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Original Article

Performance Assessment of Complex Repairable System Comprising Two Subsystems in a Series Configuration using Copula Repair Strategy

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Abstract *- This paper discusses the reliability analysis of a complex system comprising two subsystems in series configuration together with the switching and human failure. The two subsystems, consist of three identical units in a parallel arrangement and functioning under 1-out-of-3: G operational policy. Switch work is to transfer a load of the failed unit to a new operative unit in both subsystems. The system may have an unforeseeable human failure due to which the system may cause a complete failed state. The failure rate of the units is constant and follows the exponential distribution. The two kinds of repair namely general repair and Gumbel-Hougaard copula repair are used to restore the present failed units and entire subsystem. Using Stochastic theory, differential equations, and supplementary variable approach essential features of reliability such as availability of the system, reliability of the system, MTTF, and profit analysis. It brings a different aspect to the research world to adopt multi-dimensional repair in the form of the copula. In addition, the findings of the model are useful for system engineers and maintenance managers.*

Keywords - *k-out-of-n: G/F type of redundant system, Availability, Reliability, MTTF, Switching failure, Human failure, Gumbel-Hougaard family copula distribution.*

Mathematical Subject Classification: *60 K10, 62N05.*

1. Introduction

The design of complex engineering systems, particularly in the manufacturing industry, lacks the research community for the prospect of developing new models and designing uninterruptible systems capable of attaining high-level standards of availability and reliability. Any improvement in system reliability is often followed by the imposed cost amount; the improvement in trustworthiness is defensible to the degree that the cost of system non-approachability is greater than that of the standard service rendered. In maintaining integrity and customer loyalty, reliability controls for the program play a crucial role. Redundancy is a strategy that is commonly used to boost measures of device stability and benefit sustained. It is used in the form of connected similar components in such a way that, if one element fails, the others can keep the device operational. There are three types of standby in general: Cold standby in which the standby unit is only used when the main or working unit fails. Here the inactive components have a zero-failure rate and cannot fail when standby; (ii) hot standby in which the standby device has the same fault rate as when running with the operating unit; (iii) hot standby in which the standby unit operates in the operating unit's background; It is an intermediate case and, in this situation, the unit may fail, but its failure rate is lower than that of the working group. In addition, redundancy is particularly beneficial in maintaining a certain degree of system reliability. Hence reliability and adequate performance in system configuration type k-out-of-n with at least k components out of n operating for the system to be in operational mode play an important role. To explore some relevant literature on such a designed structure, a four-transmitter telecommunications system can be modeled as a 2-out-of-4: a G system, a wide six-tire bus with four tires is a (4-out-of-6: G) system. In system reliability theory, a conclusively k-out-of-n system plays a very crucial role in proper system operation. The warm standby system model k-out-of-n has found numerous applications in the fields of reliability including reduction system monitoring, network design, power generation, transmission system, etc.

Authors Kullstam (1981) have made comprehensive efforts over the past decades to formulate and solve the reliability characteristics of k-out-of-n systems, such as availability, MTBF, and MTTR for a repairable system. Together with Coit (2001), Park and Pham (2012), Xing et al. (2012), and Ram et al. (2013) researched the performance of complex system repairable systems employing k-out-of-n G/ F, operational schemes. Malinowski (2016) analyzed the reliability of a flow network with a series-parallel-reducible structure. Levitin et al. (2013) evacuated the reliability of mixed configured series-parallel systems with random failure propagation time. The exact reliability formula for consecutive repairable k-out-of-n-:G type operative systems was demonstrated by Liang et al, (2010). Sharma and Kumar (2017) computed availability and other reliability measures of the successive k-out-of- n machining system using standby with multiple working vacations.

Eryilmaz, S, (2010), have developed formulas for consecutive k-out-of-n: F system using lifetime distribution, reliability, and properties of the k-out-of-n system with arbitrarily dependent components and mixture representations for the reliability of consecutive- k systems. Singh and Poonia (2019) have premeditated two-unit systems under correlated lifetimes under inspection employing the regenerative point technique. Levitin and Dai (2012) considered a generalized linear multi-state sliding window system proposed in case of multiple failures. The structure consists of independently linearly ordered multistate elements in this model. Rawal et al. (2013) analyzed a model of the internet data center (IDC) with the redundant server with the main mail server trickling different types of failure and two types of repairs employing copula distribution. Confirming the various operational possibilities in the network, some crucial research was performed to determine the network's different reliability features.

Many researchers around the globe use the switching device to present their work on the functionality of complex repairable systems. Authors such as Singh et al. (2013) have studied cost analysis of an engineering system involving two subsystems in a series configuration with controllers and human failure under the concept of k-out-of-n: G policy. It is worth noting that if the device is in service and operating under minor or major partial failure mode, we can employ general repair. Since the system stops working due to complete shutdown mode and therefore it must be repaired quickly for this purpose the copula repair particularly [Gumbel-Hougaard family copula] distribution must be deployed to restore the failed system see R B Nelson (2006). To cite some related work presented by some authors Singh et al. (2013), Gulati et al. (2016), Ibrahim et al. (2017), Jia et al. (2017), and Kumar et al. (2017), studied the reliability measures of systems comprising subsystems in series configurations and k-out-of-n: G/ F policy with implications of a joint probability distribution. Recently Singh et al. (2020) examined a complex system with two subsystems in a series configuration with an imperfect switching device with implications of the copula linguistic approach and have concluded that copula repair predicts better performance over general repair.

