International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT- V68I5P509
Volume 68 Issue 5, 51-60, May 2022

Original Article

Some Absolute Mean Graceful Graphs in the Context of Barycentric Subdivision

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Received: 23 March 2022 Revised: 18 May 2022 Accepted: 29 May 2022 Published: 09 June 2022

Abstract - *In this paper, we present absolute mean graceful labeling for some graphs in context of barycentric subdivision of graph. We have proved that barycentric subdivision of wheel W_n, complete bipartite graph $K_{m,n}$, helm H_n, sunlet graph S_n, jelly fish graph $J_{n,n}$, alternate quadrilateral snake AQ_n are absolute mean graceful graphs.*

Keywords - *Labeling, Graceful Labeling, Absolute Mean Graceful Labeling.*

1. Introduction

Labeling of graph is the assignment of values to vertices or edges or both subject in certain conditions. A. Rosa[2] initiated the concept of labeling with the name of β-valuation. S. Golomb[3] named such labeling as graceful labeling. Kaneria and Chudasama[6] introduced graph labeling namely absolute mean graceful labeling. We begin with a simple, connected and undirected graph $G = (V, E)$ with p vertices and q edges. for all terminology and notations, we follow F. Harary[1]. First of all we recall some definitions, which are used in this paper.

Definition 1.1 : A function f is called graceful labeling for a graph G, if $f: V(G) \longrightarrow \{0, 1, 2, \dots, q\}$ is injective and the induced function $f^*: E(G) \longrightarrow \{1, 2, \dots, q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge $e = uv \in E(G)$. A graph G is called graceful graph, if it admits a graceful labeling.

Definition 1.2 : A function f is said to be absolute mean graceful labeling of a graph G, if $f: V(G) \longrightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$ is injective and edge labeling function $f^*: E(G) \longrightarrow \{1, 2, \dots, q\}$ defined as $f^*(e) = \left\lceil \frac{|f(u) - f(v)|}{2} \right\rceil$ is bijective, for every edge $e = uv \in E(G)$. A graph G is called absolute mean graceful graph, if it admits an absolute mean graceful labeling.

Definition 1.3 : The barycentric subdivision of graph is obtained by inserting a vertex of degree two into every edge of original graph. Barycentric subdivision of helm H_4 is shown in Fig. 1.

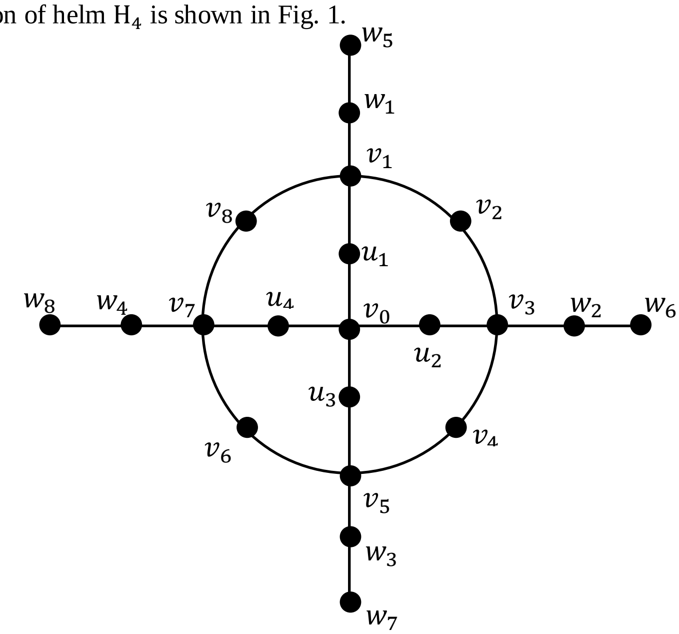

Fig. 1 Barycentric subdivision of helm H_4

Kaneria and Chudasama[5,6] proved that path graphs P_n, cycles C_n, complete bipertite graphs $K_{m,n}$, grid graphs $P_m \times P_n$, step grid graphs St_n and double step grid graphs DSt_n are absolute mean graceful graphs and they also proved that path union of finite copies in trees T, path graphs P_n, cycles C_n, complete bipertite graphs $K_{m,n}$, grid graphs $P_m \times P_n$, step grid graphs St_n and double step grid graphs DSt_n are absolute mean graceful graphs. Akbari, Kaneria and Parmar[7] proved that jewel graph $\mathrm{J_n}$, jewel graph without prime edge $\mathrm{J_n^*}$, extended jewel graph $\mathrm{EJ_n}$, jelly fish graph $\mathrm{J_{m,n}}$, jelly fish graph without prime edge $\mathrm{J_{m,n}^*}$, extended jelly fish graph $\mathrm{EJ_{m,n}}$ are absolute mean graceful graphs. for comprehensive learning of graph labeling, we reffered Gallian[4].

2. Main Results

Theorem 2.1 : The barycentric subdivision of wheel W_n is absolute mean graceful graph.

