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Original Article

Domination Zagreb Indices of a Book Graph and Stacked Book Graph

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Abstract - *Domination Zagreb indices are the parameters defined using sum and product of degrees of vertices, joining an edge, in a graph using domination concept. In this paper we establish the relation for Domination Zagreb indices of book graph and Stacked book graph with its component graphs and also find their corresponding polynomials.*

Keywords – *Domination, Zagreb indices, Book graph, Stacked book graph.*

1. Introduction

The concepts of connectivity in Chemical Graph Theory, which define relationships between the structure of a molecule and its properties. One important parameter, topological index, which characterizes molecular graph and remain invariant under graph automorphism are called Zagreb Indices. These parameters introduced by Gutman I [5] are defined by using sum and product of degrees of vertices joining an edge. Here we are taking the domination set is a subset. . Consider a subset of E(G) denoted as E(G)={E(G)∈(u,v) /u ∈ D& v∈ V − D / d(u) = d(D) and d(v) = d(V-D) }. We are using the domination concept in Zagreb indices. The Zagreb indices, hyper Zagreb indices and their corresponding polynomials we use definitions given by Gutman [5]. And also we refer to [8] and [9]. Zagreb and hyper Zagreb indices of Book graph and Stacked book graph were proved by Kavitha B N and Indrani Pramod Kelkar [9].

1) $DM_1(G) = \sum_{D,V-D\in E(G)}[\deg(D) + \deg(V - D)]$
2) $DM_1(G,x) = \sum_{D,V-D\in E(G)} x^{[\deg(D)+\deg(V-D)]}$
3) $DM_2(G) = \sum_{D,V-D\in E(G)}[\deg(D)\deg(V - D)]$
4) $DM_2(G,x) = \sum_{D,V-D\in E(G)} x^{[\deg(D)\deg(V-D)]}$

The Domination hyper Zagreb indices and their corresponding Polynomials

5) $DHM_1(G) = \sum_{D,V-D\in E(G)}[\deg(D) + \deg(V - D)]^2$
6) $DHM_1(G,x) = \sum_{D,V-D\in E(G)} x^{[\deg(D)+\deg(V-D)]^2}$
7) $DHM_2(G) = \sum_{D,V-D\in E(G)}[\deg(D)\deg(V - D)]^2$
8) $DHM_2(G,x) = \sum_{D,V-D\in E(G)} x^{[\deg(D)\deg(V-D)]^2}$

In this paper we present results for Domination Zagreb indices, Domination hyper Zagreb indices and their polynomials for product graphs Book graph B_m and Stacked book graph $B_{m,n}$. We prove the domination Zagreb indices B_m and $B_{m,n}$ is greater than or equal to Zagreb indices B_m and $B_{m,n}$. We prove the domination hyper Zagreb indices B_m and $B_{m,n}$ is greater than or equal to hyper Zagreb indices B_m and $B_{m,n}$.

2. Domination Zagreb Indices Book Graph

Book Graph is a cross product of Star S_{m+1} and path P_2 where$m \geq 3$. The domination number of book graph is 2. We refer to Kavitha B N and Indrani Pramod Kelkar [6] & [7] . Dominating set is $D = \{(v, w_1), (v, w_2)\}$, $V - D = \{(u_i, w_1), (u_i, w_2)\}$ where $i = \{1,2,3, \dots \dots m\}$ from 6. The edge set condition $v_1 \in D$, $v_2 \in V - D$. The edge set B_m contain edges of one types $v_1v_2 \in E(B_m)$ $\therefore \left|E_{D,V-D\in E(B_m)}\right| = \left|E_{v_1v_{2\in E(B_m)}}\right| = \left|E_{m+1,2}\right| = 2m$

