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Original Article

Generalized Multiplicative Indices on Certain Chemical Networks

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Abstract - *The evaluation of some indices for chemical networks, which includes the first and the second Zagreb index, generalized multiplicative indices by which we can project the stability or other properties of networks, such as n-dimensional silicate networks* $SL(n)$*, chain silicate networks* (C_n)*, hexagonal networks* (HX_n)*, oxide networks*(O_n)*, cellular networks*$HC(n)$*, and Sierpinski networks* $S(K_p, m)$*. The graphical analysis is used to plot the graphs and see the effects of our results on the considered parameters.*

Keywords - *Silicate networks, Chain silicate networks, Hexagonal networks, Oxide networks, Cellular networks, Sierpinski networks.*

1. Introduction

Every graph is represented by secure collateral interdependency constructions that are made up of vertices and edges, each of these vertices correlate to their relevant nodes and interconnecting connections by edges. These networks are networks are famed firmly to measure in theoretical terms [1]. The comparative standards of numerous networks alternate with the application of hardware technology, scheme of data routing, functions of computations, arrangements of information, and many other frameworks, structure, or implementation parameters. Researchers in collateral clarifications are thus influenced to put forward latest or advanced interconnection links, claiming the well being and contribution production assessments in separate circumstances [2].

Few of the connection network configurations are sketched and others take from nature such as complete binary trees, butterfly, grids, hypercubes and Benes networks are some of the structures [2]. For instance Honeycomb, grid and Hexagonal networks carry coincidence to molecular or atomic structures. We name them as natural structures as shown in figure 2. In a molecular graph the vertices signifies the atoms and edges connecting any two vertices represents the bonds. In chemistry the term valency and the vertex degree of a graph are closely associated with each other. A numerical value that is used to specify some properties of the molecular graph is termed as the graph index or topological index.

In chemical graph theory analysing molecular graphs of the molecular structure is a continuous attention; an attempt to excel in better comprehension of molecular structure. A topological index called the Wiener index was introduced by Wiener in the year 1947 [3]. In an acyclic saturated hydrocarbon, Winer index is the sum of distances of paths between any two carbon atoms. From the establishment of Winer index, it has been deduced to number of structures and utilized inregression models which has been used in the study of quantitative structure-activity relationship (QSAR) [4-7].

In a molecular graph the vertices signifies the atoms and edges connecting any two vertices represents the bonds. In chemistry the term valency and the vertex degree of a graph are closely associated with each other. A numerical value that is used to specify some properties of themolecular graph is termed as the graph index or topological index. Mathematical chemistry is the amalgamation of both chemistry as well as graph theory which emphasis on finding graph indices of a molecular graph which associates well with chemical properties of the molecular graph of the chemical compounds. Many graph indices has been studied and examined in theoreticalchemistry that these graph indices have various applications in QSPR/QSAR [8, 9, 18, 19].

Let G be a simple, finite, connected graph comprising $V(G)$as the vertex set and $E(G)$ edge set. The term $d(v)$ of a vertex v is called the degree and it is number of vertices incident to v. The vertices u and v connected by an edge is represented by uv. For other graph terminologies and notations. Consider a graph G, with cardinality of vertex set equal to p and edge set equal to q. Let $d(u)$ denotes the degree of an arbitrary vertex u in G. The minimum degree of any arbitrary vertex in $V(G)$ is represented by $\delta(G)$. For other graph terminologies we refer [10].

Zagreb form of first and second multiplicative indices are indices connected to the foundation of Wiener [11]. $M_1(G) = \prod_{u\in V(G)} d(u)^2$ and $M_2(G) = \prod_{uv\in E(G)} d(u)d(v)$ and Narumi-Katayama index $NK(G) = \prod_{v\in V(G)} d(v)$[8].

In the field of computational chemistry most of the research is based on the indices which are similar to the Wiener index [12-15].The Zagreb form of multiplicative indices for trees was determined and its features was studied by Gutman from this he obtained thedistinct

trees that has both highest and lowest values for $M_1(G)$and $M_2(G)$ in the year 2011 [12].Results on Gutman's result were generalized to k-trees by Wang and Wei [15].

$$W^s{}_1(G) = \prod_{u\in V(G)} d(u)^s$$

Note for $s = 1,2$ it represents Zagrebindex and Narumi-Katayama respectively.Eliasi et al.,determined an advanced version of the first Zagreb index based on the grounds of theeffective analysis of Zagreb multiplicativeindices [16].

$$M^*{}_1(G) = \prod_{uv\in E(G)} \big(d(u) + d(v)\big)$$

The first author introduced the first and second hyper-Zagreb indices of a graph, putting forward the conviction of indexing the edge set in the paper [17]. They are defined as

$$H_1(G) = \prod_{uv\in E(G)} \big(d(u) + d(v)\big)^2 \text{and} H_2(G) = \prod_{uv\in E(G)} \big(d(u)d(v)\big)^2$$

In this paper, we establish the general first and secondZagreb multiplicative indices of a graph G.

$$M^a{}_1(G) = \prod_{uv\in E(G)} \big(d(u) + d(v)\big)^a \text{and} M^a{}_2(G) = \prod_{uv\in E(G)} \big(d(u)d(v)\big)^a.$$

In the following sections we establish the multiplicative Zagreband the general multiplicative Zagreb indices for networks such as n-dimensional silicatenetworks $SL(n)$, chain silicate networks (C_n), hexagonalnetworks(HX_n), oxide networks (O_n),cellular networks $HC(n)$, and Sierpinski networks $S(K_p, m)$.

2. Results for Silicate Networks

The silicates are immense, themost fascinating and the most complexclass of minerals which are available to us undoubtedly. The fundamental chemicalcomponent of silicates is the tetrahedron(SiO_4).In a sheet of silicates, it comprises of ring of tetrahedrons these tetrahedrons are associated by nodes of oxygen to other tetrahedrons of a sheet like structure in a two dimensional plane.

Mixture of both the combination of metal carbonates or metal oxides and the sand gives the silicates. Necessarily, a silicate accommodatestetrahedron(SiO_4). The corner nodes of tetrahedron(SiO_4) constitute ions of oxygen and the center node constitute the ions of silicon in the field of chemistry. Same thing can be called as oxygen vertex and silicon vertex in graph theory as in Figure 1.

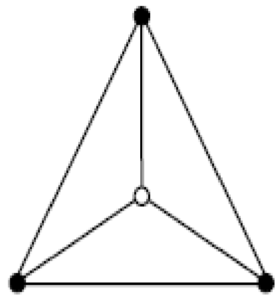


Fig. 1 Tetrahedron(SiO_4) where corner vertex is oxygen ions and center is the silicon ions

These minerals are procured byamalgamation oftwo tetrahedronswith the oxygen verticesof distinct kinds of silicates continuously. We can obtain the differentkinds of structures of silicates from themethods of arrangements oftetrahedron:they may occur as discrete un-associated entities,as associated finite number of arrays, as the chains ofone-dimensions, as 3-dimensional frameworks or as 2-dimensional sheets. Few of theunits of silicates are shownin figures 2 and 3. These are named assheet silicates, pyrosilicates,cyclic silicates,orthosilicates and chain silicates.

Fig. 2 Different forms of silicates

Basicorthosilicates component holds a separate SiO_4units.When two tetrahedron SiO_4 are combined with a common oxygen vertex, we havepyrosilicates .When these tetrahedras arelinearly organized,then we have chainsilicates as in Figure 2.