2. Model description and notations

2.1. System description

Conferring to the listed examined literature in the introduction, the scheme consisting of the k-out-of-n: G. was not evaluated by anyone among the authors. We examined the performance of a repairable hot standby system with two subsystems (namely subsystem-1 and 2) in a series configuration to bridge this gap. Each subsystem has three parallel units operating under 1-out-of-3: good policy in a parallel configuration. For the proper functioning of the system, the units of both subsystems connected to the switch can be unstable at the time of need, and the switching time is immediate. If an operating unit fails, it is replaced immediately by a standby unit using the available switch. In addition, during service, the device could face unexpected human failure due to the mishandling of the subsystem. For the operation of the system, there are four types of potential states: perfect operation, minor partial failure, major partial failure, and maximum failure. It is assumed that failure rates in both operating and standby units have exponential distributions. Using the supplementary variables and consequences of Laplace transformation, system reliability is evaluated to test different characteristics such as transition state probabilities, system availability, system reliability, MTTF, and benefits analysis. This paper's system is structured as follows.

We checked the relevant work presented in different papers in Section 1. The summary of the system description along with assumptions and notations is described in section 2; the description of the state is given in section 3, while the system configuration and transition diagram are provided in section 4. Mathematical modeling using differential equations is discussed in section 5. The system performance analysis results such as reliability, availability, MTTF, and expected profit margin were simulated by considering some specific cases presented in Section 6. We have explained a summary of our results in Section 7. With the support of MAPLE (software), explicit expressions for reliability characteristics are obtained. The state definition of the system under investigation is provided in Table 1, and the transition state diagram is given in Figure 1.

2.2. Assumptions

The following assumptions are made in this paper:

1. The subsystem-1 / subsystem-2 works successfully until one or more than one unit is in good working condition, i.e., 1-out-of-3:G policy.
2. Both the subsystems have a switching device, which in the system is unreliable and the function of the switch is to change failed units to operative units. Switch failure may treat as a complete failed state.
3. There may be an unpredictable human failure to the system at any time (t).
4. The system has four states: Good, minor partially failed, major partially failed, and utterly failed.
5. The units in both the subsystems are in active mode as a hot standby and ready to start within a slight time after the failure of any unit in the subsystems.
6. The repairman is available full-time and ready to restore minor and major faults.
7. All failure rates are constant and follow the exponential distribution.

8. In a complete failed situation system restore using copula distribution.
9. The repaired unit is trickled as a new and it is ready to perform the task as required.

2.3. Notations

s , t	Laplace transform / Time scale variable
λ_1/μ_1	The failure rate of each unit in subsystem-1/subsystem-2.
$\lambda_{s_1}/\lambda_{s_2}$	Failure rate of switch for the subsystem-1/subsystem-2.
λ_h	Failure rate related to the catastrophic failure mode.
$\varphi_1(x)/\psi_1(y)$	Repair rate of one unit in subsystem-1/subsystem-2.
$\varphi_2(x)/\psi_2(y)$	Repair rate of two units in subsystem-1/subsystem-2.
$P(t)/\ \ \bar{P}(s)$	State transition / Laplace transform of state transition probability.
$P_i(x,t)$	The probability that the system is in the state S_i for $i=1$ to 8 and the system is under repair with elapsed repair time is x, t x repaired variable and t is time variable.
$E_p(t)$	Expected profit in the interval [0, t)
$K_1/\ K_2$	Revenue generation/ service cost per unit time, respectively.
$\mu_0(x)$	An expression of the joint probability from failed state S_i to good state S_0 according to the Gumbel-Hougaard family copula is given as $\mu_0(x) = C_\theta\{u_1(x), u_2(x)\} = exp[x^\theta + \{log\ \varphi\ (x)\}^\theta]^{\frac{1}{\theta}}$ where $u_1(x) = \varphi(x)$ and $u_2(x) = e^x$ here θ is the parameter $1 < \theta < \infty$.

3. System configuration and state transition diagram

The state transition diagram in Fig 1. In the transition diagram, S_0 is the perfect state, S_1 and S_4 are minor partially failed, S_2 and S_5 are major partially failed, and S_3, S_6, S_7, and $S_8$are failed states. Due to the failure of a maximum of one unit from subsystem-1 or 2, the transitions approach minors partially failed states S_1 and S_4, and if two units failed in subsystem-1 or 2, the transitions approach to major partially failed states S_2 and S_5. The state S_3 is a complete failed state due to the failure of any three units in either of the subsystems. The states S_6, S_7, and S_8 are completely failed states due to switching and human failure.