Proof : Let G be the barycentric subdivision of wheel W_n. Let $v_1, v_2, \dots, v_{2n}$ be rim vertices of G. Let $u_1, u_2, \dots, u_n$ be internal vertices of G. Let v_0 be the apex vertex of G.
i.e. $V(G) = \{v_1 v_2, \dots, v_{2n}\} \cup \{u_1, u_2, \dots, u_n\} \cup \{v_0\}$
and $E(G) = \{v_i v_{i+1} / 1 \le i < 2n\} \cup \{v_1 v_{2n}\} \cup \{v_0 u_i / 1 \le i \le n\} \cup \{v_{2i-1} u_i / 1 \le i \le n\}$
to obtain vertex labeling function $f: V(G) \longrightarrow \{0, \pm 1, \pm 2, \dots, \pm 4n\}$, we take following cases.

Case-I: $n \equiv 0 \pmod 2$

$$f(v_i) = \begin{cases} (-1)^i \,(q+2-2i) & ; if\ i = 1, 2, \dots, n+1 \\ (-1)^i \,(2i-1) & ; if\ i = n+2, n+3, \dots, 2n \\ n+1 & ; if\ i = 0 \end{cases}$$

$$f(u_i) = \begin{cases} 2i-2 & ; if\ \ i = 1, 2, 3, \dots, \dfrac{n+2}{2} \\ n-2i+1 & ; if\ i = \dfrac{n+4}{2}, \dfrac{n+6}{2}, \dots, n \end{cases}$$

Case-II: $n \equiv 1 \pmod 2$

$$f(v_i) = \begin{cases} (-1)^i \,(q+2-2i) & ; if\ i = 1, 2, \dots, n+1 \\ (-1)^i \,(2i-1) & ; if\ i = n+2, n+3, \dots, 2n \\ n+1 & ; if\ i = 0 \end{cases}$$

$$f(u_i) = \begin{cases} 2i-2 & ; if\ \ i = 1, 2, 3, \dots, \dfrac{n+1}{2} \\ n-2i+2 & ; if\ i = \dfrac{n+3}{2}, \dfrac{n+5}{2}, \dots, n \end{cases}$$

By defined pattern of function f, we can se that f is one-one.
Now we shall prove that induced function f^* is a bijection. First of all we obtain range of f^*.
for all cases,

$\{ f^*(v_0 u_i) / 1 \le i \le n\} = \{1, 2, \dots, n\}$, $\{f^*(v_{2i-1} u_i) / 1 \le i \le n\} = (n+1, n+2, \dots, 2n\}$, and

$\{f^*(v_i v_{i+1}) / 1 \le i < 2n\} \cup \{v_1 v_{2n}\} = (2n+1, 2n+2, \dots, 4n\}$,

i.e. $f^*\big(E(G)\big) = \{1, 2, \dots, q = 4n\}$

Hence, f^* is onto. Further, domain of f^* and range of f^* have same cardinality, gives f^* is one-one. Therefore, f^* is bijection. Thus, f is absolute mean graceful labeling for G.
Therefore, barycentric subdivision of wheel is absolute mean graceful graph.

Illustration 2.2 : Absolute mean graceful labeling for barycentric subdivision of wheel W_6 with $p = 19$ and $q = 24$ is shown in following Fig. 2.

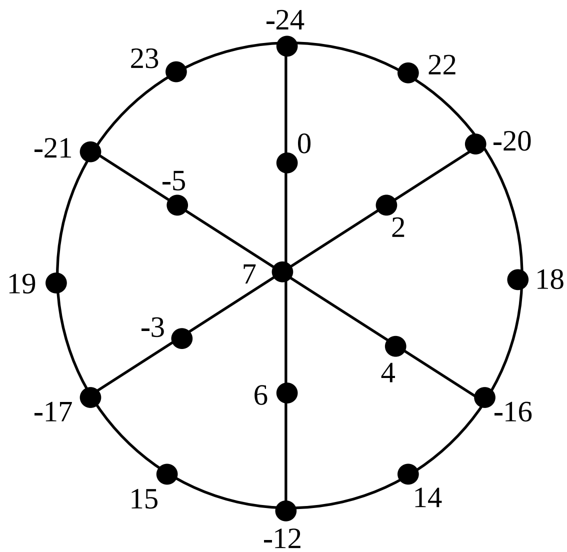

Fig. 2 Absolute mean graceful labeling for barycentric subdivision of wheel W_6

Theorem 2.3 : The barycentric subdivision of complete bipartite graph $K_{m,n}$ is absolut mean graph.
Proof : Let G be the barycentric subdivision of complete bipartite graph $K_{m,n}$.
Let $V(K_{m,n}) = V_1 \cup V_2$ be the bipartition of vertex set of complete bipartite graph $K_{m,n}$. Let $\{u_i/1 \le i \le m\}$ denote the vertices of V_1 and $\{v_j/1 \le j \le n\}$ denote the vertices of V_2. Let $\{w_{ij}/1 \le i \le m, 1 \le j \le n\}$ be the vertices formed by barycentric subdivision of $K_{m,n}$, where w_{ij} is the vertex adjacent to u_i and v_j, $i \in \{1,2,\dots,m\}, j \in \{1,2,\dots,n\}$.
i.e. $V(G) = \{u_1,u_2,\dots,u_m\} \cup \{v_1,v_2,\dots,v_n\} \cup \{w_{ij}/1 \le i \le m, 1 \le j \le n\}$ and
$E(G) = \{u_i w_{ij}/1 \le i \le m, 1 \le j \le n\} \cup \{w_{ij} v_j/1 \le i \le m, 1 \le j \le n\}$.
The vertex labeling function $f: V(G) \longrightarrow \{0, \pm 1, \pm 2, \dots, \pm 2mn\}$ defined as follows.
$f(u_i) = 4i - q - 2 \qquad ; if\ i = 1,2,\dots,m$