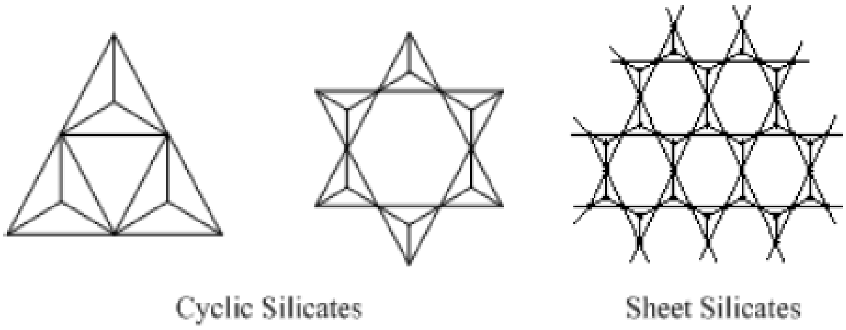


Fig. 3 Cyclic and sheet silicates

There are many ways how a silicate network can be formulated. We formulate a silicate network from a honeycomb structure. We take a network of honeycomb $HC(n)$ of dimension $n.$ On each of the vertices of $HC(n)$ set these silicon ions.Take each of the edges of $HC(n)$ once and sub-divide. On each of the new vertices formed, place the oxygen ions. On each of the two degree silicon ions infuse $6n$ pendant edges on $HC(n)$, later at the pendent vertices set the oxygen ions as in figure 4(a). For eachsiliconionlink the three oxygen ions which are adjacent such that they form a tetrahedron structure as in figure 4(b).The network then obtained iscalled as the silicate network of n dimension, represented by $SL(n)$. The graph infigure 4(b) is a silicate network ofdimension two.

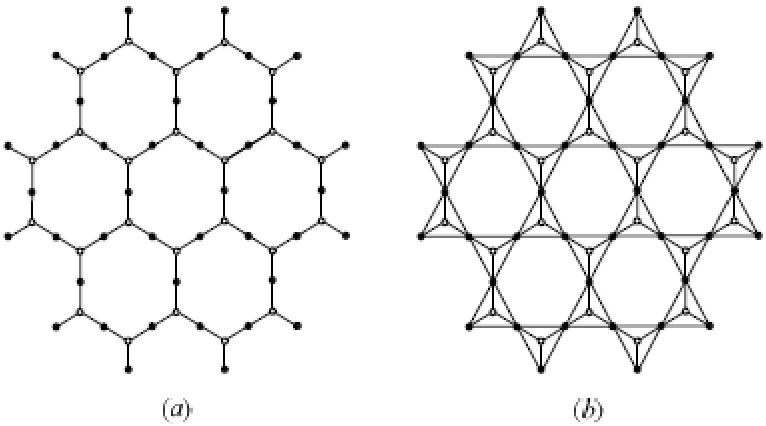


Fig. 4 Construction of silicate networks

The cardinality of vertex set in $SL(n)$is $(15n^2 + 3n)$ and edge set is $36n^2$. $SL(n)$is the silicate networkof $n$dimensionis a bi-regular graph. The number of degree three vertices is equal to $(6n^2 + 6n)$and number of degree six vertices is equal to $(9n^2 - 3n)$vertices of degree 6.

There are three kinds of edges possible related to the degree of vertices on each edge of a silicate network. The following tablegives the three types and gives the numberof edges in each type.

Table 1. Edge partition of $SL(n)$

Sl No.	$d(u)d(v)\backslash uv \in E(G)$	No. of edges
1.	(3,3)	6n
2.	(3,6)	$18n^2+6n$
3.	(6,6)	$18n^2-12n$

Theorem 2.1. Let G be the graph of a $SL(n)$. Then

i). $M_1(G) = (3)^{2(6n^2+6n)}(6)^{2(9n^2-3n)}$

ii).$M_2(G) = (9)^{6n}(18)^{18n^2+6n}(36)^{18n^2-12n}$.

iii). $M^*{}_1(G) = (6)^{6n}(9)^{18n^2+6n}(12)^{18n^2-12n}$.

iv). $H_1(G) = (36)^{6n}(81)^{18n^2+6n}(144)^{18n^2-12n}$

v). $H_2(G) = (81)^{6n}(324)^{18n^2+6n}(1296)^{18n^2-12n}$.

vi).$M^{(a)}{}_1(G) = (6)^{a(6n)}(9)^{a(18n^2+6n)}(12)^{a(18n^2-12n)}$.

vii).$M^{(a)}{}_2(G) = (9)^{a(6n)}(18)^{a(18n^2+6n)}(36)^{a(18n^2-12n)}$.

Proof:

Let G be a graph. Using table 1

i). $M_1(G) = \prod_{u\in V(G)} d(u)^2$

$M_1(G) = (3)^{2(6n^2+6n)}(6)^{2(9n^2-3n)}$

ii). $M_2(G) = \prod_{uv\in V(G)} d(u)d(v)$

$M_2(G) = (9)^{6n}(18)^{18n^2+6n}(36)^{18n^2-12n}$.

iii). $M^*{}_1(G) = \prod_{uv\in E(G)}(d(u)+d(v))$

$M^*{}_1(G) = (6)^{6n}(9)^{18n^2+6n}(12)^{18n^2-12n}$.

iv).$H_1(G) = \prod_{uv\in E(G)}\big(d(u)+d(v)\big)^2$

$H_1(G) = (36)^{6n}(81)^{18n^2+6n}(144)^{18n^2-12n}$.

v). $H_2(G) = \prod_{uv\in E(G)}\big(d(u)d(v)\big)^2$

$H_2(G) = (81)^{6n}(324)^{18n^2+6n}(1296)^{18n^2-12n}$.

vi). $M^{(a)}{}_1(G) = \prod_{uv\in E(G)}\big(d(u)+d(v)\big)^a$

$M^{(a)}{}_1(G) = (6)^{a(6n)}(9)^{a(18n^2+6n)}(12)^{a(18n^2-12n)}$.

vii). $M^{(a)}{}_2(G) = \prod_{uv\in E(G)}\big(d(u).d(v)\big)^a$

$M^{(a)}{}_2(G) = (9)^{a(6n)}(18)^{a(18n^2+6n)}(36)^{a(18n^2-12n)}$.

We plot the graph of $SL(n)$.

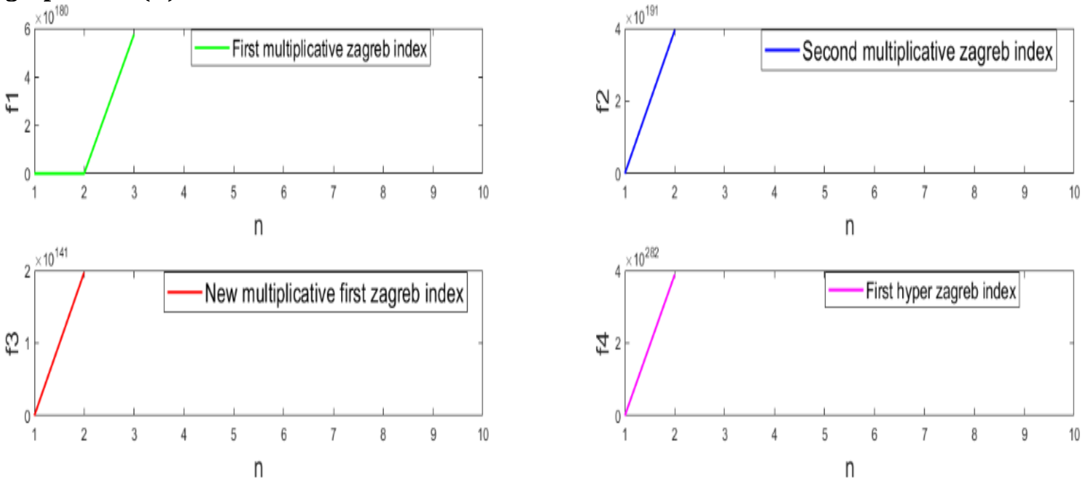


Fig. 5 Plots of $SL(n)$ where $f1(n) = M_1(G), f2(n) = M_2(G), f3(n) = M_1^*(G)$ and $f4(n) = H_1(G)$.

3. Results for Chain silicate networks

We find tetrahedralSiO_4 component in every silicates available. It is an extremely vital and intricate mineral. Silicates are the composition of sand with carbonates of metal or by combining metal oxides. These are the building blocks for the standard rock-forming minerals.Now, we studywith reference to another component of thesilicate networks, known as the chain silicateC_n networks.$C_n$has $(3n + 1)$ vertices and $6n$ number of edges. Based on the properties of graphical representation of vertex degree C_n, the partition of vertices contains $(2n + 2)$ number of degree 3 and $(n - 1)$ number of degree 6 vertices.

