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Integrated supply chain model with pricing strategies and controllable processing lead time under VMI-CS policy

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Abstract *- This paper considers a supply chain inventory model with one vendor and multiple buyers under a vendor managed inventory with consignment stock agreement between the vendor and buyer. The supply lead time between the vendor and the buyer is considered as controllable at a cost. The expected linear demand of each buyer is assumed in which the demand depends on the selling price and lead time. A numerical example and sensitivity analysis are used to demonstrate the proposed model with the objective of maximizing total profit.*

Keywords **-** *Supply chain, controllable lead time, vendor managed inventory, consignment stock, price-sensitive demand*.

1. Introduction

Nowadays, different types of business are emerging and as a result, new products are introduced in the market everyday. The main reason for this is growing population and the products they use. Thus, a wide variety of products are emerging based on innovative ideas. In every business, the manufactured product crosses many places, many people and eventually reaches the buyer, which is called a supply chain.

A supply chain is defined by some as the configuration of entities, i.e. suppliers, warehouses, manufacturers, retailers/buyers, etc., that either directly or indirectly fulfil customers' requests by providing them with products or services. Inventory management is a part of supply chain that monitors the movement of goods from the initial production stage to the final stage of reaching their consumers. The key feature of inventory management is to maintain a clear record of each new or leaving a warehouse. In addition, the basic concept of inventory management include purchasing inventory, storing inventory, making a profit form purchasing inventory. Over the years, companies have developed several inventory management programmes such as consignment, vendor managed inventory, and so on. Vendor managed inventory(VMI)
is not a new concept but many retailers have not adopted it as part of their business strategy. However, when properly a salesperson implemented the VMI strategy can bring significant benefits to the business. VMI is where the vendor manages the buyer's inventory and the buyer shares information with a vendor. The vendor maintains an agreed inventory level of a specific product. Similarly, consignment stock(CS) plays a vital role in inventory management to maximize the supply chain's profitability. Moreover, in the CS agreement signed between the vendor and the buyer under which the inventory ownership is in the hand of the vendor until it is sold. Therefore, we considered a supply chain model between the vendor and the buyer following VMI and CS contract policies.

2. Literature review

This section describes the literary contribution of key concepts considered in this model. Supply chain management (SCM) is described as the management of own of products and services, beginning with the origin of the product and ending with the consumption of the product. Amidst today's competitive industry, its strategies are considered the backbone of business enter prizes. SCM is a technique to coordinate all the players in supply chain such as manufacturers, distributors, retailers, and customers, the main vision of the supply chain is to attain the minimum total cost (maximum total profit).

Supply Chain Management (SCM) is an integrated system for managing production, wholesale, retail, and logistics operations. Goyal [11] was the first person to introduce a seller-buyer integrated supply chain model. Lu [18] developed a two-level supply chain model between a single seller and multiple retailers. Lo et al. [17] described the integration of the production inventory model between the vendor and the buyer. This type of supply (SC) model attains great attention among many researchers (see Jha and Sanker [12] and Kumar et al.[16]). Batarfi et al. [2] proposed a two channel supply model with pricing and inventory decisions under the learning and forgetting concept. Chen and Su [8] illustrated a coordinated supply chain model with CS agreement under online and online business practices.

Lead time plays a crucial role in building consumers trust in a company. Yi and Sarker [24] are the first researchers who analyzed the CS policy model under controllable lead time. Jha and Sanker [13] developed the production inventory model by considering the crashing cost for multi-buyer. Mandal and Giri [19] have examined an integrated supply chain model between a single-vendor multi-buyer with varying lead time and quality improvement. Sarker et al. [20] developed a model between single-vendor multi-buyer with varying production rate and controllable lead time and the deteriorating products-inventory model with varying demand and lead time is also illustrated by Sarkar et al. [21]. Castellano et al. [7] have studied single vendor-multiple buyer supply chain model with the distribution-free approach under controllable lead time. Sharma et al. [22] analyzed the supply chain model with deteriorating products under varying lead time. Karthick and Uthayakumar [14] have examined the optimization strategy for an imperfect production play of cost savings and manageable lead time.

Pricing is the crucial factor in maximizing profits in the supply chain ensuring the **s**upply and demand are in sync. Therefore, the literature on the benefits that can occur if the demand for a commodity depends on its price is given below. Alfares and Ghaithan [1] have developed a pricing supply chain model with price-dependent demand. Zhao et al. [25] have examined a two-echelon supply chain model with pricing decisions of complementary items. Bieniek [4] have presented a vendor and retailer managed consignment inventory model with additive price-dependent demand. Dey et al. [9] has addressed an integrated inventory model to implements selling price dependent demand investment. In our proposed model, we consider how to increase demand for a product based on price and make better decisions in the supply chain.

