Some Absolute Mean Graceful Graphs in the Context of Barycentric Subdivision

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Abstract - In this paper, we present absolute mean graceful labeling for some graphs in context of barycentric subdivision of graph. We have proved that barycentric subdivision of wheel \( W_n \), complete bipartite graph \( K_{m,n} \), helm \( H_n \), sunlet graph \( S_n \), jelly fish graph \( J_{n,n} \), alternate quadrilateral snake \( AQ_n \) are absolute mean graceful graphs.

Keywords - Labeling, Graceful Labeling, Absolute Mean Graceful Labeling.

1. Introduction

Labeling of graph is the assignment of values to vertices or edges or both subject in certain conditions. A. Rosa[2] initiated the concept of labeling with the name of \( \beta \)-valuation. S. Golomb[3] named such labeling as graceful labeling. Kaneria and Chudasama[6] introduced graph labeling namely absolute mean graceful labeling. We begin with a simple, connected and undirected graph \( G = (V, E) \) with \( p \) vertices and \( q \) edges. for all terminology and notations, we follow F. Harary[1]. First of all we recall some definitions, which are used in this paper.

Definition 1.1 : A function \( f \) is called graceful labeling for a graph \( G \), if \( f: V(G) \rightarrow \{0, 1, 2, \ldots, q\} \) is injective and the induced function \( f^*: E(G) \rightarrow \{1, 2, \ldots, q\} \) defined as \( f^*(e) = |f(u) - f(v)| \) is bijective for every edge \( e = uv \in E(G) \). A graph \( G \) is called graceful graph, if it admits a graceful labeling.

Definition 1.2 : A function \( f \) is said to be absolute mean graceful labeling of a graph \( G \), if \( f: V(G) \rightarrow \{0, \pm 1, \pm 2, \ldots, \pm q\} \) is injective and edge labeling function \( f^*: E(G) \rightarrow \{1, 2, \ldots, q\} \) defined as \( f^*(e) = \left\lfloor \frac{|f(u) - f(v)|}{2} \right\rfloor \) is bijective, for every edge \( e = uv \in E(G) \). A graph \( G \) is called absolute mean graceful graph, if it admits an absolute mean graceful labeling.

Definition 1.3 : The barycentric subdivision of graph is obtained by inserting a vertex of degree two into every edge of original graph. Barycentric subdivision of helm \( H_4 \) is shown in Fig. 1.
Kaneria and Chudasama\[5,6\] proved that path graphs $P_n$, cycles $C_n$, complete bipartite graphs $K_{m,n}$, grid graphs $P_m \times P_n$, step grid graphs $S_{t_n}$, and double step grid graphs $D_{S_{t_n}}$ are absolute mean graceful graphs and they also proved that path union of finite copies in trees $T$, path graphs $P_n$, cycles $C_n$, complete bipartite graphs $K_{m,n}$, grid graphs $P_m \times P_n$, step grid graphs $S_{t_n}$, and double step grid graphs $D_{S_{t_n}}$ are absolute mean graceful graphs. Akbari, Kaneria and Parmar\[7\] proved that jewel graph $J_n$, jewel graph without prime edge $J_n^*$, extended jewel graph $EJ_n$, jelly fish graph $J_{m,n}$, jelly fish graph without prime edge $J_{m,n}^*$, extended jelly fish graph $EJ_{m,n}$ are absolute mean graceful graphs. For comprehensive learning of graph labeling, we referred Gallian\[4\].

2. Main Results

**Theorem 2.1** : The barycentric subdivision of wheel $W_n$ is absolute mean graceful graph.

**Proof** : Let $G$ be the barycentric subdivision of wheel $W_n$. Let $v_1, v_2, \ldots, v_{2n}$ be rim vertices of $G$. Let $u_1, u_2, \ldots, u_n$ be internal vertices of $G$. Let $v_0$ be the apex vertex of $G$. i.e. $V(G) = \{v_1, v_2, \ldots, v_{2n}\} \cup \{u_1, u_2, \ldots, u_n\} \cup \{v_0\}$ and $E(G) = \{v_iv_{i+1}/1 \leq i \leq 2n\} \cup \{v_iv_{2n}\} \cup \{v_0u_i/1 \leq i \leq n\} \cup \{v_{2i-1}u_i/1 \leq i \leq n\}$ to obtain vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, \ldots, \pm 4n\}$, we take following cases.