Take a single tetrahedron which as pyramid structure with a triangular base, at the four corners of tetrahedron set the oxygen atoms and the silicon atomwhich are placed in between equally spaced oxygen atoms. The tetrahedron thus formed is a silicate tetrahedron, which is given in Figure (6a), and when we join tetrahedron one after the otherin a linear way we obtain a single silicate chain in a row, as shown in Figure (6b). The edges of the chain silicateC_n networks is divided into three types of vertex based division on valencies,as A_1, A_2andA_3. For $n = 1, A_1$ contains 6 edges ij,where $d_i = 3$ and $d_j = 3, A_2$ contains 0 edges ij, where $d_i = 3$ and $d_j = 6, A_3$ contains 0 edges ij, where $d_i = 6$ and $d_j = 6$. For $n \geq 2, A_1$ contains $(n + 4)$ edges ij, $d_i = 3$ and $d_j = 3$, A_2 contains $(4n - 2)$ edges ij, $d_i = 3$ and $d_j = 6, A_3$ contains $(n - 2)$ edges ij, $d_i = 6$ and $d_j = 6$.

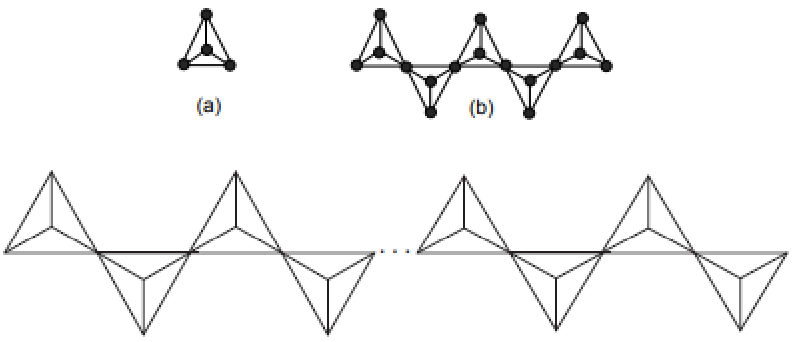


Fig. 6 Graph of C_n

Table 2. Edge partition of C_n

Sl No.	$d(u)d(v)\backslash uv \in E(G)$	No. of edges
1.	(3,3)	$(n + 4)$
2.	(3,6)	$(4n - 2)$
3.	(6,6)	$(n - 2)$

Theorem 3.1. Let G be the graph of C_n. Then

i). $M_1(G) = (3)^{2(2n+2)}(6)^{2(n-1)}$

ii).$M_2(G) = (9)^{(n+4)}(18)^{(4n-2)}(36)^{(n-2)}$.

iii). $M^*{}_1(G) = (6)^{(n+4)}(9)^{(4n-2)}(12)^{(n-2)}$.

iv). $H_1(G) = (36)^{(n+4)}(81)^{(4n-2)}(144)^{(n-2)}$

v). $H_2(G) = (81)^{(n+4)}(324)^{(4n-2)}(1296)^{(n-2)}$

vi). $M^{(a)}{}_1(G) = (6)^{a(n+4)}(9)^{a(4n-2)}(12)^{a(n-2)}$.

vii). $M^{(a)}{}_2(G) = (9)^{a(n+4)}(18)^{a(4n-2)}(36)^{a(n-2)}$.

Proof:

For $n = 1$, it is easy to calculate.

We consider for $n > 1$

Let G be a graph. Using table 2

i). $M_1(G) = \prod_{u\in V(G)} d(u)^2$

$M_1(G) = (3)^{2(2n+2)}(6)^{2(n-1)}$

ii). $M_2(G) = \prod_{uv\in V(G)} d(u)d(v)$

$M_2(G) = (9)^{(n+4)}(18)^{(4n-2)}(36)^{(n-2)}$.

iii). $M^*{}_1(G) = \prod_{uv\in E(G)}(d(u) + d(v))$

$M^*{}_1(G) = (6)^{(n+4)}(9)^{(4n-2)}(12)^{(n-2)}$.

iv).$H_1(G) = \prod_{uv\in E(G)}\big(d(u) + d(v)\big)^2$

$H_1(G) = (36)^{(n+4)}(81)^{(4n-2)}(144)^{(n-2)}$.

v). $H_2(G) = \prod_{uv\in E(G)}\big(d(u)d(v)\big)^2$

$H_2(G) = (81)^{(n+4)}(324)^{(4n-2)}(1296)^{(n-2)}$.

vi). $M^{(a)}{}_1(G) = \prod_{uv\in E(G)}\big(d(u) + d(v)\big)^a$

$M^{(a)}{}_1(G) = (6)^{a(n+4)}(9)^{a(4n-2)}(12)^{a(n-2)}$.

vii). $M^{(a)}{}_2(G) = \prod_{uv\in E(G)}\big(d(u).d(v)\big)^a$

$M^{(a)}{}_2(G) = (9)^{a(n+4)}(18)^{a(4n-2)}(36)^{a(n-2)}$.

