}, \frac{c}{0.1}) \rangle$; $I = \langle \xi, (\frac{a}{0.2}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.2}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.2}) \rangle$ and $\mathcal{F} = \{I, I^c, H^c, G^c, 1_N\}$. Then τ is a NT and consider the N-point $a_{0.3,0.2,0.1}$. Then \mathcal{F} is N $\beta\omega$ -convergent to a N-point $a_{0.3,0.2,0.1}$.

Proposition 4.1. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is almost contra-N $\beta\omega$ -continuous function and each N-filter base B in (X, τ) is N $\beta\omega$ -converging to $x_{r,s,t}$, then N-filter base $f(B)$ is NRC-convergent to the point $f(x_{r,s,t})$.

Proof. Let $x_{r,s,t} \in (X, \tau)$ and B be any N-filter base in (X, τ) and N $\beta\omega$ -converging to $x_{r,s,t}$. Since f is almost contra-N $\beta\omega$ -continuous, subsequently, for any NR-closed G in (Y, σ) including $f(x_{r,s,t})$, there exists N $\beta\omega$ -open H in (X, τ) consisting $x_{r,s,t}$. Therefore $f(H) \subseteq G$. As B is N $\beta\omega$ -convergent to $x_{r,s,t}$ and H is N $\beta\omega$ -open consisting $x_{r,s,t}$, there occurs $P \in B$ such that $P \subseteq H$. This means that $f(P) \subseteq G$ and consequently the N-filter base $f(B)$ is NRC-convergent to $f(x_{r,s,t})$.

Definition 4.3. A space (X, τ) is called a N $\beta\omega$ -connected, if (X, τ) can't be expressed as union of two disjoint non-empty N $\beta\omega$ -open sets.

Example 4.4. Let $X = \{a, b, c\}$, $\tau = \{0_N, G, 1_N\}$ and $(X, \tau) = \{0_N, G, H, I, G^c, H^c, I^c, 1_N\}$ where $G = \langle \xi, (\frac{a}{0.8}, \frac{b}{0.8}, \frac{c}{0.8}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7}), (\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}) \rangle$; $H = \langle \xi, (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}) \rangle$; $I = \langle \xi, (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9}), (\frac{a}{0.8}, \frac{b}{0.8}, \frac{c}{0.8}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}) \rangle$. Then (X, τ) is N $\beta\omega$ -connected

Theorem 4.2. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an almost contra-N $\beta\omega$ -continuous and surjection with (X, τ) is N $\beta\omega$ -connected space, then (Y, σ) is N-connected.

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an almost contra-N $\beta\omega$ -continuous and surjection with (X, τ) is N $\beta\omega$ connected. Suppose (Y, σ) is not N-connected. Accordingly, there exist disjoint N-open sets G and H such that $(Y, \sigma) = G \cup H$. Then G and H are N-clopen in (Y, σ) . Since f is almost contra-N $\beta\omega$ -continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are N $\beta\omega$ -open sets in (X, τ) . In addition $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint, non-empty and $(X, \tau) = f^{-1}(G) \cup f^{-1}(H)$ which is the contradiction to the fact that (X, τ) is N $\beta\omega$ -connected space. Hence, (Y, σ) is N-connected.

Definition 4.4. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called weakly almost contra-N $\beta\omega$ -continuous if for each N-point $x_{r,s,t}$ in (X, τ) and each NR-closed set V of (Y, σ) containing $f(x_{r,s,t})$, there exists a N $\beta\omega$ -open set U in (X, τ) , such that $cl_N(f(U)) \subseteq V$.

Definition 4.5. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called as N($\beta\omega, P$)-open if the image of each N $\beta\omega$ -open set is NP- open.

Theorem 4.3. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a weakly almost contra-N $\beta\omega$ -continuous and N($\beta\omega, P$)-open, then f is almost contra-N $\beta\omega$ -continuous.

Proof . Let $x_{r,s,t}$ be any N-point in (X, τ) and V be a NR-closed set containing $f(x_{r,s,t})$. Since f is weakly almost contra-N $\beta\omega$ -continuous, there exist a N $\beta\omega$ -open set U in (X, τ) containing $x_{r,s,t}$ so as $cl_N(f(U)) \subseteq V$. Since f is a N($\beta\omega, P$)-open, $f(U)$ is a NP-open set in (Y, σ) and $f(U) \subseteq int_N(cl_N(f(U))) \subseteq V$. This shows f is almost contra-N $\beta\omega$ -continuous.

5. Conclusion

In this paper, we defined the concepts of neutrosophic contra irresolute beta omega continuous mapping and studied the properties of this mapping. Furthermore, we have defined and analyzed the concept of neutrosophic almost contra beta omega continuous mapping. Moreover, we have studied neutrosophic beta omega convergence.

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