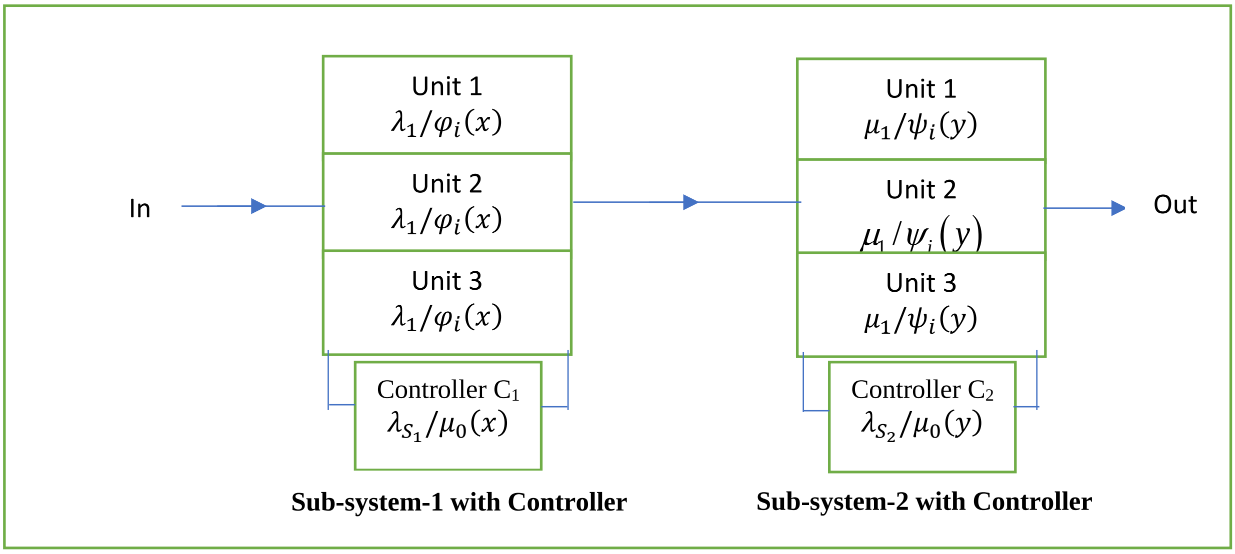

Fig. 1(a) System structure

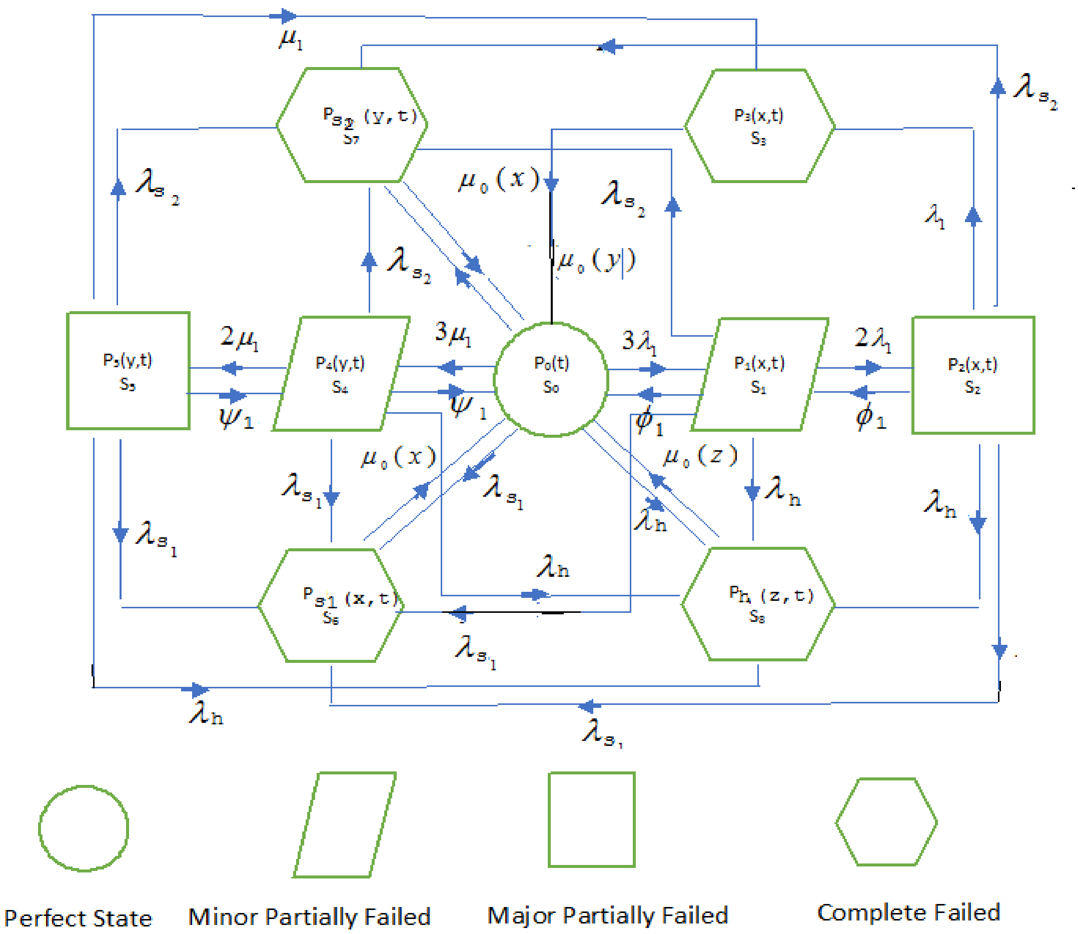

Fig. 1(b). State transition diagram of the model

4. State description

The state explanation of the model is that S_0 is a state where both the subsystems are in good working condition. S_1 and S_4 are the states where the system is in minor partially failure mode, while S_2 and S_5 are indicating that the system is in major partially failure mode, and the repair is employed, states S_3, S_6, S_7 and S_8 are the total failure mode. Repair is being employed using the Gumbel-Hougaard (GH)family copula.

Table 1. State Description

State	Description
S_0	This is a perfect state, in which units of both subsystems are in good working condition.
S_1	The indicated state is an operative state with minor degraded mode after the failure of any one unit in subsystem 1.
S_2	The indicated state is a degraded and operational state after the failure of any two units in subsystem-1, but both units of subsystem-2 are in a good functional state. The system is under repair.
S_4	This indicated a degraded and functioning state after the failure of any one unit in subsystem-2, but all the units of subsystem-1 are in a good operational state. The system is under repair.
S_5	The indicated state is degraded but is in operative nature due to the failure of any two units in subsystem-2, but all the units of subsystem-1 are in a good operational state. The system is under repair.
S_3 S_6 S_7 S_8	All these states represent a complete failure state when the system is in shut down mode and the system is under repair using the Gumbel-Hougaard family copula distribution.