Case-I: $n \equiv 0 \pmod{2}$

$$f(v_i) = \begin{cases} (-1)^i (q + 2 - 2i) & \text{if } i = 1, 2, \ldots, n + 1 \\ (-1)^i (2i - 1) & \text{if } i = n + 2, n + 3, \ldots, 2n \\ n + 1 & \text{if } i = 0 \end{cases}$$

$$f(u_i) = \begin{cases} 2i - 2 & \text{if } i = 1, 2, 3, \ldots, \frac{n + 2}{2} \\ n - 2i + 1 & \text{if } i = \frac{n + 4}{2}, \frac{n + 6}{2}, \ldots, n \end{cases}$$

Case-II: $n \equiv 1 \pmod{2}$

$$f(v_i) = \begin{cases} (-1)^i (q + 2 - 2i) & \text{if } i = 1, 2, \ldots, n + 1 \\ (-1)^i (2i - 1) & \text{if } i = n + 2, n + 3, \ldots, 2n \\ n + 1 & \text{if } i = 0 \end{cases}$$

$$f(u_i) = \begin{cases} 2i - 2 & \text{if } i = 1, 2, 3, \ldots, \frac{n + 1}{2} \\ n - 2i + 2 & \text{if } i = \frac{n + 3}{2}, \frac{n + 5}{2}, \ldots, n \end{cases}$$

By defined pattern of function $f$, we can see that $f$ is one-one. Now we shall prove that induced function $f^*$ is a bijection. First of all we obtain range of $f^*$. For all cases,

$$\{ f^*(v_0u_i)/1 \leq i \leq n \} = \{1, 2, \ldots, n\}, \{ f^*(v_{2i-1}u_i)/1 \leq i \leq n \} = \{n + 1, n + 2, \ldots, 2n\}, \text{ and}$$

$$\{ f^*(v_iv_{i+1})/1 \leq i < 2n\} \cup \{v_1v_{2n}\} = \{2n + 1, 2n + 2, \ldots, 4n\},$$

i.e. $f^*(E(G)) = \{1, 2, \ldots, q = 4n\}$

Hence, $f^*$ is onto. Further, domain of $f^*$ and range of $f^*$ have same cardinality, gives $f^*$ is one-one. Therefore, $f^*$ is bijection. Thus, $f$ is absolute mean graceful labeling for $G$.

Therefore, barycentric subdivision of wheel is absolute mean graceful graph.
Illustration 2.2: Absolute mean graceful labeling for barycentric subdivision of wheel $W_{6}$ with $p = 19$ and $q = 24$ is shown in following Fig. 2.

Fig. 2 Absolute mean graceful labeling for barycentric subdivision of wheel $W_{6}$

Theorem 2.3: The barycentric subdivision of complete bipartite graph $K_{m,n}$ is absolute mean graph.

Proof: Let $G$ be the barycentric subdivision of complete bipartite graph $K_{m,n}$.

Let $V(K_{m,n}) = V_1 \cup V_2$ be the bipartition of vertex set of complete bipartite graph $K_{m,n}$. Let $\{u_i/1 \leq i \leq m\}$ denote the vertices of $V_1$ and $\{v_j/1 \leq j \leq n\}$ denote the vertices of $V_2$. Let $\{w_{ij}/1 \leq i \leq m, 1 \leq j \leq n\}$ be the vertices formed by barycentric subdivision of $K_{m,n}$, where $w_{ij}$ is the vertex adjacent to $u_i$ and $v_j$, $i \in \{1, 2, ..., m\}, j \in \{1, 2, ..., n\}$.

i.e. $V(G) = \{u_1, u_2, ..., u_m\} \cup \{v_1, v_2, ..., v_n\} \cup \{w_{ij}/1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(G) = \{u_iw_{ij}/1 \leq i \leq m, 1 \leq j \leq n\} \cup \{w_{ij}v_j/1 \leq i \leq m, 1 \leq j \leq n\}$.

The vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, ..., \pm 2mn\}$ defined as follows.

$f(u_i) = 4i - q - 2$ ; if $i = 1, 2, ..., m$

$f(v_j) = \begin{cases} 4mj - 2mn + 1 & \text{if } j = 1, 2, ..., n - 1 \\ -2mn & \text{if } j = n \end{cases}$

$f(w_{ij}) = 2(i + jm - m)$ ; \forall \ i, j

By defined pattern of function $f$, we can see that $f$ is one-one.