$$f(v_j) = \begin{cases} 4mj - 2mn + 1 & ; if\ j = 1,2,\dots,n-1 \\ -2mn & ; if\ j = n \end{cases}$$

$f(w_{ij}) = 2(i + jm - m) \quad ; \forall\, i,j$
By defined pattern of function f, we can see that f is one-one.
Now we shall prove that induced function f^* is a bijection. First of all we obtain range of f^*.
$\{f^*\{w_{ij}v_j/1 \le j < n, 1 \le i \le m\} = \{1,2,\dots,m(n-1)\}$
$f^*(w_{ij}v_j)/j = n, 1 \le i \le m\} = \{2mn - m + 1, 2mn - m + 2, \dots, 2mn\}$
$f^*(u_i w_{ij})/1 \le i \le m, 1 \le j \le n\} = \{m(n-1) + 1, m(n-1) + 2, \dots, 2mn - m\}$
i.e. $f^*(E(G)) = \{1,2,\dots,q = 2mn\}$

Hence, f^* is onto. Further, domain of f^* and range of f^*have same cardinality, gives f^* is one-one. Therefore, f^* is bijection. Thus, f is absolute mean graceful laebling for G.

Therefore, barycentric subdivision of complete bipartite graph is absolute mean graceful graph.

Illustration 2.4 : Absolute mean graceful labeling for barycentric subdivision of complete bipartite graph $K_{3,4}$ with $p = 19$ and $q = 24$ is shown in following Fig. 3.

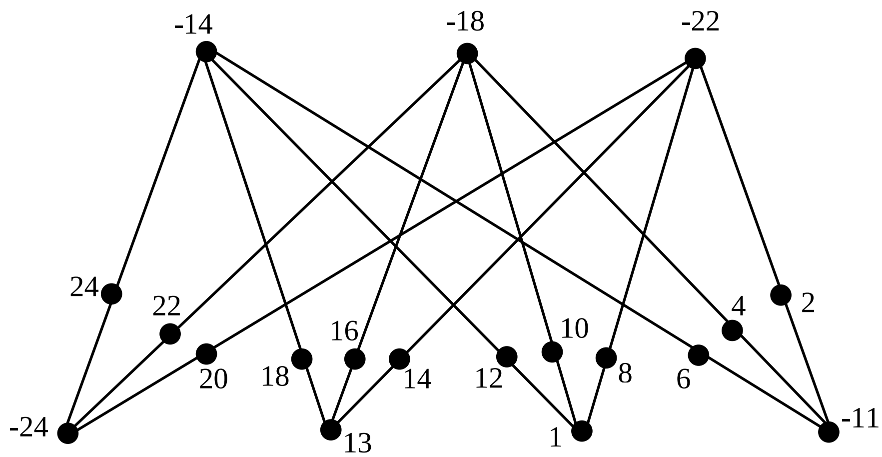

Fig. 3 Absolute mean graceful labeling for barycentric subdivision of complete bipartite graph $K_{3,4}$

Theorem 2.5 : The barycentric subdivision of Helm H_n is absolute mean graceful graph.

Proof : Let G be the barycentric subdivision of helm H_n. Let $v_1, v_2, \ldots, v_{2n}$ be rim vertices on cycle of G. Let v_0 be the apex vertex of G. Let $w_1, w_2, \ldots, w_{2n}$ be vertices formed by barycentric subdivision of H_n, which is shown in figure.

i.e. $V(G) = \{v_1, v_2, \ldots, v_{2n}, u_1, u_2, \ldots, u_n, w_1, w_2, \ldots, w_{2n}, v_0\}$ and

$E(G) = \{v_0 u_i / 1 \le i \le n\} \cup \{v_{2i-1} u_i / 1 \le i \le n\} \cup \{v_i v_{i+1} / 1 \le i < 2n\} \cup \{v_1 v_{2n}\} \cup \{v_{2i-1} w_i / 1 \le i \le n\}$
$\cup \{w_i w_{i+n} / 1 \le i \le n\}$

to obtain vertex labeling function $f: V(G) \longrightarrow \{0, \pm 1, \pm 2, \ldots, \pm 6n\}$, we take following cases.