5. Formulation of the mathematical model

By a probability of considerations and permanency stochastic theory arguments, one can obtain the undermentioned set of differential equations allied with the present mathematical model.

$$\left[\frac{\partial}{\partial t} + 3\lambda_1 + 3\mu_1 + \lambda_{s_1} + \lambda_{s_2} + \lambda_h\right] P_0(t) = \left[\int_0^\infty \varphi_1(x)\, P_1(x,t)dx + \int_0^\infty \psi_1(y)\, P_4(y,t)dy + \int_0^\infty \mu_0(x)\, P_3(x,t)dx + \int_0^\infty \mu_0(x)\, P_{s_1}(x,t)dx + \int_0^\infty \mu_0(y)\, P_{s_2}(y,t)dy + \int_0^\infty \mu_0(z)\, P_h(z,t)dz\right] \quad (1)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2} + \lambda_h + \varphi_1(x)\right] P_1(x,t) = 0 \quad (2)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_1 + \lambda_{s_1} + \lambda_{s_2} + \lambda_h + \varphi_2(x)\right] P_2(x,t) = 0 \quad (3)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right] P_3(x,t) = 0 \quad (4)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + 2\mu_1 + \lambda_{s_1} + \lambda_{s_2} + \lambda_h + \psi_1(y)\right] P_4(y,t) = 0 \quad (5)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_1 + \lambda_{s_1} + \lambda_{s_2} + \lambda_h + \psi_2(y)\right] P_5(y,t) = 0 \quad (6)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right] P_{s_1}(x,t) = 0 \quad (7)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_0(y)\right] P_{s_2}(y,t) = 0 \quad (8)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu_0(z)\right] P_h(z,t) = 0 \quad (9)$$

Boundary conditions

$$P_1(0,t) = 3\lambda_1 P_0(t) \quad (10)$$

$$P_2(0,t) = 2\lambda_1 P_1(0,t) = 6\lambda_1^2 P_0(t) \quad (11)$$

$$P_4(0,t) = 3\mu_1 P_0(t) \quad (12)$$

$$P_5(0,t) = 2\mu_1 P_4(0,t) = 6\mu_1^2 P_0(t) \quad (13)$$

$$P_3(0,t) = \lambda_1 P_2(0,t) + \mu_1 P_5(0,t) = 6(\lambda_1^3 + \mu_1^3) P_0(t) \quad (14)$$

$$P_{s_1}(0,t) = \lambda_{s_1}[P_0(t) + P_1(0,t) + P_2(0,t) + P_4(0,t) + P_5(0,t)] \quad (15)$$

$$P_{s_2}(0,t) = \lambda_{s_2}[P_0(t) + P_1(0,t) + P_2(0,t) + P_4(0,t) + P_5(0,t)] \quad (16)$$

$$P_h(0,t) = \lambda_h[P_0(t) + P_1(0,t) + P_2(0,t) + P_4(0,t) + P_5(0,t)] \quad (17)$$

Initials conditions

$P_0(0) = 1$, and $P_i(x,0) = o, i = o, 1,2,3,4$ *and also* $P_{s_1}(x,0) = 0, P_{s_2}(x,0) = 0$ (18)

Laplace transformation of equations (1) to (17) and using equation (18), one may obtain $\left[s + 3\lambda_1 + 3\mu_1 + \lambda_{s_1} + \lambda_{s_2} + \lambda_h\right]\bar{P}_0(s) = 1 + \int_0^\infty \varphi_1(x)\bar{P}_1(x,s)dx + \int_0^\infty \psi_1(y)\,\bar{P}_4(y,s)dy + \int_0^\infty \mu_0(x)\,\bar{P}_3(x,s)dx + \int_0^\infty \mu_0(x)\,\bar{P}_{s_1}(x,s)dx + \int_0^\infty \mu_0(y)\,\bar{P}_{s_2}(y,s)dy + \int_0^\infty \mu_0(z)\,\bar{P}_h(z,s)dz]$ (19)

$$\left[s + \frac{\partial}{\partial x} + 2\lambda_1 + \lambda_{s_1} + \lambda_{s_2} + \lambda_h + \varphi_1(x)\right] \bar{P}_1(x,s) = 0 \quad (20)$$

$$\left[s + \frac{\partial}{\partial x} + \lambda_1 + \lambda_{s_1} + \lambda_{s_2} + \lambda_h + \varphi_2(x)\right] \bar{P}_2(x,s) = 0 \quad (21)$$

$$\left[s + \frac{\partial}{\partial x} + \mu_0(x)\right] \bar{P}_3(x,s) = 0 \quad (22)$$

$$\left[s + \frac{\partial}{\partial y} + 2\mu_1 + \lambda_{s_1} + \lambda_{s_2} + \lambda_h + \psi_1(y)\right] \bar{P}_4(y,s) = 0 \quad (23)$$

$$\left[s+\frac{\partial}{\partial y}+\mu_1+\lambda_{s_1}+\lambda_{s_2}+\lambda_h+\psi_2(y)\right]\bar{P}_5(y,s)=0 \quad (24)$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\bar{P}_{s_1}(x,s)=0 \quad (25)$$

$$\left[s+\frac{\partial}{\partial y}+\mu_0(y)\right]\bar{P}_{s_2}(y,s)=0 \quad (26)$$

$$\left[s+\frac{\partial}{\partial z}+\mu_0(z)\right]\bar{P}_h(z,s)=0 \quad (27)$$

$$\bar{P}_1(0,s)=3\lambda_1\bar{P}_0(s) \quad (28)$$

$$\bar{P}_2(0,s)=6\lambda_1^2\bar{P}_0(s) \quad (29)$$

$$\bar{P}_4(0,s)=3\mu_1\bar{P}_0(s) \quad (30)$$

$$\bar{P}_5(0,s)=6\mu_1^2\bar{P}_0(s) \quad (31)$$

$$\bar{P}_3(0,s)=\lambda_1\bar{P}_2(0,s)+\mu_1\bar{P}_5(0,s)=6(\lambda_1^3+\mu_1^3)\bar{P}_0(s) \quad (32)$$

$$\bar{P}_{s_1}(0,s)=\lambda_{s_1}[1+3(\lambda_1+\mu_1)+6(\lambda_1^2+\mu_1^2)]\bar{P}_0(s) \quad (33)$$

$$\bar{P}_{s_2}(0,s)=\lambda_{s_2}[1+3(\lambda_1+\mu_1)+6(\lambda_1^2+\mu_1^2)]\bar{P}_0(s) \quad (34)$$

$$\bar{P}_h(0,s)=\lambda_h[1+3(\lambda_1+\mu_1)+6(\lambda_1^2+\mu_1^2)]\bar{P}_0(s) \quad (35)$$