Theorem 2.1: The Domination first Zagreb indices and there polynomial for B_m are

$$DM_1(B_m) = 2m^2 + 6m$$
$$DM_1(B_m,x) = 2mx^{m+3}$$

Proof: The Domination first Zagreb indices and polynomial of B_m are defined as

$$DM_1(B_m) = \sum_{D,V-D\in E(B_m)}[\deg(D) + \deg(V - D)]$$
$$= \sum_{v_1v_2\in E(B_m)}[\deg(v_1) + \deg(v_2)]$$
$$v_1 \in D \;\&\; v_2 \in V - D$$

$$DM_1(B_m,x) = \sum_{D,V-D\in E(B_m)} x^{[\deg(D)+\deg(V-D)]}$$
$$= \sum_{v_1v_2\in E(B_m)} x^{[\deg(v_1)+\deg(v_2)]}$$
$$= \sum_{E_{m+1,2}} x^{[\deg(v_1)+\deg(v_2)]}$$

$= \sum_{E_{m+1,2}}[\deg(v_1) + \deg(v_2)]$ $\quad = 2mx^{[m+1+2]}$
$= 2m[m+3]$ $\quad = 2mx^{m+3}$
$= 2m^2 + 6m$

Corollary 2.2: we refer to 9 1) $DM_1(B_m) \le M_1(B_m)$
2) $DM_1(B_m, x) \le M_1(B_m, x)$

Theorem 2.3: The Domination Second Zagreb indices and there polynomial for B_m are

$$DM_2(B_m) = 4m^2 + 4m$$
$$DM_2(B_m, x) = 2mx^{2m+2}$$

Proof: The Domination Second Zagreb indices and polynomial of B_m are defined as

$DM_2(B_m) = \sum_{D,V-D\in E(B_m)}[\deg(D)\deg(V-D)]$ $\quad DM_2(B_m, x) = \sum_{D,V-D\in E(B_m)} x^{[\deg(D)\deg(V-D)]}$
$= \sum_{v_1v_2\in E(B_m)}[\deg(v_1)\deg(v_2)]$ $\quad = \sum_{v_1v_2\in E(B_m)} x^{[\deg(v_1)\deg(v_2)]}$
$v_1 \in D \,\&\, v_2 \in V-D$ $\quad = \sum_{E_{m+1,2}} x^{[\deg(v_1)\deg(v_2)]}$
$= \sum_{E_{m+1,2}}[\deg(v_1)\deg(v_2)]$ $\quad = 2mx^{[(m+1)2]}$
$= 2m[(m+1)2]$ $\quad = 2mx^{2m+2}$
$= 4m^2 + 4m$

Corollary 2.4: we refer to 9 1) $DM_2(B_m) \le M_2(B_m)$
2) $DM_2(B_m, x) \le M_2(B_m, x)$

Theorem 2.5: The Domination first hyper Zagreb indices and there polynomial for B_m are

$$DHM_1(B_m) = 2m^3 + 12m^2 + 18m$$
$$DHM_1(B_m, x) = 2mx^{[m^2+6m+9]}$$

Proof: The Domination First hyper Zagreb indices and polynomial of B_m are defined as

$DHM_1(B_m) = \sum_{D,V-D\in E(B_m)}[\deg(D) + \deg(V-D)]^2$ $\quad DHM_1(B_m, x) = \sum_{D,V-D\in E(B_m)} x^{[\deg(D)+\deg(V-D)]^2}$
$= \sum_{D,V-D\in E(B_m)}[\deg(v_1) + \deg(v_2)]^2$ $\quad = \sum_{v_1v_2\in E(B_m)} x^{[\deg(v_1)+\deg(v_2)]^2}$
$v_1 \in D \,\&\, v_2 \in V-D$ $\quad = \sum_{E_{m+1,2}} x^{[\deg(v_1)+\deg(v_2)]^2}$
$= \sum_{E_{m+1,2}}[\deg(v_1) + \deg(v_2)]^2$ $\quad = 2mx^{[m+3]^2}$
$= 2m[m+1+2]^2$ $\quad = 2mx^{[m^2+6m+9]}$
$= 2m[m+3]^2$
$= 2m^3 + 12m^2 + 18m$

Corollary 2.6: we refer to 9 1) $DHM_1(B_m) \le HM_1(B_m)$
2) $DHM_1(B_m, x) \le HM_1(B_m, x)$