Case-I: $n \equiv 0 (\text{mod } 2)$

$$f(v_i) = \begin{cases} (-1)^i\,(q + 2 - 2i) & ; if\ i = 1, 2, \ldots, n+1 \\ (-1)^i\,(2i + 2n - 1) & ; if\ i = n+2, n+3, \ldots, 2n \\ -(n+1) & ; if\ i = 0 \end{cases}$$

$$f(u_i) = \begin{cases} 2n + 2i - 3 & ; if\ i = 1, 2, 3, 4, \ldots, \frac{n+2}{2} \\ 3n - 2i + 1 & ; if\ i = \frac{n+4}{2}, \frac{n+6}{2}, \ldots, n \end{cases}$$

$$f(w_i) = \begin{cases} 2\mathrm{i} - 2 & ; if\ \mathrm{i} = 1, 2, \ldots, \frac{n+2}{2} \\ \mathrm{n} - 2\mathrm{i} + 1 & ; if\ \mathrm{i} = \frac{n+4}{2}, \frac{n+6}{2}, \ldots, n \\ 1 & ; if\ i = \mathrm{n} + 1 \\ 2\mathrm{n} - 2i + 2 & ; if\ \mathrm{i} = \mathrm{n}+2, \mathrm{n}+3, \ldots, \frac{3\mathrm{n}+2}{2} \\ 2\mathrm{i} - 3\mathrm{n} - 1 & ; if\ \mathrm{i} = \frac{3n+4}{2}, \frac{3n+6}{2}, \ldots, 2n \end{cases}$$

Case-II: $n \equiv 1 (\text{mod } 2)$

$$f(v_i) = \begin{cases} (-1)^i\,(q+2-2i) & ; if\ i = 1,2,\dots,n+1 \\ (-1)^i\,(2i+2n-1) & ; if\ i = n+2, n+3, \dots, 2n \\ -n & ; if\ i = 0 \end{cases}$$

$$f(u_i) = \begin{cases} 2n+2i-2 & ; if\ \ i = 1,2,3,\dots,\dfrac{n+1}{2} \\ 3n-2i+2 & ; if\ i = \dfrac{n+3}{2}, \dfrac{n+5}{2}, \dots, n \end{cases}$$

$$f(w_i) = \begin{cases} 2\mathrm{i}-2 & ; if\ \mathrm{i} = 1,2,\dots,\dfrac{n+1}{2} \\ \mathrm{n}-2\mathrm{i}+2 & ; if\ \mathrm{i} = \dfrac{n+3}{2}, \dfrac{n+5}{2}, \dots, n \\ -2\mathrm{n} & ; if\ i = \mathrm{n}+1 \\ 2\mathrm{n}-2i+2 & ; if\ \mathrm{i} = \mathrm{n}+2, \mathrm{n}+3, \dots, \dfrac{3\mathrm{n}+1}{2} \\ 2\mathrm{i}-3\mathrm{n}-2 & ; if\ \mathrm{i} = \dfrac{3n+3}{2}, \dfrac{3n+5}{2}, \dots, 2\mathrm{n} \end{cases}$$

By defined pattern of function f, we can see that f is one-one

Now we shall prove that induced function $f^*$is a bijection. First of all we obtain range of f^*.

for all cases,

$\{f^*(v_0u_i)/1 \le i \le n\} = (n+1, n+2, \dots, 2n\}$, $\{f^*(w_iw_{n+i})/1 \le i \le n\} = \{1, 2, \dots, n\}$,

$\{f^*(w_iv_{2i-1})\ /\ 1 \le i \le n\} = \{\ 2n+1, 2n+2, \dots, 3n\}$, $\{f^*(v_{2i-1}u_i)/1 \le i \le n\} = \{3n+1, 3n+2, \dots, 4n\}$,

$\{f^*(v_iv_{i+1})/1 \le i < 2n\} \cup \{v_1v_{2n}\} = \{4n+1, 4n+2, \dots, 6n\}$

i.e. $f^*\big(E(G)\big) = \{1, 2, \dots, q = 6n\}$

Hence, f^* is onto. Further, domain of f^* and range of f^* have same cardinality, gives f^* is one-one. Therefore, f^* is bijection. Thus, f is absolute mean graceful labeling for G.

Therefore, barycentric subdivision of helm is absolute mean graceful graph.

Illustration 2.6 : Absolute mean graceful labeling for barycentric subdivision of helm H_4 with $p = 21$ and $q = 24$ is shown in Fig. 4.

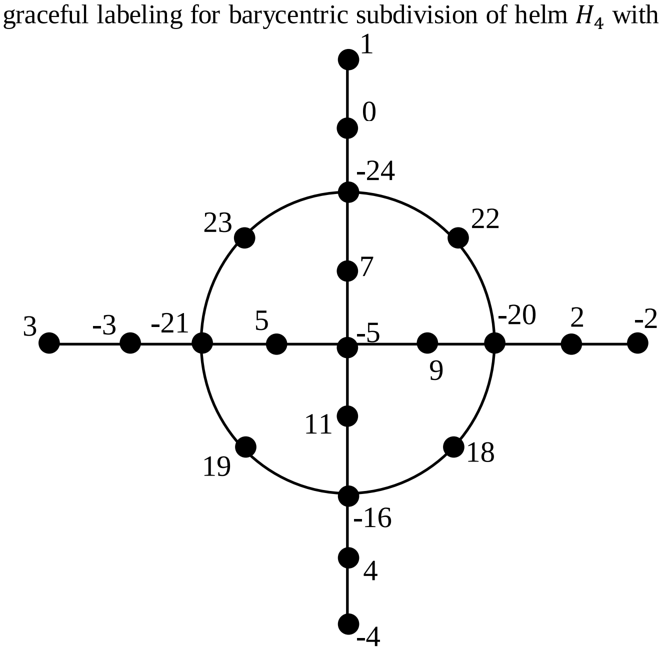

Fig. 4 Absolute mean graceful labeling for barycentric subdivision of helm H_4.