Now solving the equations (19) (27) with the boundary conditions, (28)- (35) one may get

$$\bar{P}_0(s)=\frac{1}{D(s)} \quad (36)$$

$$\bar{P}_1(s)=\frac{3\lambda_1}{D(s)}\frac{1}{\left(s+2\lambda_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\emptyset_1\right)} \quad (37)$$

$$\bar{P}_2(s)=\frac{6\lambda_1^2}{D(s)}\frac{1}{\left(s+\lambda_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\emptyset_2\right)} \quad (38)$$

$$\bar{P}_3(s)=\frac{6\left(\lambda_1^3+\mu_1^3\right)}{D(s)}\frac{1}{s+\mu_0} \quad (39)$$

$$\bar{P}_4(s)=\frac{3\mu_1}{D(s)}\frac{1}{\left(s+2\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\varphi_1\right)} \quad (40)$$

$$\bar{P}_5(s)=\frac{6\mu_1^2}{D(s)}\frac{1}{\left(s+\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\varphi_2\right)} \quad (41)$$

$$\bar{P}_{s_1}(s)=\frac{\lambda_{s_1}}{D(s)}[1+3\lambda_1+3\mu_1+6(\lambda_1^2+\mu_1^2)]\frac{1}{s+\mu_0} \quad (42)$$

$$\bar{P}_{s_2}(s)=\frac{\lambda_{s_2}}{D(s)}[1+3\lambda_1+3\mu_1+6(\lambda_1^2+\mu_1^2)]\frac{1}{s+\mu_0} \quad (43)$$

$$\bar{P}_h(s)=\frac{\lambda_h}{D(s)}[1+3\lambda_1+3\mu_1+6(\lambda_1^2+\mu_1^2)]\frac{1}{s+\mu_0} \quad (44)$$

Where:

$$D(s)=s+3\lambda_1+3\mu_1+\lambda_{s_1}+\lambda_{s_2}+\lambda_h-3\lambda_1\bar{S}_{\varphi_1}\left(s+2\lambda_1+\lambda_{s_1}+\lambda_{s_2}+\lambda_h\right)-3\mu_1\bar{S}_{\psi_1}\left(s+2\mu_1+\lambda_{s_1}+\lambda_{s_2}+\lambda_h\right)-6(\lambda_1^3+\mu_1^3)\bar{S}_{\mu_0}(s)-\left(\lambda_{s_1}+\lambda_{s_2}+\lambda_h\right)1+3\lambda_1+3\mu_1+6(\lambda_1^2+\mu_1^2)\bar{S}_{\mu_0}(s)$$

The Sum of Laplace transformations of the state transitions, for operative states and failed states at any time, is given as.

$$\bar{P}_{up}(s)=\bar{P}_0(s)+\bar{P}_1(s)+\bar{P}_2(s)+\bar{P}_4(s)+\bar{P}_5(s) \quad (45)$$

$$\bar{P}_{down}(s)=1-\bar{P}_{up}(s) \quad (46)$$

6. Analytical Study

6.1. System Availability Analysis

1. Repair follows two types of distributions general and (GH) family copula distribution, we have

Setting $\bar{S}_{\varphi_1}(s) = \frac{\varphi_1}{s+\varphi_1}, \bar{S}_{\psi_1}(s) = \frac{\psi_1}{s+\psi_1}, \bar{S}_{\mu_0}(s) = \frac{exp[x^\theta + \{log\,\varphi(x)\}^\theta]^{\frac{1}{\theta}}}{s + exp[x^\theta + \{log\,\varphi(x)\}^\theta]^{\frac{1}{\theta}}}$

Assigning the specific values $\lambda_1 = 0.02, \mu_1 = 0.03, \lambda_{s_1} = 0.021, \lambda_{s_2} = 0.022, \lambda_h = 0.025, \theta = 1, x = 1, \varphi_1 = 1, \psi_1 = 1$ in (45), computing inverse Laplace transform, with Maple 17 software one can obtain the following availability expression of the system. Here we have considered the following cases:

(a) When both the subsystems have switching device, we obtain,

$$P_{up}(t) = 0.030148e^{-2.8040t} + 0.024319e^{-1.2900t} - 0.003139e^{-1.1309t} - 0.011268e^{-1.0955t} - 0.021207e^{-1.0481t} - 0.029779e^{-1.0383t} + 1.007386e^{-0.0093t} - 0.001840e^{-1.0880t} + 0.005382e^{-1.0980t} \quad (47a)$$

(b) When subsystem-2 does not have a switching device, i.e.$\lambda_{s_2} = 0$, we obtain,

$$P_{up}(t) = -0.001895e^{-1.0660t} + 0.005182e^{-1.0760t} + 0.020739e^{-2.7765t} + 0.027661e^{-1.2728t} - 0.003152e^{-1.1089t} - 0.011093e^{-1.0734t} - 0.021381e^{-1.0261t} - 0.030907e^{-1.0162t} + 1.014845e^{-0.0104t} \quad (47b)$$

(c) When both subsystems 1 and 2 do not have a switching device, i.e.$\lambda_{s_1} = \lambda_{s_2} = 0$, we obtain,

$$P_{up}(t) = -0.001948e^{-1.0450t} + 0.005009e^{-1.0550t} + 0.011482e^{-2.7501t} + 0.031176e^{-1.2564t} - 0.003163e^{-1.0879t} - 0.010942e^{-1.0522t} - 0.021381e^{-1.0261t} - 0.030907e^{-1.0162t} + 1.014845e^{-0.0104t} \quad (47c)$$

For different values of time-variable $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90$ and 100 units of time, one may get different values $P_{up}(t)$ with the help of (47a-47c), as presented in table-2 and the corresponding figure-2.