In the supply chain, some strategies are essential to attract customers and make more profit, based on which both VMI and CS policy appear to be the best strategies. Braglia and Zavanella [6] were the first researchers who proposed the inventory model under the consignment stock policy between the single vendor and a single buyer. Gharaei et al. [10] designed the Vendor Managed Inventory (VMI) with the CS policy model and sharing multiple items between the single-vendor multi-buyer under greenhouse emissions penalty. Ben-Daya et al. [5] developed a VMI-CS policy supply chain model between a single manufacturer and multiple buyer. Srinivas and Rao [23] have analyzed the supply chain model with genetic algorithm under CS policy between a single manufacturer and multiple buyers. Both VMI and CS policy have different natures. However, researchers have proven that they can increase profits by incorporating it into their model. With these, it seems that when VMI and CS agreements are put together it will reduce un necessary costs and help to add products among customers on time.

Contributions of various study articles from the existing literature are given in Table 1, It is clear that there has been very little research on lead time dependent demand. And, in particular, it is notated that the literature regarding selling price dependent studies are limited. Moreover, to best of our knowledge, none of the supply chain models has been reported in the literature on selling price and lead time dependent demand under Vendor Managed Inventory and Consignment stock (VMI-CS) policy with controllable lead time.

The remainder of this paper is organized as follows. Section 3 provides the necessary notations and assumptions for the creation of this model. Section 4 comprises the VMI-CS policy based mathematical formulation with controllable lead time. The solution procedure is created in section 5. Section 6 deals with numerical examples to illustrate the results. In Section 7, sensitivity analysis of the optimal solution with respect to major parameters is carried out. Some managerial insights are obtained and illustrated in section 8. Finally, we draw some conclusions and provide some suggestions for future research in the last section.