We plot the graph of C_n

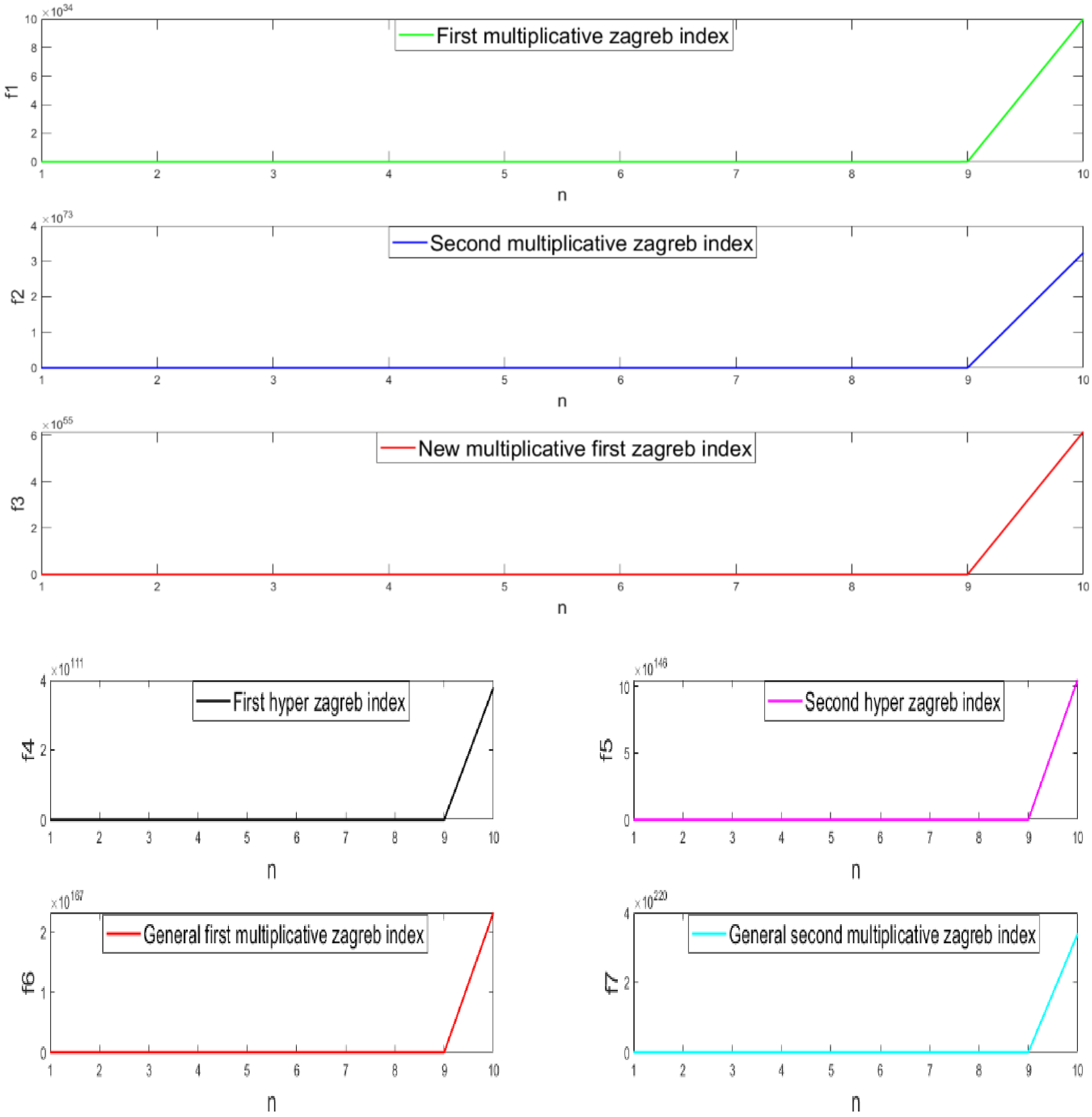


Fig. 7 Plots of C_n where $f1(n) = M_1(G)$, $f2(n) = M_2(G)$, $f3(n) = M_1^*(G)$, $f4(n) = H_1(G)$, $f5(n) = H_2(G)$, $f6(n) = M^{(a)}{}_1(G)$ and $f7(n) = M^{(a)}{}_2(G)$

4. Results for Oxide network

Oxide networks are very essential in the survey of silicate networks. Oxide network is obtained from silicon network, by removing siliconvertices such a network is named as oxide network. O_n represents oxide network of n-dimension. O_n has $(9n^2 + 3n)$ vertices and $18n^2$ edges also O_n has $6n$ degree two vertices and $(9n^2 - 3n)$ number of degree four vertices.

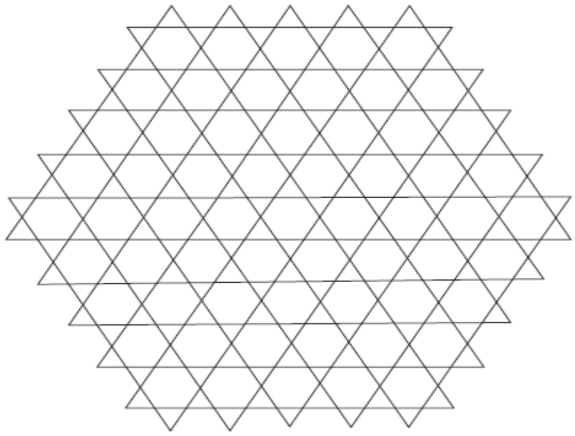


Fig. 8 Graph of O_n

Table 3. Edge partition of O_n

Sl No.	$d(u)d(v)\backslash uv \in E(G)$	No. of edges
1.	(2,4)	$12n$
2.	(4,4)	$18n^2 - 12n$

Theorem 4.1. Let *G* be the graph of O_n. Then

i). $M_1(G) = (2)^{2(6n)}(4)^{2(9n^2-3n)}$.

ii). $M_2(G) = (8)^{(12n)}(16)^{(18n^2-12n)}$.

iii). $M^*{}_1(G) = (6)^{(12n)}(8)^{(18n^2-12n)}$.

iv). $H_1(G) = (36)^{(12n)}(64)^{(18n^2-12n)}$.

v). $H_2(G) = (64)^{(12n)}(256)^{(18n^2-12n)}$.

vi). $M^{(a)}{}_1(G) = (6)^{a(12n)}(8)^{a(18n^2-12n)}$.

vii). $M^{(a)}{}_2(G) = (8)^{a(12n)}(16)^{a(18n^2-12n)}$.

Proof:

Let G be a graph. Using table 3

i). $M_1(G) = \prod_{u \in V(G)} d(u)^2$

$M_1(G) = (2)^{2(6n)}(4)^{2(9n^2-3n)}$.

ii). $M_2(G) = \prod_{uv \in V(G)} d(u)d(v)$

$M_2(G) = (8)^{(12n)}(16)^{(18n^2-12n)}$.

iii). $M^*{}_1(G) = \prod_{uv \in E(G)}(d(u) + d(v))$

$M^*{}_1(G) = (6)^{(12n)}(8)^{(18n^2-12n)}$.

iv). $H_1(G) = \prod_{uv \in E(G)}\big(d(u) + d(v)\big)^2$

$H_1(G) = (36)^{(12n)}(64)^{(18n^2-12n)}$.

v). $H_2(G) = \prod_{uv \in E(G)}\big(d(u)d(v)\big)^2$

$H_2(G) = (64)^{(12n)}(256)^{(18n^2-12n)}$.

vi). $M^{(a)}{}_1(G) = \prod_{uv \in E(G)}\big(d(u) + d(v)\big)^a$

$M^{(a)}{}_1(G) = (6)^{a(12n)}(8)^{a(18n^2-12n)}$.

vii). $M^{(a)}{}_2(G) = \prod_{uv\in E(G)}\big(d(u).d(v)\big)^a$

$M^{(a)}{}_2(G) = (8)^{a(12n)}(16)^{a(18n^2-12n)}$.