Now we shall prove that induced function $f^*$ is a bijection. First of all we obtain range of $f^*$.

$f^*[w_{ij}/1 \leq i < n, 1 \leq j \leq m] = \{1, 2, ..., m(n - 1)\}$

$f^*[w_{ij}/j = n, 1 \leq i \leq m] = \{2mn - m + 1, 2mn - m + 2, ..., 2mn\}$

$f^*[u_iw_{ij}/1 \leq i \leq m, 1 \leq j \leq n] = \{m(n - 1) + 1, m(n - 1) + 2, ..., 2mn - m\}$

i.e. $f^*[E(G)] = \{1, 2, ..., q = 2mn\}$

Hence, $f^*$ is onto. Further, domain of $f^*$ and range of $f^*$ have same cardinality, gives $f^*$ is one-one. Therefore, $f^*$ is bijection. Thus, $f$ is absolute mean graceful labeling for $G$.

Therefore, barycentric subdivision of complete bipartite graph is absolute mean graceful graph.
Illustration 2.4: Absolute mean graceful labeling for barycentric subdivision of complete bipartite graph $K_{3,4}$ with $p = 19$ and $q = 24$ is shown in following Fig. 3.

Fig. 3 Absolute mean graceful labeling for barycentric subdivision of complete bipartite graph $K_{3,4}$

Theorem 2.5: The barycentric subdivision of Helm $H_n$ is absolute mean graceful graph.

Proof: Let $G$ be the barycentric subdivision of helm $H_n$. Let $v_1, v_2, ..., v_{2n}$ be rim vertices on cycle of $G$. Let $v_0$ be the apex vertex of $G$. Let $w_1, w_2, ..., w_{2n}$ be vertices formed by barycentric subdivision of $H_n$, which is shown in figure.

i.e. $V(G) = \{v_1, v_2, ..., v_{2n}, u_1, u_2, ..., u_n, w_1, w_2, ..., w_{2n}, v_0\}$ and

$E(G) = \{v_0u_i/1 \leq i \leq n\} \cup \{v_{2i-1}u_i/1 \leq i \leq n\} \cup \{v_i, v_{i+1}/1 \leq i < 2n\} \cup \{v_1v_{2n}\} \cup \{v_{2i-1}w_i/1 \leq i \leq n\}
\cup \{w_0w_{i+n}/1 \leq i \leq n\}$

to obtain vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, ..., \pm 6n\}$, we take following cases.

Case-I: $n \equiv 0(\text{mod } 2)$

$f(v_i) = \begin{cases} 
(1)^i(q + 2 - 2i) & \text{if } i = 1, 2, ..., n + 1 \\
(1)^i(2i + 2n - 1) & \text{if } i = n + 2, n + 3, ..., 2n \\
-(n + 1) & \text{if } i = 0
\end{cases}$

$f(u_i) = \begin{cases} 
2n + 2i - 3 & \text{if } i = 1, 2, 3, 4, ..., \frac{n + 2}{2} \\
3n - 2i + 1 & \text{if } i = \frac{n + 4}{2}, \frac{n + 6}{2}, ..., n
\end{cases}$

$f(w_i) = \begin{cases} 
2i - 2 & \text{if } i = 1, 2, ..., \frac{n + 2}{2} \\
-n + 2i + 1 & \text{if } i = \frac{n + 4}{2}, \frac{n + 6}{2}, ..., n
\end{cases}$

Case-II: $n \equiv 1(\text{mod } 2)$

$f(v_i) = \begin{cases} 
(1)^i(q + 2 - 2i) & \text{if } i = 1, 2, ..., n + 1 \\
(1)^i(2i + 2n - 1) & \text{if } i = n + 2, n + 3, ..., 2n
\end{cases}$

$f(u_i) = \begin{cases} 
2n + 2i - 3 & \text{if } i = 1, 2, 3, 4, ..., \frac{n + 2}{2} \\
3n - 2i + 1 & \text{if } i = \frac{n + 4}{2}, \frac{n + 6}{2}, ..., n
\end{cases}$