Table 2. Availability variation with respect to time

Time (t)	a	b	c
0	1.0000	1.0000	1.0000
10	0.9173	0.9141	0.9116
20	0.8354	0.8234	0.8132
30	0.7607	0.7417	0.7254
40	0.6928	0.6681	0.6471
50	0.6309	0.6019	0.5772
60	0.5745	0.5421	0.5149
70	0.5232	0.4883	0.4593
80	0.4764	0.4399	0.4097
90	0.4339	0.3962	0.3655
100	0.3951	0.3569	0.3261

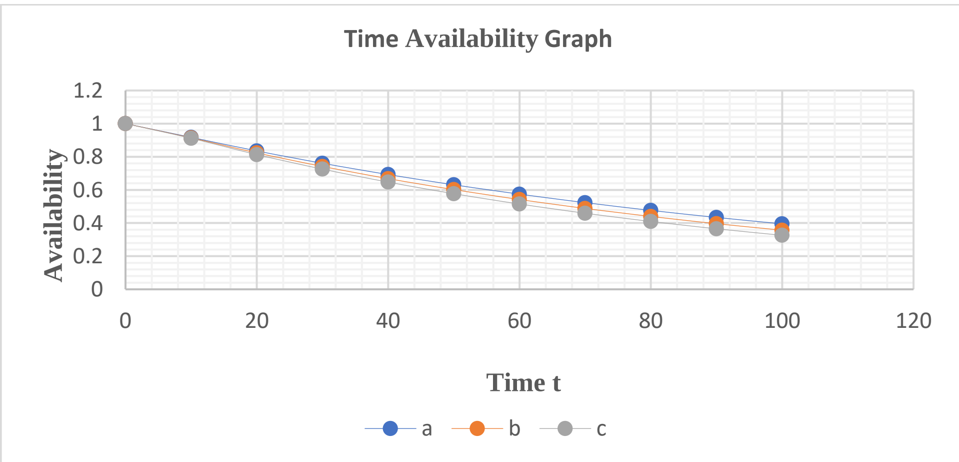

Fig. 2 Availability as a function of time

6.2 System Reliability analysis

Reliability is the probabilistic measure of a non-repairable system. Therefore, treating all repair rates equal to zero and obtaining inverse Laplace transform in (45), we get an expression for the reliability of the system after taking the failure rates as $\lambda_1 = 0.02, \mu_1 = 0.03, \lambda_{c_1} = 0.021, \lambda_{c_1} = 0.022,\ \lambda_{c_T} = 0.025$ considered the same cases like availability, we have

(a) When both the subsystems have switching device, we obtain,

$$R_i(t) = 0.049568e^{-0.0880t} + 12.572026e^{-0.1280t} + 0.141705e^{-1.1309t} + 2.15877e^{-0.1080t} - 4.3843410^{-41}e^{-1.468t}(3.1754\ 10^{41}\ cosh(1.3332t) + 3.1887\ 10^{41}\ cosh(1.3332t)) \quad (48a)$$

(b) When subsystem-2 does not have switching device i.e. $\lambda_{s_2} = 0$, we obtain,

$$R_i(t) = 1.095203e^{-0.0860t} + 0.031914e^{-0.0660t} + 2.619204e^{-0.1060t} + \ 0.083085e^{-0.0760t} - 5.649054\ 10^{-37}e^{-1.4571t}(5.0086\ 10^{36}\ cosh(1.3175t) + 5.0799\ 10^{36}\ cosh(1.3175t))\ \ (48b)$$

(c) When both subsystems 1 and 2 do not have a switching device, i.e.$\lambda_{s_1} = \lambda_{s_2} = 0$, we obtain.

$$R_i(t) = 0.748776e^{-0.0650t} + 0.023915e^{-0.0450t} + 1.502406e^{-0.0850t} + 0.059838e^{-0.0550t} - 2.780612\ 10^{-35}e^{-1.4466t}(4.8008\ 10^{34}\ cosh(1.3024t) +\ 4.8808\ 10^{34}\ cosh(1.3024t)) \qquad (48c)$$

For different values of time-variable $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90$ and 100 units of time, one may get different values of reliability $R(t)$ with the help of (48a-48c), as shown in table-3 and the corresponding figure-3.

Table 3. Computed values of reliability corresponding to the different cases

Time (t)	a	b	c
0	1.0000	1.0000	1.0000
10	0.6832	0.7209	0.7645
20	0.3105	0.3626	0.4329
30	0.1222	0.1616	0.2237
40	0.0449	0.0684	0.1121
50	0.0159	0.0283	0.0558
60	0.0055	0.0116	0.0278
70	0.0018	0.0047	0.0140
80	0.0006	0.0019	0.0071
90	0.0002	0.0008	0.0037
100	0.0001	0.0003	0.0019