***Theorem 2.7*:** The barycentric subdivision of sunlet graph S_n is absolut mean graceful graph.
Proof : Let G be the barycentric subdivision of sunlet graph S_n. Let $v_1, v_2, \dots, v_{2n}$ be rim vertices on cycle of G. Let $w_1, w_2, \dots, w_{2n}$ be vertices formed by barycentric subdivision of sunlet graph S_n.
i.e. $V(G) = \{v_1, v_2, \dots, v_{2n}, w_1, w_2, \dots, w_{2n}\}$ and
$E(G) = \{v_i v_{i+1} / 1 \le i < 2n\} \cup \{v_1 v_{2n}\} \cup \{v_{2i-1} w_i / 1 \le i \le n\} \cup \{w_i w_{n+i} / 1 \le i \le n\}$
to obtain vertex labeling function $f: V(G) \longrightarrow \{0, \pm 1, \pm 2, \dots, \pm 4n\}$, we take following cases.

Case-I: $n \equiv 0(\text{mod } 2)$

$$f(v_i) = \begin{cases} (-1)^i\,(q+2-2i) & ; if\ i = 1,2,3,4,\dots,n+1 \\ (-1)^i\,(2i-1) & ; if\ i = n+2, n+3, \dots, 2n \end{cases}$$

$$f(w_i) = \begin{cases} 2\mathrm{i}-2 & ; if\ \mathrm{i} = 1,2,\dots,\dfrac{n+2}{2} \\ \mathrm{n}-2\mathrm{i}+1 & ; if\ \mathrm{i} = \dfrac{n+4}{2}, \dfrac{n+6}{2}, \dots, n \\ 1 & ; if\ i = \mathrm{n}+1 \\ 2\mathrm{n}-2i+2 & ; if\ \mathrm{i} = \mathrm{n}+2, \mathrm{n}+3, \dots, \dfrac{3\mathrm{n}+2}{2} \\ 2\mathrm{i}-3\mathrm{n}-1 & ; if\ \mathrm{i} = \dfrac{3n+4}{2}, \dfrac{3n+6}{2}, \dots, 2\mathrm{n} \end{cases}$$

Case-II: $n \equiv 1(\text{mod } 2)$

$$f(v_i) = \begin{cases} (-1)^i\,(q+2-2_i) & ; if\ i = 1,2,3,4,\dots,n+1 \\ (-1)^i\,(2i-1) & ; if\ i = n+2, n+3, \dots, 2n \end{cases}$$

$$f(w_i) = \begin{cases} 2\mathrm{i}-2 & ; if\ \mathrm{i} = 1,2,\dots,\dfrac{n+1}{2} \\ \mathrm{n}-2\mathrm{i}+2 & ; if\ \mathrm{i} = \dfrac{n+3}{2}, \dfrac{n+5}{2}, \dots, n \\ -2\mathrm{n} & ; if\ i = \mathrm{n}+1 \\ 2\mathrm{n}-2i+2 & ; if\ \mathrm{i} = \mathrm{n}+2, \mathrm{n}+3, \dots, \dfrac{3\mathrm{n}+1}{2} \\ 2\mathrm{i}-3\mathrm{n}-2 & ; if\ \mathrm{i} = \dfrac{3n+3}{2}, \dfrac{3n+5}{2}, \dots, 2\mathrm{n} \end{cases}$$

By defined pattern of function f, we can see that f is one-one.
Now we shall prove that induced function f^* is a bijection. First of all we obtain range of f^* .
for all cases,
$\{f^*(v_i v_{i+1}) / 1 \le i < 2n\} \cup \{v_1 v_{2n}\} = (2n+1, 2n+3, \dots, 4n\}$, $\{f^*(w_i w_{n+i}) / 1 \le i \le n\} = \{1, 2, \dots, n\}$,
$\{f^*(w_i v_{2i-1}) / 1 \le i \le n\} = \{n+1, n+2, \dots, 2n\}$
i.e. $f^*\big(E(G)\big) = \{1, 2, \dots, q = 4n\}$

Hence, f^* is onto. Further, domain of f^* and range of f^* have same cardinality, gives f^* is one-one. Therefore, f^* is bijection.
Thus, f is absolute mean graceful labeling for G.
Therefore, barycentric subdivision of sunlet graph is absolute mean graceful graph.

Illustration 2.8 : Absolute mean graceful labeling for barycentric subdivision of sunlet graph S_5 with $p = 20$ and $q = 20$ is shown in following Fig. 5.