$f(w_i) = \begin{cases} 
2n - 2i + 2 & \text{if } i = n + 2, n + 3, ..., \frac{3n + 2}{2} \\
2i - 3n - 1 & \text{if } i = \frac{3n + 4}{2}, \frac{3n + 6}{2}, ..., 2n
\end{cases}$
Now we shall prove that induced function $f^*$ is a bijection. First of all we obtain range of $f^*$. for all cases,
\[
\{f^*(v_iu_i)/1 \leq i \leq n\} = \{n + 1, n + 2, ..., 2n\}, \{f^*(w_iw_{n+i})/1 \leq i \leq n\} = \{1, 2, ..., n\}, \\
\{f^*(w_iw_{2i-1})/1 \leq i \leq n\} = \{2n + 1, 2n + 2, ..., 3n\}, \{f^*(v_i+1v_i)/1 \leq i \leq n\} = \{3n + 1, 3n + 2, ..., 4n\}, \\
\{f^*(v_i+v_{2n}) = \{4n + 1, 4n + 2, ..., 6n\} \\
\text{i.e. } f^*(E(G)) = \{1, 2, ..., n\} \\
\text{Hence, } f^* \text{ is onto. Further, domain of } f^* \text{ and range of } f^* \text{ have same cardinality, gives } f^* \text{ is one-one. Therefore, } f^* \text{ is bijection. Thus, } f \text{ is absolute mean graceful labeling for } G.
\]
Therefore, barycentric subdivision of helm is absolute mean graceful graph.

**Illustration 2.6**: Absolute mean graceful labeling for barycentric subdivision of helm $H_4$ with $p = 21$ and $q = 24$ is shown in Fig. 4.

![Fig. 4 Absolute mean graceful labeling for barycentric subdivision of helm $H_4$.](image-url)
**Theorem 2.7:** The barycentric subdivision of sunlet graph $S_n$ is absolut mean graceful graph.

**Proof:** Let $G$ be the barycentric subdivision of sunlet graph $S_n$. Let $v_1, v_2, ..., v_{2n}$ be vertices formed by barycentric subdivision of sunlet graph $S_n$.

i.e. $V(G) = \{v_1, v_2, ..., v_{2n}, w_1, w_2, ..., w_{2n}\}$ and $E(G) = \{v_{4i+1}v_{4i+1}/1 \leq i < 2n\} \cup \{v_{4i+2}v_{4i+2}/1 \leq i \leq n\} \cup \{w_{2i+1}w_{2i+1}/1 \leq i \leq n\}$

to obtain vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, ..., \pm 4n\}$, we take following cases.

**Case-I:** $n \equiv 0(\text{mod} \ 2)$

$f(v_i) = \begin{cases} (-1)^i (q + 2 - 2i) & \text{if } i = 1, 2, 3, 4, ..., n + 1 \\ (-1)^i (2i - 1) & \text{if } i = n + 2, n + 3, ..., 2n \end{cases}$

$f(w_i) = \begin{cases} 2i - 2 & \text{if } i = 1, 2, ..., \frac{n + 2}{2} \\ n - 2i + 1 & \text{if } i = \frac{n + 4}{2}, \frac{n + 6}{2}, ..., n \\ 1 & \text{if } i = n + 1 \\ 2n - 2i + 2 & \text{if } i = n + 2, n + 3, ..., \frac{3n + 2}{2} \\ 2i - 3n - 1 & \text{if } i = \frac{3n + 4}{2}, \frac{3n + 6}{2}, ..., 2n \\ \end{cases}$

**Case-II:** $n \equiv 1(\text{mod} \ 2)$

$f(v_i) = \begin{cases} (-1)^i (q + 2 - 2i) & \text{if } i = 1, 2, 3, 4, ..., n + 1 \\ (-1)^i (2i - 1) & \text{if } i = n + 2, n + 3, ..., 2n \end{cases}$

$f(w_i) = \begin{cases} 2i - 2 & \text{if } i = 1, 2, ..., \frac{n + 1}{2} \\ n - 2i + 2 & \text{if } i = \frac{n + 3}{2}, \frac{n + 5}{2}, ..., n \\ -2n & \text{if } i = n + 1 \\ 2n - 2i + 2 & \text{if } i = n + 2, n + 3, ..., \frac{3n + 1}{2} \\ 2i - 3n - 2 & \text{if } i = \frac{3n + 3}{2}, \frac{3n + 5}{2}, ..., 2n \\ \end{cases}$

By defined pattern of function $f$, we can see that $f$ is one-one.