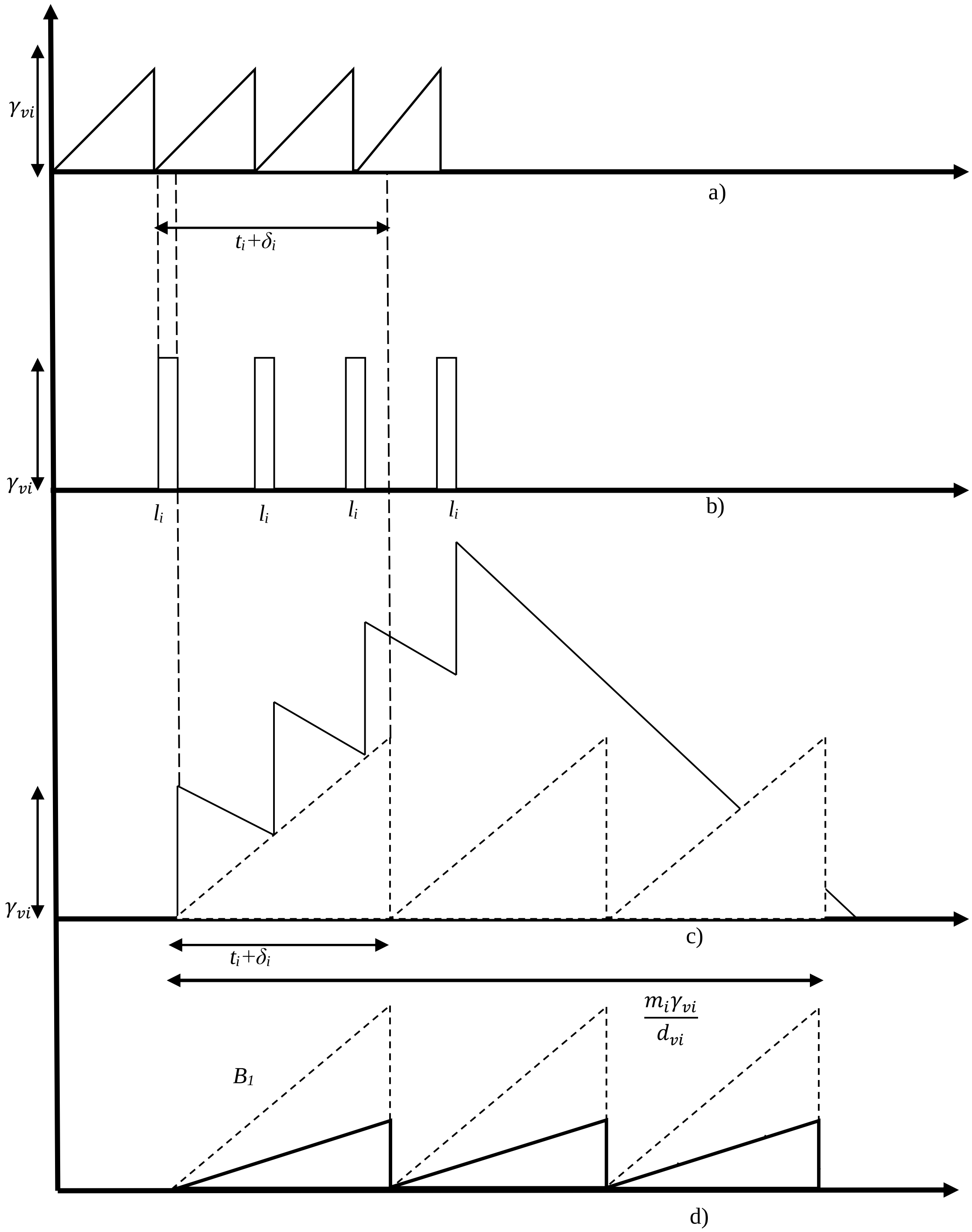

Fig. 1 Inventory pattern of consignment stock policy under controllable lead time.
(a) Vendor's inventory (b) Transit inventory (c) Inventory of buyer (d) Financial behaviour of i^{th} buyer

Table 1. Summary of research gap analysis.

Reference	Supply Chain	VMI-CS Policy	Multi buyer	Controllable lead time	Selling price and lead time dependent demand
Batarfi et al. [2]	√				
Castellano et al.[7]	√		√	√	
Batarfi et al. [3]	√	√			√
Chen and Su [8]	√				
Gharaei et al. [10]		√	√		
Jha and Sanker [13]	√		√	√	
Karthick and Uthayakumar [14]		√			
Mandal and Giri [19]	√		√	√	
Yi and Sarker [24]	√			√	
Present Model	√	√	√	√	√

3. Notations and assumptions

The following notations and assumption are considered to build this model.

3.1. Notations

Parameters

d_{vi} Demand rate of i^{th} buyer, i=1,2....z (units/year)
α Primary demand, $\alpha > 0$ (units/year)
λ_{vi} Coefficient of price elasticity of item for i^{th} buyer (unit^2/\$/year)
μ_{vi} Sensitivity of delivery lead time of demand d_{vi} (customer/day)
P_r Production rate, $P_r > \alpha$ (units/year)
C_{pr} Production cost for i^{th} item (\$/year)
ω_{vi} Wholesale price of the item to the i^{th} buyer (\$/unit)
S_v Setup cost (\$/setup)
A_{bi} Ordering cost for the item of i^{th} buyer (\$/order)
h^{pf} Physical & Financial holding cost for the vendor (\$/unit/year)
h_{vi}^{f} Financial holding cost for a unit of item at the i^{th} buyer's side paid by the vendor (\$/unit/year)
h_{bi}^{p} Physical holding cost for i^{th} buyer (\$/unit/year)
h_{di}^{p} Physical holding cost of i^{th} buyer in transit (\$/unit/year)
t_i Fixed transportation cost for i^{th} buyer (\$/shipment)
δ_i Variable transportation cost for ^{ith} buyer (\$/unit)
T_c Cycle time (year)
$B(l_i)$ Lead time crashing cost for i^{th} buyer (\$/year)

Decision variables

m_i No of shipments for i[th] buyer (integer)
C_{vi} Selling price of a item for i[th] buyer ($/unit)
γ_{vi} Shipment size of i[th] buyer (units/shipment)
l_i Lead time length of i[th] buyer (year)

3.2. Assumptions

(1) The production rate of i[th] item per year is considered as finite, and it should be greater than the primary demand rate of item for the i[th] buyer (i.e., $P_r > \alpha$) to avoid any shortages (see, for instance, Yi and Sarker [24]).

(2) The system inventory is continuously reviewed, and the shortages is not allowed.

(3) The holding cost of the vendor is divided into two parts namely financial and physical. Therefore vendor's holding cost is h^{pf} and unit holding cost for i[th] buyer in transit is $h_{di} = h_{di}^{p} + h_{vi}^{f}$.