Theorem 2.7: The Domination second hyper Zagreb indices and there polynomial for B_m are

$$DHM_2(B_m) = 8m(m^2 + m + 1)$$
$$DHM_2(B_m, x) = 2mx^{[4m^2+8m+4]}$$

Proof: The Second hyper Zagreb indices and polynomial of B_m are defined as

$DHM_2(B_m) = \sum_{D,V-D\in E(B_m)}[\deg(D)\deg(V-D)]^2$ $\quad DHM_2(B_m, x) = \sum_{D,V-D\in E(B_m)} x^{[\deg(D)\deg(V-D)]^2}$
$= \sum_{D,V-D\in E(B_m)}[\deg(v_1)\deg(v_2)]^2$ $\quad = \sum_{v_1v_2\in E(B_m)} x^{[\deg(v_1)\deg(v_2)]^2}$
$v_1 \in D \,\&\, v_2 \in V-D$ $\quad = \sum_{E_{m+1,2}} x^{[\deg(v_1)+\deg(v_2)]^2}$
$= \sum_{E_{m+1,2}}[\deg(v_1)\deg(v_2)]^2$ $\quad = 2mx^{[2m+2]^2}$
$= 2m[(m+1)2]^2$ $\quad = 2mx^{[4m^2+8m+4]}$
$= 8m(m^2 + m + 1)$

Corollary 2.9: we refer to 9 1) $DHM_2(B_m) \le HM_2(B_m)$
2) $DHM_2(B_m, x) \le HM_2(B_m, x)$

3. Domination Zagreb Indices Stacked Book graph

Stacked Book Graph is a cross product of Star S_{m+1} and path P_n where $m \geq 3, n > 2$. The domination number of book graph is n. We refer to [6] & [7]. Dominating set is $D = \{(v, w_i), (v, w_{i+1})\}$ for $i = 1,2,3, \dots \dots \dots n-1$, $V - D = \{(u_i, w_1), (u_i, w_2), \dots \dots \dots (u_i, w_n)\}$ For $i = 1,2,3, \dots \dots \dots m$ from 3.3.1

The edge set condition $x \in D$, $y \in V - D$. $xy \in E(B_m)$

The edge set $B_{m,n}$ contain edges of two types

$E_{2,m+1} = \{ v_2 v_{m+1} \in E(B_{m,n}); v_2 \in D, v_{m+1} \in V - D$ $|E_{2,m+1}| = 2m$

$E_{3,m+2} = \{ v_2 v_{m+2} \in E(B_{m,n}); v_2 \in D, v_{m+2} \in V - D$ $|E_{2,m+2}| = m(n-2)$

Theorem 3.1: The Domination first Zagreb indices and there polynomial for $B_{m,n}$ are

$$DM_1(B_{m,n}) = m^2 n + 4mn - 2m$$
$$DM_1(B_{m,n}, x) = 2mx^{m+3} + m(n-2)x^{[m+4]}$$

Proof: The Domination first Zagreb indices and polynomial of $B_{m,n}$ are defined as

$$DM_1(B_{m,n}) = \sum_{D,V-D \in E(B_{m,n})}[\deg(D) + \deg(V-D)]$$
$$= \sum_{xy \in E(B_{m,n})}[\deg(x) + \deg(y)]$$
$$x \in D \,\&\, y \in V - D$$
$$= \sum_{E_{2,m+1}}[\deg(x) + \deg(y)] + \sum_{E_{2,m+2}}[\deg(x) + \deg(y)]$$
$$= [2m][2 + m + 1] + [m(n-2)][2 + m + 2]$$
$$= m^2 n + 4mn - 2m$$

$$DM_1(B_{m,n}, x) = \sum_{D,V-D \in E(B_{m,n})} x^{[\deg(D) + \deg(V-D)]}$$
$$= \sum_{xy \in E(B_{m,n})} x^{[\deg(x) + \deg(y)]}$$
$$= \sum_{E_{2,m+1}} x^{[\deg(x) + \deg(y)]} + \sum_{E_{2,m+2}} x^{[\deg(x) + \deg(y)]}$$
$$= 2mx^{[2+m+1]} + m(n-2)x^{[2+m+2]}$$
$$= 2mx^{m+3} + m(n-2)x^{[m+4]}$$