We plot the graph of O_n

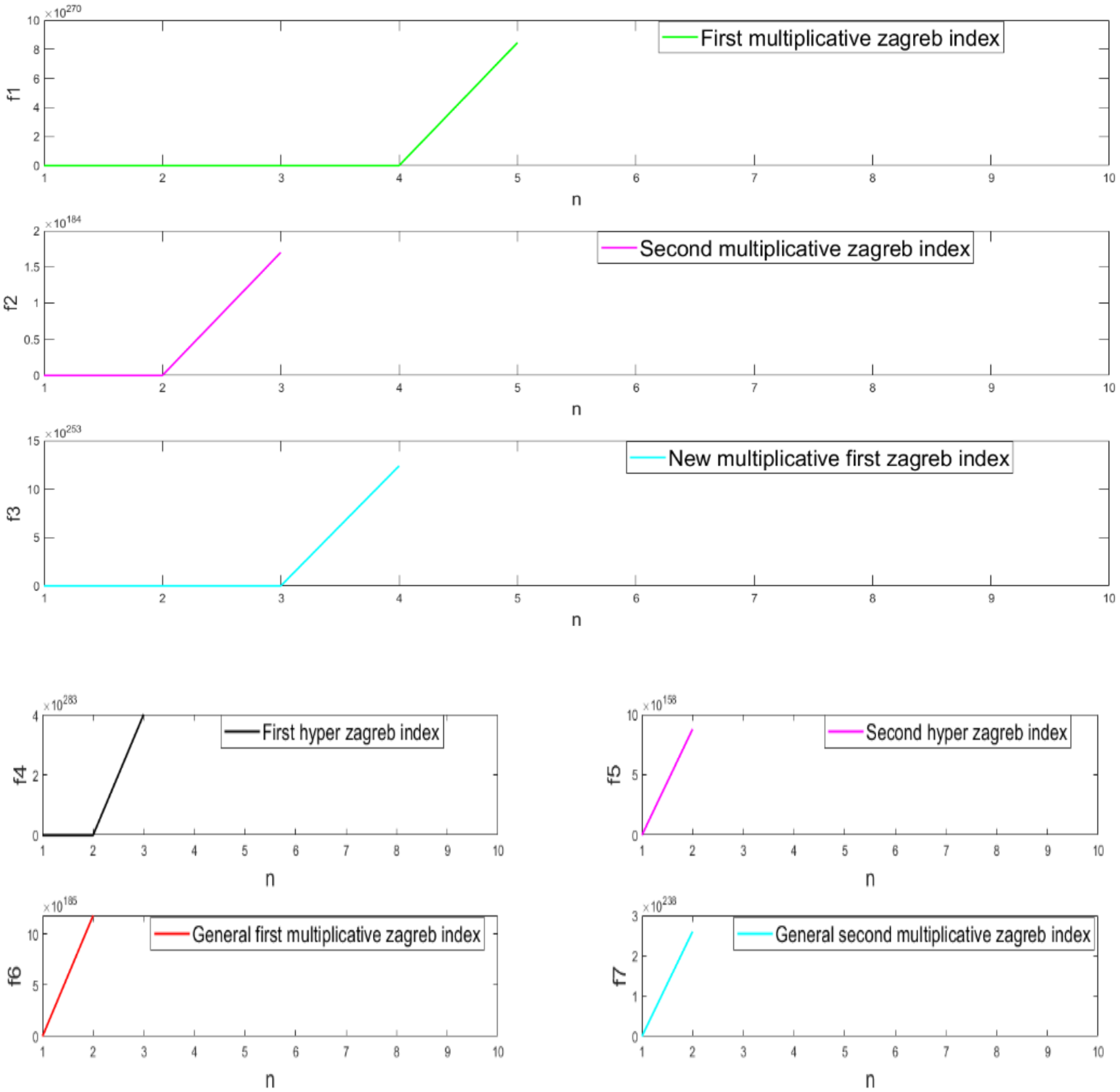


Fig. 9 Plots of O_n where $f1(n) = M_1(G), f2(n) = M_2(G), f3(n) = M_1^*(G), f4(n) = H_1(G), f5(n) = H_2(G), f6(n) = M^{(a)}{}_1(G)$ and $f7(n) = M^{(a)}{}_2(G)$.

5. Results for Hexagonal Networks

It is investigated that there occursthree kinds of lattice, which are made up of similar kind of polygons such as hexagonal, triangular and square. Triangularlattice is used to construct the hexagonal networks. HX_nRepresents the hexagonal network of ndimension, wheren represents the total number of vertices on every side of hexagon. The HX_nhas $3n^2 - 3n + 1$ vertices and $9n^2 - 15n + 6$ edges. Based on the degrees, there are three kinds of vertices. HX_nhas six vertices of degree three, $6n - 12$ vertices of degree four, and $3n^2 - 9n + 7$ vertices of degree six.

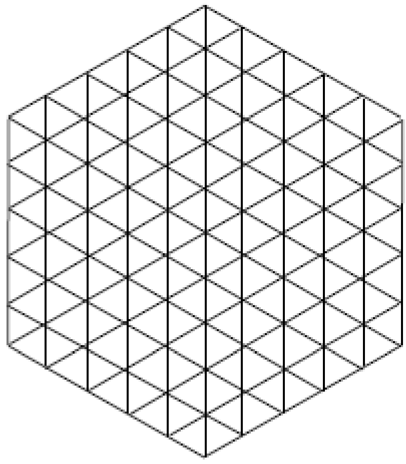


Fig. 10 Graph of HX_n

Table 4. Edge partition ofHX_n

Sl No.	$d(u)d(v)\backslash uv \in E(G)$	No. of edges
1.	(3,4)	12
2.	(3,6)	6
3.	(4,4)	$6n-18$
4.	(4,6)	$12n-24$
5.	(6,6)	$9n^2-33n+30$

Theorem 5.1. Let *G* be the graph of a HX_n. Then

i). $M_1(G) = (3)^{2(6)}(4)^{2(6n-12)}(6)^{2(3n^2-9n+7)}$.

ii).$M_2(G) = (12)^{(12)}(18)^{(6)}(16)^{(6n-18)}(24)^{(12n-24)}(36)^{(9n^2-33n+30)}$.

iii). $M^*{}_1(G) = (7)^{(12)}(9)^{(6)}(8)^{(6n-18)}(10)^{(12n-24)}(12)^{(9n^2-33n+30)}$.

iv). $H_1(G) = (49)^{(12)}(81)^{(6)}(64)^{(6n-18)}(100)^{(12n-24)}(144)^{(9n^2-33n+30)}$.

v). $H_2(G) = (144)^{(12)}(324)^{(6)}(256)^{(6n-18)}(576)^{(12n-24)}(1296)^{(9n^2-33n+30)}$.

vi). $M^{(a)}{}_1(G) = (7)^{a(12)}(9)^{a(6)}(8)^{a(6n-18)}(10)^{a(12n-24)}(12)^{a(9n^2-33n+30)}$.

vii). $M^{(a)}{}_2(G) = (12)^{a(12)}(18)^{a(6)}(16)^{a(6n-18)}(24)^{a(12n-24)}(36)^{a(9n^2-33n+30)}$.

Proof:

Let G be a graph. Using table 4

i). $M_1(G) = \prod_{u\in V(G)} d(u)^2$

$M_1(G) = (3)^{2(6)}(4)^{2(6n-12)}(6)^{2(3n^2-9n+7)}$.

ii). $M_2(G) = \prod_{uv\in V(G)} d(u)d(v)$

$M_2(G) = (12)^{(12)}(18)^{(6)}(16)^{(6n-18)}(24)^{(12n-24)}(36)^{(9n^2-33n+30)}$.

iii). $M^*{}_1(G) = \prod_{uv\in E(G)}(d(u)+d(v))$

$M^*{}_1(G) = (7)^{(12)}(9)^{(6)}(8)^{(6n-18)}(10)^{(12n-24)}(12)^{(9n^2-33n+30)}$.

iv). $H_1(G) = \prod_{uv\in E(G)}\big(d(u)+d(v)\big)^2$

$H_1(G) = (49)^{(12)}(81)^{(6)}(64)^{(6n-18)}(100)^{(12n-24)}(144)^{(9n^2-33n+30)}$.

v). $H_2(G) = \prod_{uv\in E(G)}\big(d(u)d(v)\big)^2$

$H_2(G) = (144)^{(12)}(324)^{(6)}(256)^{(6n-18)}(576)^{(12n-24)}(1296)^{(9n^2-33n+30)}$.

vi). $M^{(a)}{}_1(G) = \prod_{uv\in E(G)}\big(d(u)+d(v)\big)^a$.

$M^{(a)}{}_1(G) = (7)^{a(12)}(9)^{a(6)}(8)^{a(6n-18)}(10)^{a(12n-24)}(12)^{a(9n^2-33n+30)}$.

vii). $M^{(a)}{}_2(G) = \prod_{uv\in E(G)}\big(d(u).d(v)\big)^a$.

$M^{(a)}{}_2(G) = (12)^{a(12)}(18)^{a(6)}(16)^{a(6n-18)}(24)^{a(12n-24)}(36)^{a(9n^2-33n+30)}$.