(4) The vendor's setup cost and buyer's ordering cost, shipment size are constant/fixed and independent of the order/production quantity (see Batarfi et al. [3]. The financial holding cost of the products in the vendor's inventory is omitted from the total cost. In addition, the financial holding cost of the products stored in the buyer's warehouse will be added until the buyer pays the vendor for the products purchased.

(5) For the i[th] buyer the lead time l_i consist of n_j components which are mutually independent. The j[th] component has a minimum duration $m_{j,k}$ normal duration $n_{j,k}$ and a crashing cost per unit item $e_{j,k}$ and assume that $e_{j,1} \leq e_{j,2} \leq \ldots \leq e_{j,n_{ij}}$. The lead time components are to be crashed one at a time beginning from the least component of e_i and so on.

(6) Let $l_{i,0} \leq \sum_{k=1}^{n_{ij}} n_{j,k}$ and $l_{i,f}$ is the length of the lead time components 1,2,3,....f crashed to their minimum duration, then expression of $l_{i,f}$ is given by $l_{i,f} = l_{i,0} - \sum_{j=1}^{f}(n_{j,k} - m_{j,k})$, where f = 1,2,......$n_{ij}$ and crashing cost for the lead time per cycle is given by B(l_i) = $e_{j,f}\left(l_{i,f-1} - l_i\right) + \sum_{k=1}^{f-1} e_{j,k}\,(n_{j,k} - m_{j,k})$, $l_i \in \left[l_{i,f},\ l_{i,f-1}\right]$.

4. Mathematical Model

In this section, a mathematical model is developed to determine the inventory calculations between single vendor and multiple buyers for a preliminary purpose. The average inventory of the system is calculated as, (refer Yi and Sarker [24]).

$$\boldsymbol{I_s = \sum_{i=1}^{z} \gamma_{vi}\left(\frac{m_i}{2} - \frac{m_i d_{vi}}{2P_r} + \frac{d_{vi}}{P_r}\right) + d_{vi} l_i} .$$

- **The average inventory of buyer,**

$$\boldsymbol{I_{buyer} = \sum_{i=1}^{z} \frac{\gamma_{vi}^2\left(\frac{m_i}{2P_r} - \frac{m_i^2}{2P_r} + \frac{m_i^2}{2d_{vi}}\right)}{T_c}} .$$

Where the cycle time,

$$\boldsymbol{T_c = \frac{m_i \gamma_{vi}}{d_{vi}} = \sum_{i=1}^{z} \frac{d_{vi}}{m_i \gamma_{vi}} \times \gamma_{vi}^2 \left(\frac{m_i}{2P_r} - \frac{m_i^2}{2P_r} + \frac{m_i^2}{2d_{vi}}\right)} .$$

- **The average inventory in transit,**

$$\boldsymbol{I_{transit} = \sum_{i=1}^{z} m_i \gamma_{vi} l_i \times \frac{1}{T_c} = \sum_{i=1}^{z} \frac{d_{vi}}{m_i \gamma_{vi}} \times m_i \gamma_{vi} l_i = \sum_{i=1}^{z} d_{vi} l_i}\,.$$

- **The average inventory of buyer,**

$$I_{buyer} = I_s - I_{buyer} - I_{transit} = \sum_{i=1}^{z} \frac{\gamma_{vi} d_{vi}}{2P_r}$$

4.1. Vendor's Cost Formulation

The inventory cost associated with the vendor for z buyers is computed using the following various components.

Setup cost

Setup cost is the cost of purchasing and maintaining the equipment needed for the production stage to make the items before the production starts and configuring a machine for a production run. In this case, the cost of preparing the machine to each production of the items,

$$SC = \sum_{i=1}^{z} \frac{S_v d_{vi}}{m_i \gamma_{vi}}.$$

Physical and financial holding cost

The vendor manufactures products, stocks a specific amount of goods, and provides the required quantity to the buyer. Therefore, the cost of holding the stored inventory for z buyers,

$$PFHC = \sum_{i=1}^{z} h^{pf} \frac{\gamma_{vi} d_{vi}}{2P_r}.$$

Financial holding cost

According to the VMI-CS policy, the vendor will continue to ship the products to the buyer, and the goods received will be stored in the buyer's warehouse. The vendor's cost for holding those items in the buyer's warehouse is calculated as,

$$FHC = \sum_{i=1}^{z} h_{vi}^{f} \left(\frac{m_i \gamma_{vi}}{2} - (m_i - 1) \frac{\gamma_{vi} d_{vi}}{2P_r} \right).$$