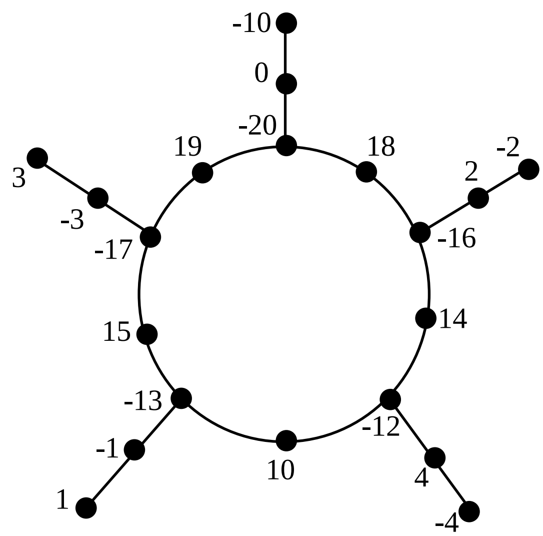

Fig. 5 Absolute mean graceful labeling for barycentric subdivision of sunlet graph S_5

Theorem 2.9 : The barycntric subdivision jelly fish graph $J_{n,n}$ is an absolute mean graceful graph.

Proof : Let G be barycentric subdivision of jelly fish graph $J_{n,n}$. Jelly fish graph is 4-cycle graph with vertices x, y, u, v including the prime edge connecting to x and y and also by appending n pendent edges to u and v.

Let $V(J_{n,n}) = \{x, y, u, v, u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ and $E(G) = \{uu_i/1 \le i \le n\} \cup \{vv_j/1 \le j \le n\} \cup \{xu, xv, yu, yv, xy\}$

Now we shall add x_1, x_2, y_1, y_2 vertices in 4-cycle, x_0 vertex in prime edge, u_{n+i} vrtex between u and u_i $(1 \le i \le n)$ and v_{n+j} vertex between v and v_j $(1 \le j \le n)$ to obtain barycentric subdivision of jelly fish graph $J_{n,n}$.

i.e. $V(G) = \{x, y, u, v, x_0, x_1, x_2, y_1, y_2\} \cup \{u_i/1 \le i \le 2n\} \cup \{v_j/1 \le j \le 2n\}$ and

$E(G) = \{uu_{n+i}/1 \le i \le n\} \cup \{u_{n+i}u_i/1 \le i \le n\} \cup \{vv_{n+j}/1 \le j \le n\} \cup \{v_{n+j}v_j/1 \le j \le n\}$

$\cup \{xx_0, yx_0, xx_1, xx_2, ux_1, vx_2, yy_1, yy_2, uy_1, vy_2\}$

The vertex labeling function $f: V(G) \longrightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$ is defined as follows.

$f(v) = q, f(u) = q - 8, f(x) = q - 3, f(y) = q - 4, f(x_0) = 14 - q, f(x_1) = 5 - q, f(x_2) = 1 - q, f(y_1) = 6 - q,$

$f(y_2) = 2 - q$

$$f(u_i) = \begin{cases} 3 + 2(i - n) & ; if\ i = 1, 2, 3, \dots, n \\ 3 - 2_i & ; if\ i = n + 1, n + 2, \dots, 2n \end{cases}$$

$$f(v_j) = \begin{cases} 2j + 8 & ; if\ j = 1, 2, 3, \dots, n \\ 10 - 2j + 2n & ; if\ j = n + 1, n + 2, \dots, 2n \end{cases}$$

By defined pattern of function f, we can see that f is one-one.

Now we shall prove that induced function $f^*$is a bijection. First of all we obtain range of f^*.

$\{f^*(v_jv_{n+j})/1 \le j \le n\} \cup \{f^*(u_iu_{n+i})/1 \le i \le n\} = \{1, 2, \dots, 2n\},$

$\{f^*(uu_{n+i})/1 \le i \le n\} \cup \{f^*(vv_{n+j})/1 \le j \le n\} = \{2n + 1, 2n + 2, \dots, 4n\}$, and

$\{f^*(xx_0), f^*(yx_0), f^*(xx_1), f^*(xx_2), f^*(ux_1), f^*(vx_2), f^*(yy_1), f^*(yy_2), f^*(uy_1), f^*(vy_2)\} = \{q - 9, q - 8, \dots, q\}$

i.e. $f^*(E(G)) = \{1, 2, \dots, q = 4n + 10\}$

Hence, f^* is onto. Further, domain of f^* and range of f^* have same cardinality, gives f^* is one-one. Therefore, f^* is bijection. Thus, f is absolute mean graceful labeling for G.
Therefore, barycentric subdivision of jelly fish graph is absolute mean graceful graph.

Illustration 2.10 : Absolute mean graceful labeling for barycentric subdivision jelly fish graph $J_{6,6}$ with $p = 33$ and $q = 33$ is shown in following Fig. 6.