We plot the graph of HX_n

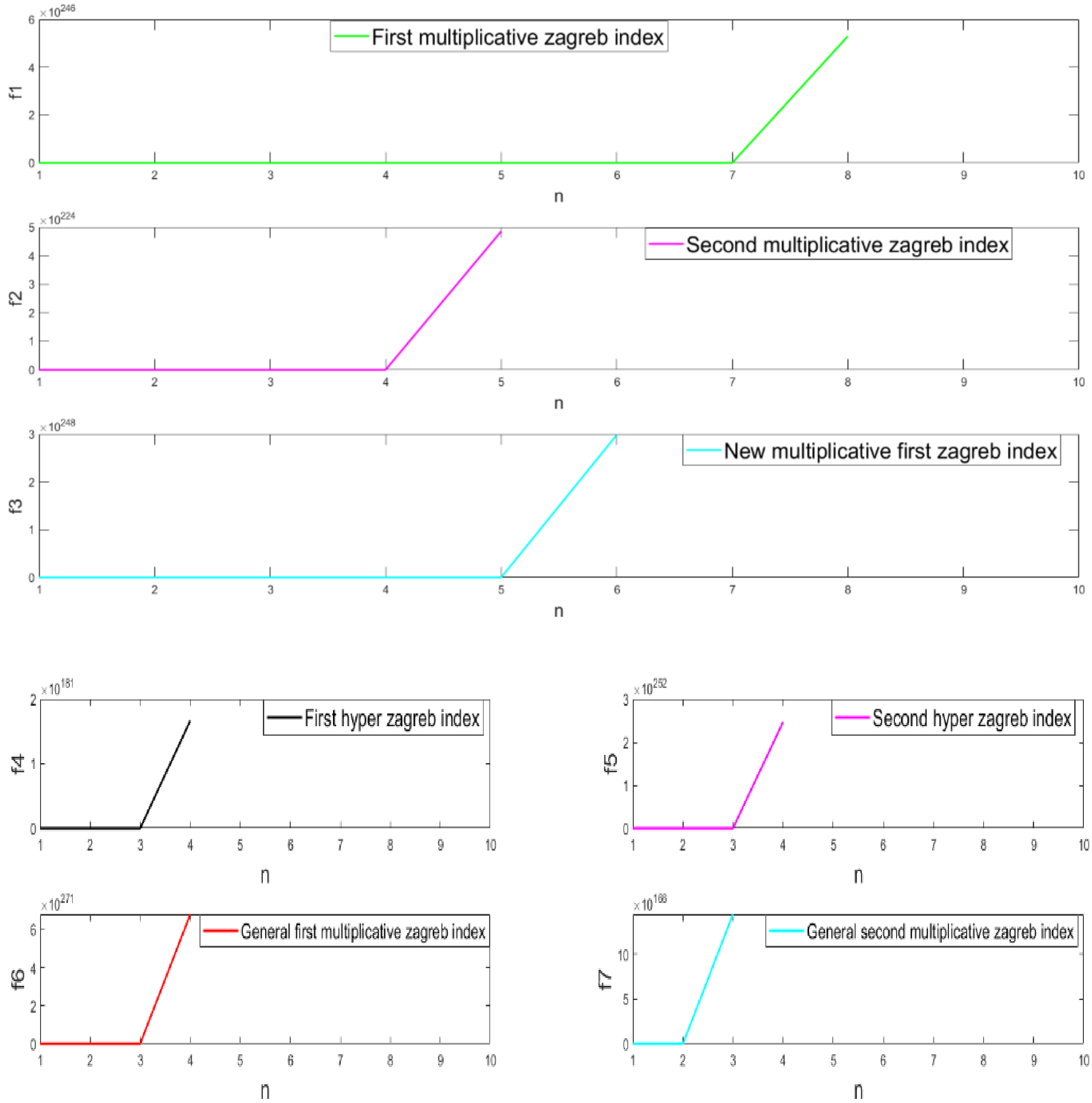


Fig. 11 Plots of HX_n where $f1(n) = M_1(G), f2(n) = M_2(G), f3(n) = M_1^*(G), f4(n) = H_1(G), f5(n) = H_2(G), f6(n) = M^{(a)}{}_1(G)$ and $f7(n) = M^{(a)}{}_2(G)$.

6. Results for Cellular Networks

It is constructed by repeated usage of lattice of hexagon structure, these networks are broadly utilized in, cellular phone base stations,computer graphics, image processing, andas the representation ofbenzenoid hydrocarbonsin chemistry. $HC(n)$is a cellular network is derived from $HC(n-1)$ by adjoining hexagon layers around the boundary layer of $HC(n-1)$. Here$n$ denotes the number of hexagons located in between boundary of $HC(n)$ and centre. $HC(n)$has $6n^2$ vertices and $9n^2-3n$ edges. By foldinghoneycomb sheets of infinite length in multiple ways we obtain carbon nanotubes otherwise a graphite sheet where$n$ is considered to be large.This networkis a dual of $HC(n+1)$bounded hexagonalnetwork. $HC(n)$network has $6n$ and $6n^2-6n$ vertices of degree two and three respectively.There arethree forms of edges derived from the degree of vertices of every edge of $HC(n)$.

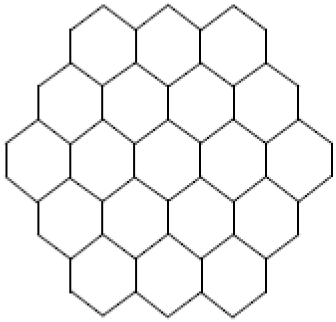


Fig. 12 Graph of $HC(n)$

Table 5. Edge partition of $HC(n)$

Sl No.	$d(u)d(v)\backslash uv \in E(G)$	**No. of edges**
1.	(2,2)	6
2.	(2,3)	$12(n-1)$
3.	(3,3)	$(9n^2-15n+6)$

Theorem 6.1. Let *G* be the graph of $HC(n)$. Then

i). $M_1(G) = (2)^{2(6n)}(3)^{2(6n^2-6n)}$.

ii).$M_2(G) = (4)^{(6)}(6)^{12(n-1)}(9)^{(9n^2-15n+6)}$.

iii). $M^*{}_1(G) = (4)^{(6)}(5)^{12(n-1)}(6)^{(9n^2-15n+6)}$.

iv). $H_1(G) = (16)^{(6)}(25)^{12(n-1)}(36)^{(9n^2-15n+6)}$.

v). $H_2(G) = (16)^{(6)}(36)^{12(n-1)}(81)^{(9n^2-15n+6)}$.

vi). $M^{(a)}{}_1(G) = (4)^{a(6)}(5)^{a(12n-12)}(6)^{a(9n^2-15n+6)}$.

vii). $M^{(a)}{}_2(G) = (4)^{a(6)}(6)^{a(12n-12)}(9)^{a(9n^2-15n+6)}$.

Proof:

Let G be a graph. Using table 5

i). $M_1(G) = \prod_{u\in V(G)} d(u)^2$

$M_1(G) = (2)^{2(6n)}(3)^{2(6n^2-6n)}$.

ii). $M_2(G) = \prod_{uv\in V(G)} d(u)d(v)$

$M_2(G) = (4)^{(6)}(6)^{12(n-1)}(9)^{(9n^2-15n+6)}$.

iii). $M^*{}_1(G) = \prod_{uv\in E(G)}(d(u)+d(v))$

$M^*{}_1(G) = (4)^{(6)}(5)^{12(n-1)}(6)^{(9n^2-15n+6)}$.

iv). $H_1(G) = \prod_{uv\in E(G)} \left(d(u)+d(v)\right)^2$

$H_1(G) = (16)^{(6)}(25)^{12(n-1)}(36)^{(9n^2-15n+6)}$.

v). $H_2(G) = \prod_{uv\in E(G)} \left(d(u)d(v)\right)^2$

$H_2(G) = (16)^{(6)}(36)^{12(n-1)}(81)^{(9n^2-15n+6)}$.

vi). $M^{(a)}{}_1(G) = \prod_{uv\in E(G)} \left(d(u)+d(v)\right)^a$.

$M^{(a)}{}_1(G) = (4)^{a(6)}(5)^{a(12n-12)}(6)^{a(9n^2-15n+6)}$.

vii). $M^{(a)}{}_2(G) = \prod_{uv\in E(G)} \left(d(u).d(v)\right)^a$.

$M^{(a)}{}_2(G) = (4)^{a(6)}(6)^{a(12n-12)}(9)^{a(9n^2-15n+6)}$.