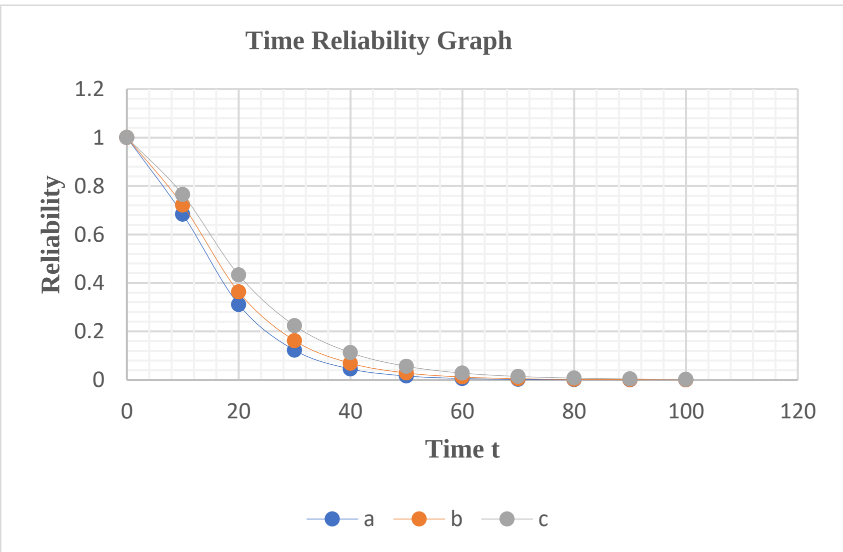

Fig. 3 Reliability as a function of time

6.3 Mean Time to Failure (MTTF)

Taking all repair rates to zero and the limit as s tends to zero in (50) for the exponential distribution; we can obtain the MTTF as:

$$MTTF = \frac{1}{A}\left[1 + \frac{3\lambda_1}{2\lambda_1+\lambda_{s_1}+\lambda_{s_2}+\lambda_h} + \frac{6\lambda_1^2}{\lambda_1+\lambda_{s_1}+\lambda_{s_2}+\lambda_h} + \frac{3\mu_1}{2\mu_1+\lambda_{s_1}+\lambda_{s_2}+\lambda_h} + \frac{6\mu_1^2}{\mu_1+\lambda_{s_1}+\lambda_{s_2}+\lambda_h}\right] \quad (50)$$

where $A = 3\lambda_1 + 3\mu_1 + \lambda_{s_1} + \lambda_{s_2} + \lambda_h$

Now taking the values of different parameters as $\lambda_1 = 0.02, \mu_1 = 0.03, \lambda_{s_1} = 0.021, \lambda_{s_2} = 0.022$, and $\lambda_h = 0.025$ and varying$\lambda_1, \mu_1, \lambda_{c_1}, \lambda_{c_2}$, and λ_{c_T} one by one respectively as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 in (50), the variation of MTTF, for failure rates, can be obtained as given table 3 and figure 3.

Table 4. Computation of MTTF corresponding to the failure rates

Failure rates	λ_1	μ_1	λ_{s_1}	λ_{s_1}	λ_h
0.01	11.2065	12.2242	11.9856	12.1127	12.5101
0.02	10.7387	11.5194	10.8417	10.9466	11.2732
0.03	10.1469	10.7387	9.8897	9.9776	10.2506
0.04	9.5608	10.0101	9.0855	9.1602	9.3916
0.05	9.0201	9.3626	8.3997	8.4619	8.6605
0.06	8.5337	8.7955	7.8031	7.8589	8.0309
0.07	8.1002	8.3002	7.2842	7.3331	7.4836
0.08	7.7143	7.8666	6.8277	6.8709	7.0035
0.09	7.3704	7.4852	6.4231	6.4615	6.5793
0.10	7.0628	7.1480	6.0623	6.0966	6.2018

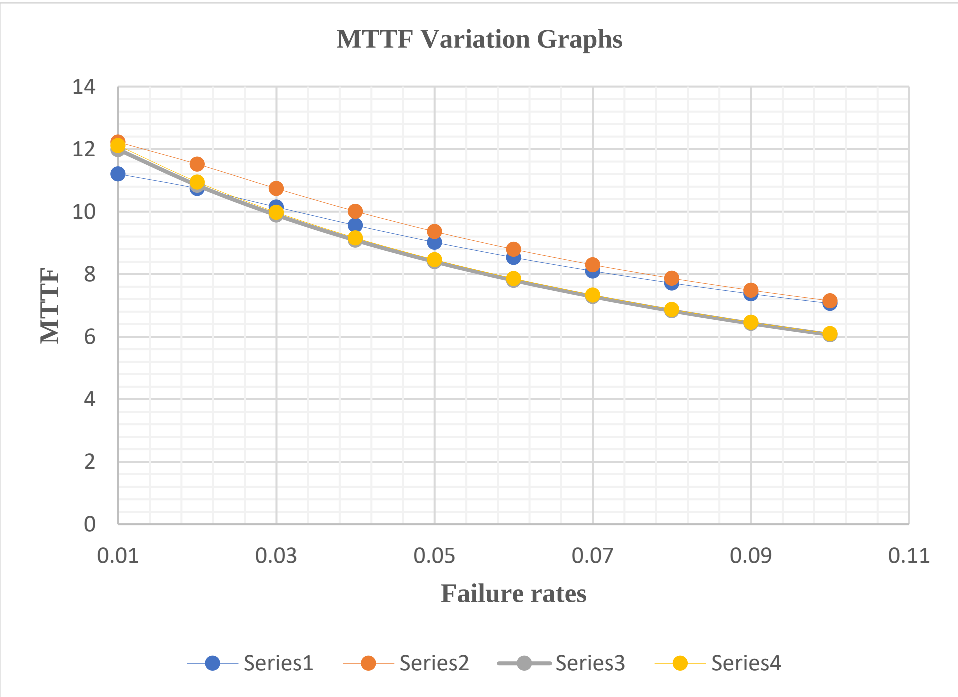

Fig. 4 MTTF as a function of failure rates

6.4 Cost Analysis

Let the service facility be always available, then the expected profit during the interval $[0, t)$ is

$$E_p(t) = K_1 \int_0^t P_{up}(t)\, dt - K_2 t \quad (56)$$

For the same set of parameters defined in (47), one can obtain (57).