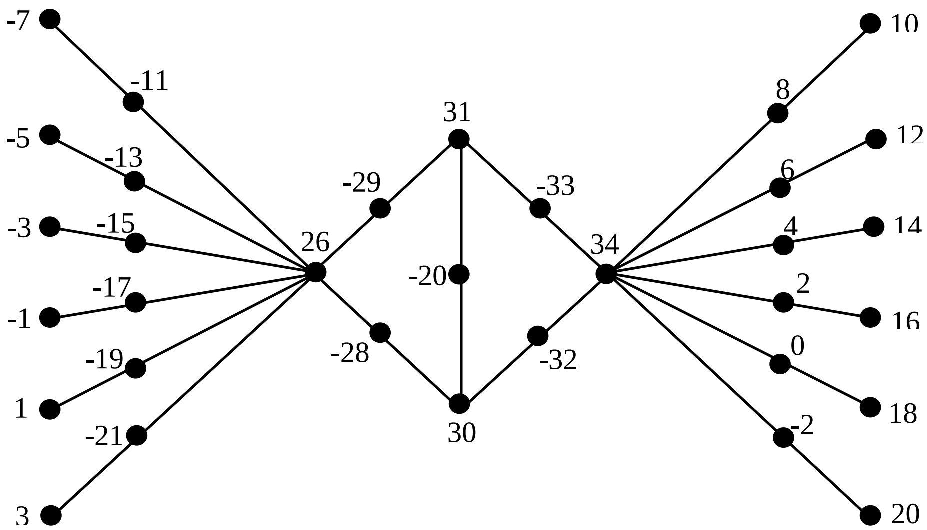

Fig. 6 Absolute mean graceful labeling for barycentric subdivision of jelly fish graph $J_{6,6}$

Theorem 2.11 : The barycentric subdivision of alternate quadrilateral snake AQ_n is absolute mean graceful graph.
Proof : Let G be the barycentric subdivision of alternate quadrilateral snake AQ_n. Let $V(AQ_n) = \{v_1, v_2, v_3, v_4, \dots\}$
Here $v_i, v_{i+1}, v_{i+2}, v_{i+3}$ are vertices for $(4i-3)^{th}$ number of snake.
Let $\{u_1, u_2, u_3, u_4, \dots\}$ are vertices insterted into quadrilateral snakes and $\{w_1, w_2, \dots\}$ are vertices insterted into alternate edge between snakes due to barycentric subdivision.
i.e. $V(G) = \{v_1, v_2, v_3, v_4, u_1, u_2, u_3, u_4, w_1, \dots\}$ $E(G) = \{v_1u_1, v_1u_4, v_2u_1, v_2u_2, v_3u_2, v_3u_3, v_4u_3, v_4u_4, v_4w_1, \dots\}$
Let k be the number of snakes. The vertex labeling function $f: V(G) \longrightarrow \{0, \pm1, \pm2, \dots, \pm q\}$ defined as follows.

$$f(v_i) = \begin{cases} q - 2i + 2 & ; if\ i = 1, 2, 3, 4 \\ q - \dfrac{5(i-1)}{2} + 1 & ; if\ i = 5, 7, \dots, 4k-1 \\ q - \dfrac{5i}{2} + 3 & ; if\ i = 6, 8, \dots, 4k-2 \\ 3 & ; if\ i = 4k \end{cases}$$

$$f(v_i) = \begin{cases} 2i - q - 1 & ; if\ i = 1, 2, 3 \\ 12 - q & ; if\ i = 4 \\ f(u_{i-4}) + 10 & ; if\ i = 5, 6, \dots, 4k-1 \\ 5 & ; if\ i = 4k \end{cases}$$

$$f(w_i) = 10 - 4k \qquad ; if\ i = 1, 2, \dots, k.$$

By defined pattern of function f, we can see that f is one-one.
Now we shall prove that induced function f^* is a bijection.
Here, $f^*(E(G)) = \{1, 2, \dots, q\}$

Hence, f^* is onto map. As domain of f^* and range of f^* have same cardinality, gives f^* is one-one. Therefore, f^* is bijection. Thus, f is an absolute mean graceful labeling for G.
Therefore, barycentric subdivision of alternate quadrilateral snake is absolute mean graceful graph.

Illustration 2.12 : Absolute mean graceful labeling for barycentric subdivision of alternate quadrilateral snake with number of snakes $k = 3$, $p = 26$ and $q = 28$ is shown in following Fig. 7.

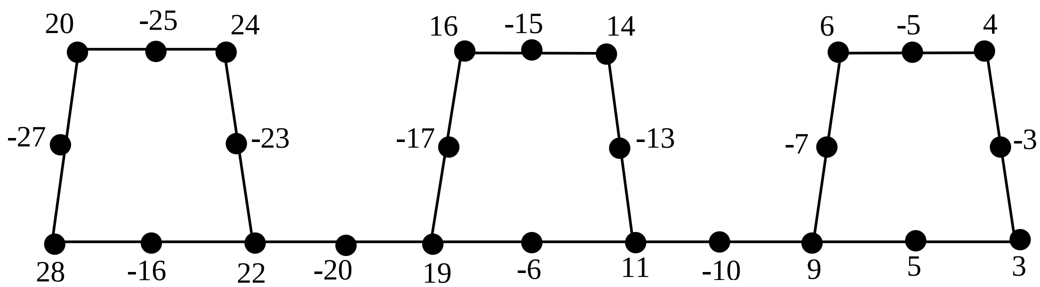

Fig. 7 Absolute mean graceful labeling for barycentric subdivision of alternate quadrilateral snake with $k = 3$

3. Conclusion

Present work contributes some new results. We discussed absolute mean gracefulness of various graphs. The labeling pattern is demonstrated by means of illustrations which is better understanding to derived results.