We plot the graph of $HC(n)$

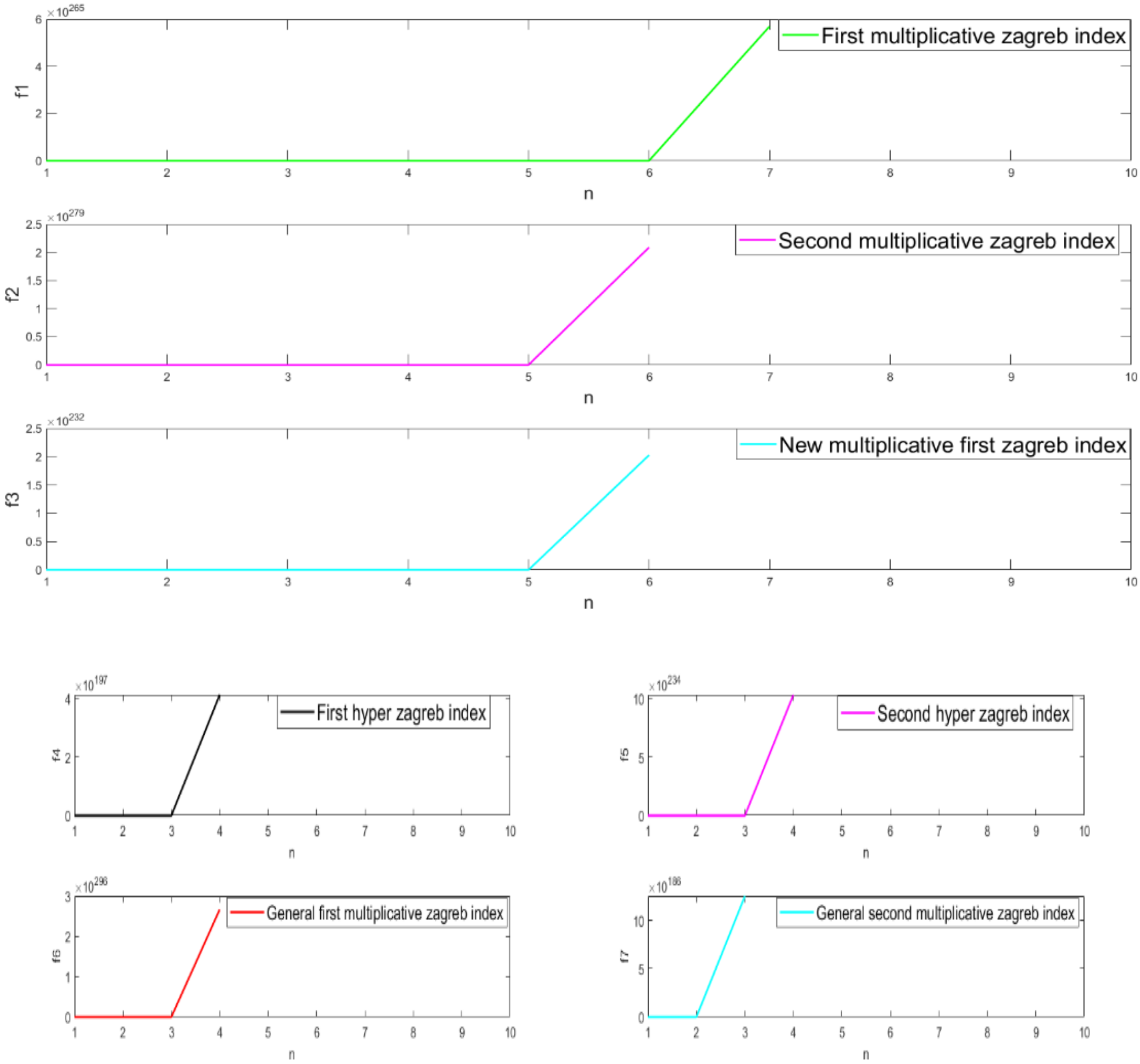


Fig 13. Plots of $HC(n)$ where $f1(n) = M_1(G), f2(n) = M_2(G), f3(n) = M_1^*(G), f4(n) = H_1(G), f5(n) = H_2(G), f6(n) = M^{(a)}{}_1(G)$ and $f7(n) = M^{(a)}{}_2(G)$.

7. Results for Sierpinski networks

To study the Sierpinskinetworks $S(K_p, m)$ where subgraphs of $S(K_p, m)$ are complete. In [32], based on the valencies every edge of $S(K_p, m)$ can be divided in to two sets of vertices A_1 and A_2. In the A_1set for all ij, we have $m(m-1)$ edges in which $d_i = m$ and $d_j = (m+1)$. Similarly, in the A_2set for all ij, we have $\frac{(m^{t+1}-2m^2+m)}{2}$edges in which $d_i = (m+1)$ and $d_j = (m+1)$.

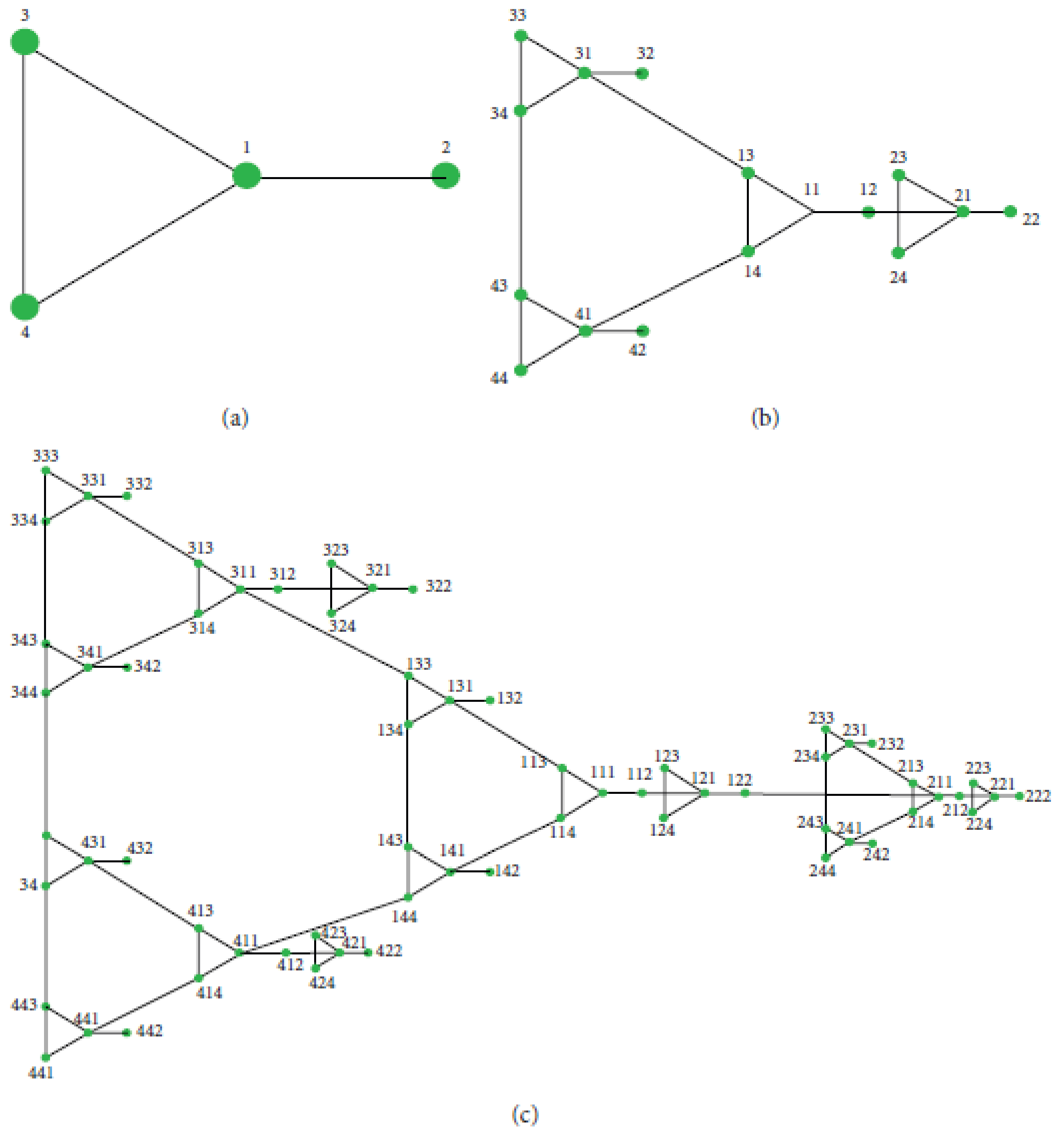


Fig. 14 Graph of $S(K_p, m)$(a) *t* =1. (b) *t*=2. (c) *t*=3.