Now we shall prove that induced function $f^*$ is a bijection. First of all we obtain range of $f^*$.

for all cases,

$\{f^*(v_{4i+1})/1 \leq i < 2n\} \cup \{v_{4i+2}v_{4i+2}/1 \leq i \leq n\} = \{1, 2, ..., n\}$

$\{f^*(w_{2i+1})/1 \leq i \leq n\} = \{n + 1, n + 2, ..., 2n\}$

i.e. $f^*(E(G)) = \{1, 2, ..., q = 4n\}$

Hence, $f^*$ is onto. Further, domain of $f^*$ and range of $f^*$ have same cardinality, gives $f^*$ is one-one. Therefore, $f^*$ is bijection.

Thus, $f$ is absolute mean graceful labeling for $G$.

Therefore, barycentric subdivision of sunlet graph is absolute mean graceful graph.
Illustration 2.8: Absolute mean graceful labeling for barycentric subdivision of sunlet graph $S_5$ with $p = 20$ and $q = 20$ is shown in following Fig. 5.

\[ \text{Fig. 5 Absolute mean graceful labeling for barycentric subdivision of sunlet graph } S_5 \]

**Theorem 2.9**: The barycentric subdivision jelly fish graph $J_{n,n}$ is an absolute mean graceful graph.

**Proof**: Let $G$ be barycentric subdivision of jelly fish graph $J_{n,n}$. Jelly fish graph is 4-cycle graph with vertices $x,y,u,v$ including the prime edge connecting to $x$ and $y$ and also by appending $n$ pendant edges to $u$ and $v$.

Let $V(J_{n,n}) = \{x,y,u,v,u_1,u_2,\ldots,u_n,v_1,v_2,\ldots,v_n\}$ and $E(G) = \{uv_i/1 \leq i \leq n\} \cup \{vv_j/1 \leq j \leq n\} \cup \{ux,xy,yv,xy\}$

Now we shall add $x_1,x_2,y_1,y_2$ vertices in 4-cycle, $x_0$ vertex in prime edge, $u_{n+1}$ vertex between $u$ and $u_i$ ($1 \leq i \leq n$) and $v_{n+j}$ vertex between $v$ and $v_j$ ($1 \leq j \leq n$) to obtain barycentric subdivision of jelly fish graph $J_{n,n}$.

i.e.

$V(G) = \{x,y,u,v,x_0,x_1,x_2,y_1,y_2\} \cup \{u_i/1 \leq i \leq 2n\} \cup \{v_j/1 \leq j \leq 2n\}$ and

$E(G) = \{uv_{n+i}/1 \leq i \leq n\} \cup \{uv_{n+i}/1 \leq i \leq n\} \cup \{vv_{n+j}/1 \leq j \leq n\} \cup \{uv_{n+i},v_j/1 \leq j \leq n\}$

$U\{xx_0,yx_0,xx_1,xx_2,ux_1,vy_1,yx_2,yy_1,yx_2,uy_1,vy_2\}$

The vertex labeling function $f:V(G) \to \{0, \pm 1, \pm 2, \ldots, \pm q\}$ is defined as follows.

$f(v) = q, f(u) = q - 8, f(x) = q - 3, f(y) = q - 4, f(x_0) = 14 - q, f(x_1) = 5 - q, f(x_2) = 1 - q, f(y_1) = 6 - q, f(y_2) = 2 - q$

$f(u_i) = \begin{cases} 3 + 2(i - n) & \text{if } i = 1, 2, 3, \ldots, n \\ 3 - 2i & \text{if } i = n + 1, n + 2, \ldots, 2n \end{cases}$

$f(v_j) = \begin{cases} 2j + 8 & \text{if } j = 1, 2, 3, \ldots, n \\ 10 - 2j + 2n & \text{if } j = n + 1, n + 2, \ldots, 2n \end{cases}$

By defined pattern of function $f$, we can see that $f$ is one-one.

Now we shall prove that induced function $f^*$ is a bijection. First of all we obtain range of $f^*$.

$\{f^*(v_{n+j})/1 \leq j \leq n\} \cup \{f^*(u_{n+i})/1 \leq i \leq n\} = \{1, 2, \ldots, 2n\}$.