References

[1] F. Harary, Graph Theory, Addition Wesley, Massachusetts, (1972).

[2] A. Rosa, On Certain Valuation of The Vertices of A Graph, Theory of Graphs (Rome, July 1966), Gordon and Breach, N.Y. and Paris. (1967) 349-355.

[3] S. W. Golomb, How to Number A Graph, Graph Theory and Computing (R. C. Read. Ed.) Academic Press, New York, (1972)23-37.

[4] J. A. Gallian, A Dynamic Survey of Graph Labeling, The Electronics Journal of Combinatorics, 22, #DS6 (2020).

[5] V. J. Kaneria, H. P. Chudasama and P. P. Andharia, Absolute Mean Graceful Labeling in Path Union of Various Graphs, Math. J. Interdiscip. Sci. 7(1)(2018)51-56.

[6] V. J. Kaneria and H. P. Chudasama, Absolute Mean Graceful Labeling In Various Graphs, Int. J. Math. and Appl. 5(4) (2017) 723-726.

[7] V. J. Kaneria, P. Z. Akbari and N. A. Parmar, Some Absolute Mean Graceful Graphs, Int. J. Math. Trends and Tech. 68(1)(2022)86-93.

[8] V.J.Kaneria and H.M.Makadia, Graceful Labeling for Double Step Grid Graph, Int. J. of Mathematics and Its Applications, 3(1) (2015)33-38.

[9] F.Szabo, Linear Algebra: An Introduction Using Mathematica, Academic Press, (2000).

[10] I.Cahit, Cordial Graphs: A Weaker Version of Graceful and Harmonious Graphs, Ars Combin., 23(1987) 201-207.

[11] V.J.Kaneria and H.M.Makadia, Graceful Labeling for Swastik Graph, Int. J. Math. Appl., 3(3-D)(2015) 25-29.

[12] B.D.Acharya and M. K. Gill, On the Index of Gracefulness of A Graph and the Gracefulness of Two-Dimensional Square Lattice Graphs, Indian J. Math., 23(1981) 81-94.

[13] V.J.Kaneria, H.M.Makadia, M.M.Jariya and Meera Meghpara, Graceful Labeling for Complete Bipartite Graphs, Applied Math. Sci., 8(103)(2014), 5099-5104.

[14] V.Yagnanarayanan and P.Vaidhyanathan, Some Intresting Applications of Graph Labeling, J. Math. Comput. Sci.,2(5) (2012)1522-1531.

[15] S.K.Vaidya, N.A.Dani, K.K.Kanani and P.L.Vihol, Cordial and 3-Equitable Labeling for Some Star Related Graphs, International Mathematical Forum, 4(2009) 1543-1553.

[16] V.J.Kaneria, H.M.Makadia and R.V.Viradia, Various Graph Operations on Semi Smooth Graceful Graphs, International Journal of Math. and Soft Computing, 6(1) (2016)57-79.

[17] V.J.Kaneria, Meera Meghpara and Maulik Khoda, Semi Smooth Graceful Labeling and Its Application to Produce α-Labeling, J. Graph Labeling, 2(2) (2016)153-160.

[18] S.K.Vaidya and N.B.Vyas, Some New Results on Mean Labeling, Int. J. of Information Science and Computer Mathematics, 6(1-2) (2012)19-29.

[19] V.J.Kaneria and M.M.Jariya, Semi Smooth Graceful Graph and Constraction of New Graceful Trees, Elixir Appl. Math., 76(2014) 28536-28538.

[20] V.J.Kaneria and M.M.Jariya, Smooth Graceful Graphs and Its Applications to Constract Graceful Graphs, Int. J. Sci. and Res., 3(8) (2014) 909-912.

[21] S.K.Vaidya, S.Srivastav, V.J.Kaneria and G.V.Ghodasara, Cordial and 3-Equitable Labeling of Star of A Cycle, Mathematics Today, 24(2008) 54-64.
[22] V. J. Kaneria, Om Teraiya and Meera Meghpara, Double Path Union of α-Graceful Graph and Its α-Labeling, J. of Graph Labeling, 2(2)(2016)107-114.
[23] V. J. Kaneria, Meera Meghpara and H. M. Makadia, Graceful Labeling for Open Star of Graphs, Inter. J. of Mathematics and Statistics Invention, 2(9)(2014)19-23.
[24] V.J.Kaneria and S.K.Vaidya, Index of Cordiality for Complete Graphs and Cycle, IJAMC, 2(4) (2010) 38-46.
[25] V.J.Kaneria, Hardik Makadia and Meera Meghpara, Cordiality of Star of the Complete Graph and a Cycle Graph $(n \cdot Kn)$, J. of Math. Research, 6(4)(2014)18-28.