Production cost

Production costs refer to the cost of producing or manufacturing an item. Also, this includes direct labour costs, direct material and overhead costs for production,

$$PC = \sum_{i=1}^{z} C_{pr} \frac{d_{vi}}{P_r}.$$

Transportation cost

The products will be sent to the buyers' in $\sum_{i=1}^{z} m_i$ shipments by the vendor. The buyer's total transportation cost is calculated as,

$$TC = \sum_{i=1}^{z} \frac{m_i d_{vi}}{\gamma_{vi}} \left(t_i + \delta_i \left(\frac{\gamma_{vi}}{m_i} \right) \right).$$

where $\sum_{i=1}^{z} t_i \frac{m_i d_{vi}}{\gamma_{vi}}$ is the buyer's fixed transportation cost, while the variable transportation cost is, $\sum_{i=1}^{z} \delta_i d_{vi}$.

Lead time crashing cost

Lead time is the interval between when an order is placed to fill the goods and when the order is received. However, to reduce the length of lead time, the crashing is used as,

$$LTCC = \sum_{i=1}^{z} \frac{d_{vi}}{\gamma_{vi}} B(l_i).$$

Transit lead time

During the period of lead time, the cost incurred for holding the inventory at the time of transport,

$$TLT = \sum_{i=1}^{z} \left(h_{di}^{p} + h_{vi}^{f} \right) d_{vi} l_i.$$

The total inventory cost function is derived by adding the equations,

$$ie., \; C_{total}(m_i , C_{vi} , \gamma_{vi} , l_i) = SC + PFHC + FHC + PC + TC + LTCC + TLT.$$

$$ie., \; C_{total}(m_i , C_{vi} , \gamma_{vi} , l_i) = \sum_{i=1}^{z} \left[\frac{S_v d_{vi}}{m_i \gamma_{vi}} + h^{pf} \frac{\gamma_{vi} d_{vi}}{2P_r} + h_{vi}^{f} \left(\frac{m_i \gamma_{vi}}{2} - (m_i - 1) \frac{\gamma_{vi} d_{vi}}{2P_r} \right) + C_{pr} \frac{d_{vi}}{P_r} + \frac{m_i d_{vi}}{\gamma_{vi}} \left(t_i + \delta_i \left(\frac{\gamma_{vi}}{m_i} \right) \right) + \frac{d_{vi}}{\gamma_{vi}} B(l_i) + \left(h_{di}^{p} + h_{vi}^{f} \right) d_{vi} l_i \right]. \quad (1)$$

Therefore, to calculate the profit $\Pi_v(m_i , C_{vi} , \gamma_{vi} , l_i)$ to be received (through z buyers), the inventory cost to the vendor has to be subtracted from the wholesale price of i^{th} buyer, which is calculated as,

$$ie., \; \Pi_v(m_i , C_{vi} , \gamma_{vi} , l_i) = \sum_{i=1}^{z} \left[\omega_{vi} d_{vi} - \left(\frac{S_v d_{vi}}{m_i \gamma_{vi}} + h^{pf} \frac{\gamma_{vi} d_{vi}}{2P_r} + h_{vi}^{f} \left(\frac{m_i \gamma_{vi}}{2} - (m_i - 1) \frac{\gamma_{vi} d_{vi}}{2P_r} \right) + C_{pr} \frac{d_{vi}}{P_r} + \frac{m_i d_{vi}}{\gamma_{vi}} \left(t_i + \delta_i \left(\frac{\gamma_{vi}}{m_i} \right) \right) + \frac{d_{vi}}{\gamma_{vi}} B(l_i) + \left(h_{di}^{p} + h_{vi}^{f} \right) d_{vi} l_i \right) \right]. \quad (2)$$

4.2. Buyer's Cost Formulation

The inventory costs associated with z buyers is computed using the following various components.