$\{f^*(uv_{n+i})/1 \leq i \leq n\} \cup \{f^*(vv_{n+j})/1 \leq j \leq n\} = \{2n + 1, 2n + 2, \ldots, 4n\}$, and

$\{f^*(xx_0), f^*(yx_0), f^*(xx_1), f^*(xx_2), f^*(ux_1), f^*(vx_2), f^*(yy_1), f^*(yy_2), f^*(uy_1), f^*(vy_2)\} = \{q - 9, q - 8, \ldots, q\}$

i.e. $f^*(E(G)) = \{1, 2, \ldots, q = 4n + 10\}$
Hence, \( f^* \) is onto. Further, domain of \( f^* \) and range of \( f^* \) have same cardinality, gives \( f^* \) is one-one. Therefore, \( f^* \) is bijection. Thus, \( f \) is absolute mean graceful labeling for \( G \).
Therefore, barycentric subdivision of jelly fish graph is absolute mean graceful graph.

**Illustration 2.10** : Absolute mean graceful labeling for barycentric subdivision jelly fish graph \( j_{6,6} \) with \( p = 33 \) and \( q = 33 \) is shown in following Fig. 6.

![Fig. 6 Absolute mean graceful labeling for barycentric subdivision of jelly fish graph \( j_{6,6} \)](image)

**Theorem 2.11** : The barycentric subdivision of alternate quadrilateral snake \( AQ_n \) is absolute mean graceful graph.

**Proof** : Let \( G \) be the barycentric subdivision of alternate quadrilateral snake \( AQ_n \). Let \( V(AQ_n) = \{v_1, v_2, v_3, v_4, \ldots\} \)

Here \( v_i, v_{i+1}, v_{i+2}, v_{i+3} \) are vertices for \((4i-3)\)th number of snake.

Let \( \{u_1, u_2, u_3, u_4, \ldots\} \) are vertices insterted into quadrilateral snakes and \( \{w_1, w_2, \ldots\} \) are vertices insterted into alternate edge between snakes due to barycentric subdivision.

i.e. \( V(G) = \{v_1, v_2, v_3, v_4, u_1, u_2, u_3, u_4, w_1, \ldots\} \) \( E(G) = \{v_1u_1, v_1u_4, v_2u_4, v_2u_2, v_3u_2, v_3u_3, v_4u_3, v_4u_4, v_4w_1, \ldots\} \)

Let \( k \) be the number of snakes. The vertex labeling function \( f: V(G) \rightarrow \{0, \pm 1, \pm 2, \ldots, \pm q\} \) defined as follows.

\[
f(v_i) = \begin{cases} 
q - 2i + 2 & \text{if } i = 1, 2, 3, 4 \\
q - \frac{5(i-1)}{2} + 1 & \text{if } i = 5, 7, \ldots, 4k - 1 \\
q - \frac{5i}{2} + 3 & \text{if } i = 6, 8, \ldots, 4k - 2 \\
q - \frac{2i}{3} & \text{if } i = 4k \\
2i - q - 1 & \text{if } i = 1, 2, 3 \\
12 - q & \text{if } i = 4 \\
f(u_{i-4}) + 10 & \text{if } i = 5, 6, \ldots, 4k - 1 \\
5 & \text{if } i = 4k \\
f(w_i) = 10 - 4k & \text{if } i = 1, 2, \ldots, k.
\end{cases}
\]

By defined pattern of function \( f \), we can see that \( f \) is one-one.
Now we shall prove that induced function \( f^* \) is a bijection.
Here, \( f^*(E(G)) = \{1, 2, \ldots, q\} \)
Hence, \( f^* \) is onto map. As domain of \( f^* \) and range of \( f^* \) have same cardinality, gives \( f^* \) is one-one. Therefore, \( f^* \) is bijection. Thus, \( f \) is an absolute mean graceful labeling for \( G \).

Therefore, barycentric subdivision of alternate quadrilateral snake is absolute mean graceful graph.

**Illustration 2.12 :** Absolute mean graceful labeling for barycentric subdivision of alternate quadrilateral snake with number of snakes \( k = 3 \), \( p = 26 \) and \( q = 28 \) is shown in following Fig. 7.

![Fig. 7 Absolute mean graceful labeling for barycentric subdivision of alternate quadrilateral snake with \( k = 3 \)](image)

3. Conclusion

Present work contributes some new results. We discussed absolute mean gracefulness of various graphs. The labeling pattern is demonstrated by means of illustrations which is better understanding to derived results.

References

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