Table 6. Edge partition of $S(K_p, m)$

Sl No.	$d(u)d(v)\backslash uv \in E(G)$	**No. of edges**
1.	$(m, m+1)$	$m(m-1)$
2.	$(m+1, m+1)$	$\frac{(m^{t+1}-2m^2+m)}{2}$

Theorem 7.1. Let G be the graph of a $S(K_p, m)$. Then

i).$M_2(G) = m^{(m^2-m)}(m+1)^{m^{(t+1)}-m^2}$.

ii). $M^*{}_1(G) = (2m+1)^{(m^2-m)}(2m+2)^{\frac{(m^{t+1}-2m^2+m)}{2}}$.

iii). $H_1(G) = (2m+1)^{2(m^2-m)}(2m+2)^{(m^{t+1}-2m^2+m)}$.

iv). $H_2(G) = (m^2+m)^{2(m^2-m)}((m+1)^2)^{(m^{t+1}-2m^2+m)}$.

v). $M^{(a)}{}_1(G) = (2m+1)^{a(m^2-m)}(2m+2)^{a\left(\frac{m^{t+1}-2m^2+m}{2}\right)}$.

vi). $M^{(a)}{}_2(G) = (m^2+m)^{a(m^2-m)}((m+1)^2)^{a\left(\frac{m^{t+1}-2m^2+m}{2}\right)}$.

Proof:

Let G be a graph. Using table 6

i). $M_2(G) = \prod_{uv\in V(G)} d(u)d(v)$

$M_2(G) = (m^2+m)^{(m^2-m)}(m+1)^{(m^{t+1}-2m^2+m)}$.

$M_2(G) = m^{(m^2-m)}(m+1)^{m^{(t+1)}-m^2}$

ii). $M^*{}_1(G) = \prod_{uv\in E(G)}(d(u)+d(v))$

$M^*{}_1(G) = (2m+1)^{(m^2-m)}(2m+2)^{\frac{(m^{t+1}-2m^2+m)}{2}}$.

iii).$H_1(G) = \prod_{uv\in E(G)}\left(d(u)+d(v)\right)^2$

$H_1(G) = (2m+1)^{2(m^2-m)}(2m+2)^{(m^{t+1}-2m^2+m)}$.

iv). $H_2(G) = \prod_{uv\in E(G)}\left(d(u)d(v)\right)^2$

$H_2(G) = (m^2+m)^{2(m^2-m)}((m+1)^2)^{(m^{t+1}-2m^2+m)}$.

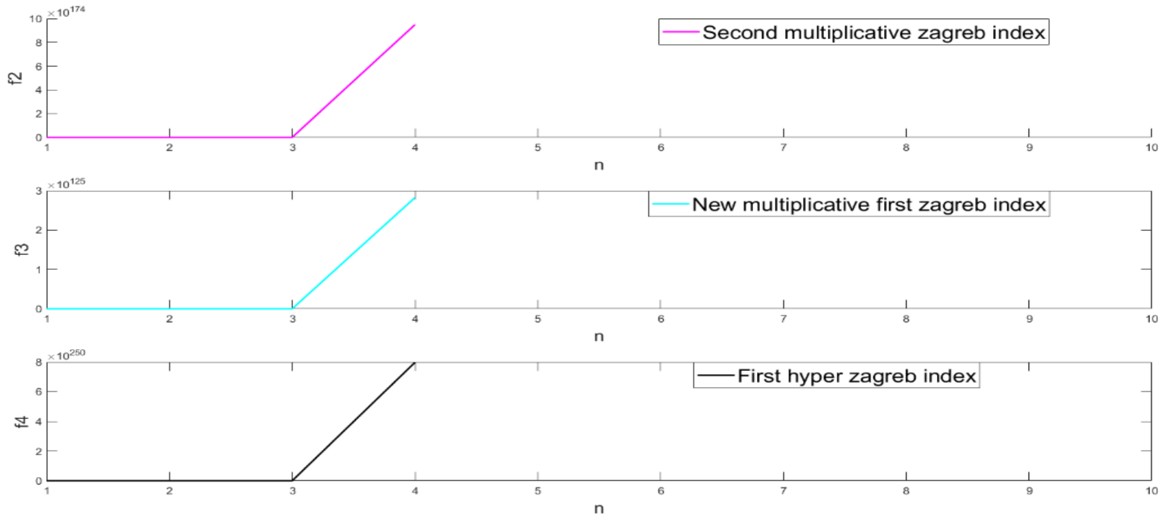
v). $M^{(a)}{}_1(G) = \prod_{uv\in E(G)}\left(d(u)+d(v)\right)^a$.

$M^{(a)}{}_1(G) = (2m+1)^{a(m^2-m)}(2m+2)^{a\left(\frac{m^{t+1}-2m^2+m}{2}\right)}$.

vi). $M^{(a)}{}_2(G) = \prod_{uv\in E(G)}\left(d(u).d(v)\right)^a$.

$M^{(a)}{}_2(G) = (m^2+m)^{a(m^2-m)}((m+1)^2)^{a\left(\frac{m^{t+1}-2m^2+m}{2}\right)}$.

We plot the graph of $S(K_p, m)$



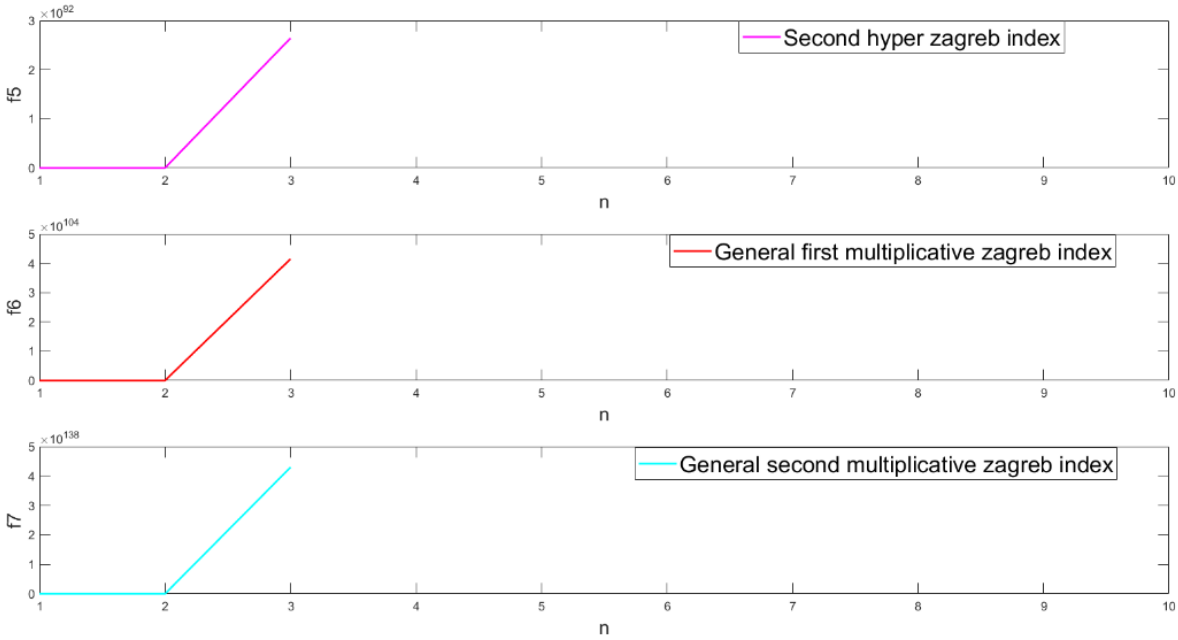


Fig. 17 Plots of $S(K_p, m)$ where $f2(n) = M_2(G), f3(n) = M_1^*(G), f4(n) = H_1(G), f5(n) = H_2(G), f6(n) = M^{(a)}{}_1(G)$ and $f7(n) = M^{(a)}{}_2(G)$.

8. Conclusion

In this paper we have obtainedthe first and the second Zagreb index, generalized multiplicative indices for chemical networks, such as n-dimensional silicatenetworks $SL(n)$, chain silicate networks (C_n), hexagonalnetworks (HX_n), oxide networks(O_n),cellular networks $HC(n)$, and Sierpinski networks $S(K_p, m)$. By using some existing computations of calculus, we retrace few degree-based topological indices from these topological indices. Topology of these networks can be reserved and studied by our obtained results. The graphical representation of six theorems shows the effects of the results obtained on the considered parameters.

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