Corollary 3.2: 1) $DM_1(B_{m,n}) \leq M_1(B_{m,n})$
2) $DM_1(B_{m,n}, x) \leq M_1(B_{m,n}, x)$

Theorem 3.3: The Domination Second Zagreb indices and there polynomial for $B_{m,n}$ are

$$DM_2(B_{m,n}) = 2m^2 n + 4mn - 4m$$
$$DM_2(B_{m,n}, x) = 2mx^{2m+2} + m(n-2)x^{[2m+4]}$$

Proof: The Domination Second Zagreb indices of $B_{m,n}$ are defined as

$$M_2(B_{m,n}) = \sum_{D,V-D \in E(B_{m,n})}[\deg(D)\deg(V-D)]$$
$$= \sum_{xy \in E(B_{m,n})}[\deg(x)\deg(y)]$$
$$x \in D \,\&\, y \in V - D$$
$$= \sum_{E_{2,m+1}}[\deg(x)\deg(y)] + \sum_{E_{2,m+2}}[\deg(x)\deg(y)]$$
$$= [2m][2(m+1)] + [m(n-2)][(2+m)2]$$
$$= 2m^2 n + 4mn - 4m$$

$$M_2(B_{m,n}, x) = \sum_{D,V-D \in E(B_m)} x^{[\deg(D)\deg(V-D)]}$$
$$= \sum_{xy \in E(B_m)} x^{[\deg(x)\deg(y)]}$$
$$= \sum_{E_{2,m+1}} x^{[\deg(x)\deg(y)]} + \sum_{E_{2,m+2}} x^{[\deg(x)\deg(y)]}$$
$$= 2mx^{[2(m+1)]} + m(n-2)x^{[2(m+2)]}$$
$$= 2mx^{2m+2} + m(n-2)x^{[2m+4]}$$

Corollary 3.4: we refer to 9 1) $DM_2(B_{m,n}) \leq M_2(B_{m,n})$
2) $DM_2(B_{m,n}, x) \leq M_2(B_{m,n}, x)$

Theorem 3.5: The Domination first hyper Zagreb indices and there polynomial for $B_{m,n}$ are

$$DHM_1(B_{m,n}) = m^3 n + 8m^2 n + 16mn - 4m^2 - 14m$$
$$DHM_1(B_{m,n}, x) = m(n-2)x^{[m+4]^2} + 2mx^{[m+3]^2}$$

Proof: The Domination first hyper Zagreb indices and polynomial of $B_{m,n}$ are defined as

$$DHM_1(B_{m,n}) = \sum_{D,V-D \in E(B_{m,n})}[\deg(D) + \deg(V-D)]^2$$
$$= \sum_{xy \in E(B_{m,n})}[\deg(x) + \deg(y)]^2$$
$$x \in D \,\&\, y \in V - D$$
$$= \sum_{E_{2,m+1}}[\deg(x) + \deg(y)]^2 + \sum_{E_{2,m+2}}[\deg(x) + \deg(y)]^2$$
$$= [2m][2 + m + 1]^2 + [m(n-2)][2 + m + 2]^2$$
$$= m^3 n + 8m^2 n + 16mn - 4m^2 - 14m$$

$$DHM_1(B_{m,n}, x) = \sum_{D,V-D \in E(B_{m,n})} x^{[\deg(D) + \deg(V-D)]^2}$$
$$= \sum_{xy \in E(B_{m,n})} x^{[\deg(x) + \deg(y)]^2}$$
$$= \sum_{E_{2,m+1}} x^{[\deg(x) + \deg(y)]^2} + \sum_{E_{2,m+2}} x^{[\deg(x) + \deg(y)]^2}$$
$$= 2mx^{[2+m+1]^2} + m(n-2)x^{[2+m+2]^2}$$
$$= m(n-2)x^{[m+4]^2} + 2mx^{[m+3]^2}$$

Corollary 3.6: we refer to 9 1) $DHM_1(B_{m,n}) \leq HM_1(B_{m,n})$
2) $DHM_1(B_{m,n},x) \leq HM_1(B_{m,n},x)$