Therefore

$E_p(t) = -0.010751e^{-2.8040t} - 0.018850e^{-1.2900t} + 0.020233e^{-1.0481t} - 107.640872e^{-0.0093t} + 0.002776e^{-1.130912t} + 0.010286e^{-1.0954t} + 0.028680e^{-1.0383t} - 0.004901e^{-1.0980t} + 0.001691e^{-1.0880t} + 107.611709 - K_2 t$ (57)

Setting $K_1 = 1$ and $K_2 = 0.6, 0.4, 0.2$ *and* 0.1 respectively, and varying $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90$ and 100 units of time, the results for expected profit can be obtained as per table-5 and figure-5.

Table 5. Profit computation for different values of time

Time t	K_2=0.6	K_2=0.4	K_2=0.2	K_2=0.1
0	0	0	0	0
10	3.5876	5.5876	7.5876	8.5876
20	6.3453	10.3453	14.3453	16.3453
30	8.3205	14.3205	20.3205	23.3205
40	9.5832	17.5832	25.5832	29.5832
50	10.1971	20.1971	30.1971	35.1971
60	10.2200	22.2200	34.2200	40.2200
70	9.7048	23.7048	37.7048	44.7048
80	8.6996	24.6996	40.6996	48.6996
90	7.2482	25.2482	43.2482	52.2482
100	5.3904	25.3904	45.3904	55.3904

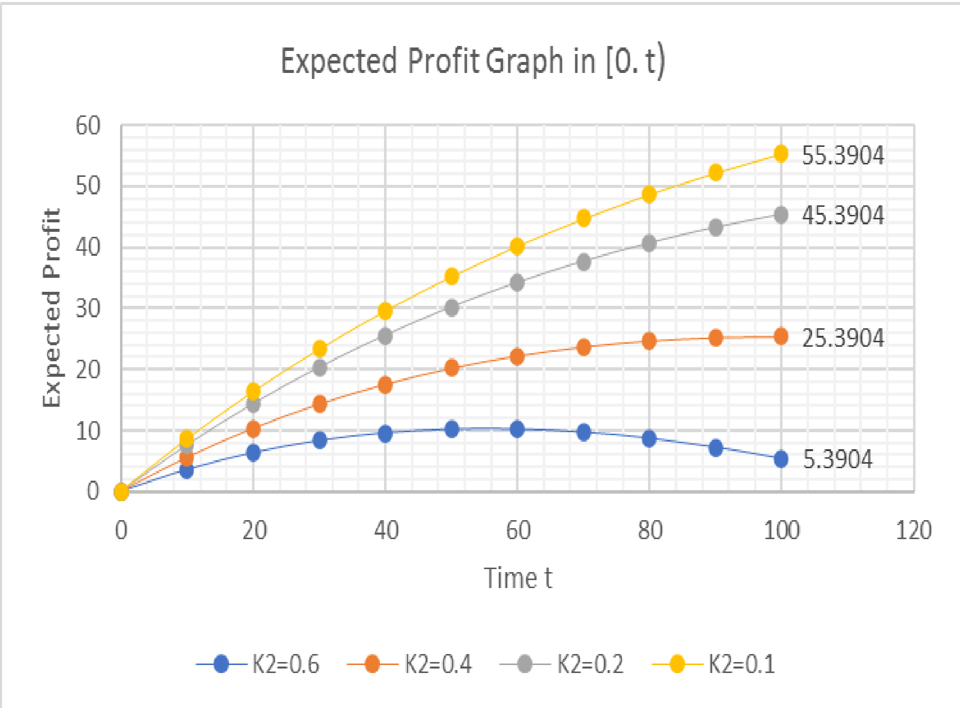

Fig. 5 Expected profit as a function of time

7. Conclusion via result analysis

This paper studies the probabilistic measures of a repairable system consisting of two subsystems in series with switching and human failure. Each subsystem is comprised of three alike units in a parallel configuration and working under 1-out-of-3: G plan. The following decisions may be drawn based on the study led in this paper:

Table-2 and Figure-2 give the analysis of the availability of the system in four different cases (Gumbel- Hougaard Copula approach, Copula approach and without switching in subsystem2, Copula approach and with switching device and same failure rate of both subsystems), when failure rates are fixed at different values to time. One can observe that the availability decreases as time t increases.

Table-3 and figure-3 give evidence for the reliability of the system at different values of the time. The graph was showing a steep fall in reliability from top to lowermost in a very short period in all four cases based on the failure rate of units.

From table-2 and 3, one can observe that corresponding values of availability are higher than the values of reliability, which highlights the requirement of systematic repair for any complex systems for healthier performance.
Table-4 and figure-4 yield the MTTF of the system concerning variation$\lambda_1, \mu_1, \lambda_{s_1}, \lambda_{s_2}$, and λ_h. It can see that the MTTF of the system reduces with the increasing values of all the parameters. MTTF was found to be the highest forμ_1. Thus, MTTF of the system in all possible cases is decreasing as failure rates $\lambda_1, \mu_1, \lambda_{s_1}, \lambda_{s_2}$, and $\lambda_h$increasing from 0.01 to 0.10.

An acute examination from table 5 and figure 5 reveals that expected profit increases as service cost K_2 decreases, while the revenue cost per unit time is fixed at K_1=1. The calculated expected profit is maximum for K_2= 0.1 and minimum for K_2=0.6. We observe that as service cost decreases, profit increase with the variation of time. In general, for low service costs, the expected profit is high in comparison to the high service cost.

In proper maintenance review, decision, and performance assessment, the model built in this paper was found to be highly advantageous. The evaluation of the full reliability and functionality of the device under investigation is another potential future task.

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