Theorem 3.7: The Domination second hyper Zagreb indices and there polynomial for $B_{m,n}$ are

$$DHM_2(B_{m,n}) = 4m^3n + 16m^2n + 16mn - 16m^2 - 24m$$
$$DHM_2(B_{m,n},x) = m(n-2)x^{[m+4]^2} + 2mx^{[m+3]^2}$$

Proof: The Domination second hyper Zagreb indices and polynomial of $B_{m,n}$ are defined as

$$DHM_2(B_{m,n}) = \sum_{D,V-D\in E(B_{m,n})}[deg(D)deg(V-D)]^2$$
$$= \sum_{xy\in E(B_{m,n})}[deg(x)\,deg(y)]^2$$
$$x \in D \,\&\, y \in V-D$$
$$= \sum_{E_{2,m+1}}[deg(x)\,deg(y)]^2 + \sum_{E_{2,m+2}}[deg(x)\,deg(y)]^2$$
$$= [2m][2(m+1)]^2 + [m(n-2)][2(m+2)]^2$$
$$= 4m^3n + 16m^2n + 16mn - 16m^2 - 24m$$

$$DHM_2(B_{m,n},x) = \sum_{D,V-D\in E(B_m)} x^{[deg(D)\,deg(V-D)]^2}$$
$$= \sum_{xy\in E(B_{m,n})} x^{[deg(x)\,deg(y)]^2}$$
$$= \sum_{E_{2,m+1}} x^{[deg(x)\,deg(y)]^2} + \sum_{E_{2,m+2}} x^{[deg(x)\,deg(y)]^2}$$
$$= 2mx^{[2(m+1)]^2} + m(n-2)x^{[2(m+2)]^2}$$
$$= m(n-2)x^{4[m+2]^2} + 2mx^{4[m+1]^2}$$

Corollary 3.8: we refer to 9 1) $DHM_2(B_{m,n}) \leq HM_2(B_{m,n})$
2) $DHM_2(B_{m,n},x) \leq HM_2(B_{m,n},x)$

4. Result

The Domination Zagreb indices and polynomial of Book Graph B_m and Stacked book graph $B_{m,n}$

Table 1.

Sl No	Zagreb Indices	Polynomial
1	$DM_1(B_m) = 2m^2 + 6m$	$DM_1(B_m,x) = 2mx^{m+3}$
2	$DM_2(B_m) = 4m^2 + 4m$	$DM_2(B_m,x) = 2mx^{2m+2}$
3	$DM_1(B_{m,n}) = m^2n + 4mn - 2m$	$DM_1(B_{m,n},x) = 2mx^{m+3} + m(n-2)x^{[m+4]}$
4	$DM_2(B_{m,n}) = 2m^2n + 4mn - 4m$	$DM_2(B_{m,n},x) = 2mx^{2m+2} + m(n-2)x^{[2m+4]}$

The Domination hyper Zagreb indices and polynomial of Book Graph B_m and Stacked book graph $B_{m,n}$

Table 2.

Sl No	Zagreb Indices	Polynomial
1	$DHM_1(B_m) = 2m^3 + 12m^2 + 18m$	$DHM_1(B_m,x) = 2mx^{[m^2+6m+9]}$
2	$DHM_2(B_m) = 8m(m^2 + m + 1)$	$DHM_2(B_m,x) = 2mx^{[4m^2+8m+4]}$
3	$DHM_1(B_{m,n}) = m^3n + 8m^2n + 16mn - 4m^2 - 14m$	$DHM_1(B_{m,n},x) = m(n-2)x^{[m+4]^2} + 2mx^{[m+3]^2}$
4	$DHM_2(B_{m,n}) = 4m^3n + 16m^2n + 16mn - 16m^2 - 24m$	$DHM_2(B_{m,n},x) = m(n-2)x^{[m+4]^2} + 2mx^{[m+3]^2}$

5. Conclusion

1. We observed that the domination Zagreb indices B_m and $B_{m,n}$ is greater than or equal to Zagreb indices B_m and $B_{m,n}$.
2. We observed that the domination hyper Zagreb indices B_m and $B_{m,n}$ is greater than or equal to hyper Zagreb indices B_m and $B_{m,n}$.

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