Ordering cost

The cost required by z buyers to process the order from the vendor is said to be an ordering cost,

$$OC = \sum_{i=1}^{z} A_{bi} \frac{d_{vi}}{\gamma_{vi}}.$$

Purchasing cost

The cost incurs for purchasing the items from the vendor,

$$PC = \sum_{i=1}^{z} \omega_{vi}\, d_{vi} \,.$$

Physical holding cost

The vendor will sent the needed amount of products to the buyer and incur the financial cost of holding those products,

$$PHC = \sum_{i=1}^{z} h_{bi}^{p}\left(\frac{m_i\gamma_{vi}}{2} - \left((m_i - 1)\frac{\gamma_{vi}d_{vi}}{2P_r}\right)\right).$$

The inventory cost associated with z buyers is calculated by adding the ordering cost, the purchasing cost and the physical holding cost.

$$ie.,\ C_{total}(C_{vi}\,,\gamma_{vi}) = OC + PC + PHC.$$

$$ie.,C_{total}(C_{vi}\,,\gamma_{vi}) = \sum_{i=1}^{z}\left[A_{bi}\frac{d_{vi}}{\gamma_{vi}} + \omega_{vi}d_{vi} + h_{bi}^{p}\left(\frac{m_i\gamma_{vi}}{2} - \left((m_i - 1)\frac{\gamma_{vi}d_{vi}}{2P_r}\right)\right)\right]. \qquad (3)$$

The profit of z buyers $\Pi_b(C_{vi}\,,\gamma_{vi})$is calculated by subtracting the inventory cost of z buyers from the selling of the item to the consumer, which is,

$$C_{total}(C_{vi}\,,\gamma_{vi}) = \sum_{i=1}^{z}\left[C_{vi}\,d_{vi} - \left[A_{bi}\frac{d_{vi}}{\gamma_{vi}} + \omega_{vi}d_{vi} + h_{bi}^{p}\left(\frac{m_i\gamma_{vi}}{2} - \left((m_i - 1)\frac{\gamma_{vi}d_{vi}}{2P_r}\right)\right)\right]\right]. \qquad (4)$$

By combining Equations (2) and (4), the integrated profit function of the supply chain can be obtained.

4.2. Model Formulation

The vendor ships the item to each buyer in equal shipments of size γ_{vi} and sell at the wholesale price ω_{vi}. The buyer sells the item to the customer/consumer at a retail price C_{vi} and pays the vendor only when the items are withdrawn from inventory/warehouse. The vendor continues producing and shipping the item until the buyer's inventory reaches a maximum level. The integrated profit function of the supply chain with the consideration of i^{th} buyer is obtained as,

$$\Pi_{vbi}(m_i\,, C_{vi}\,, \gamma_{vi}\,, l_i) = \left[d_{vi}\times\left(C_{vi} - \frac{(S_v + m_iA_{bi})}{m_i\gamma_{vi}} - h^{pf}\frac{\gamma_{vi}}{2P_r} + \left(h_{vi}^{f}\ +\ h_{bi}^{p}\right)\times\left((m_i-1)\frac{\gamma_{vi}}{2P_r}\right) - \frac{C_{pr}}{P_r} - \frac{m_i}{\gamma_{vi}}\left(t_i + \delta_i\left(\frac{\gamma_{vi}}{m_i}\right)\right) - \frac{B(l_i)}{\gamma_{vi}} - \left(h_{di}^{p}\ +\ h_{vi}^{f}\right)l_i\right) - \left(h_{vi}^{f}\ +\ h_{bi}^{p}\right)\left(\frac{m_i\gamma_{vi}}{2}\right)\right]. \qquad (5)$$

However, in this model formulation, the demand rate d_{vi} is assumed to be a linear function which is depends on the selling price and the lead time.

i.e., $d_{vi} = \alpha - \lambda_{vi}\gamma_{vi} - \mu_{vi}l_i$ then Equation 5 can be written as,

$$\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i) = \Bigg[(\alpha - \lambda_{vi}\gamma_{vi} - \mu_{vi}l_i) \times \Bigg(C_{vi} - \frac{(S_v + m_i A_{bi})}{m_i\gamma_{vi}} - h^{pf}\frac{\gamma_{vi}}{2P_r} + \left(h_{vi}^f + h_{bi}^p\right) \times \left((m_i - 1)\frac{\gamma_{vi}}{2P_r}\right) - \frac{C_{pr}}{P_r} - \frac{m_i}{\gamma_{vi}}\left(t_i + \delta_i\left(\frac{\gamma_{vi}}{m_i}\right)\right) - \frac{B(l_i)}{\gamma_{vi}} - \left(h_{di}^p + h_{vi}^f\right)l_i\Bigg) - \left(h_{vi}^f + h_{bi}^p\right)\left(\frac{m_i\gamma_{vi}}{2}\right)\Bigg]. \quad (6)$$

Therefore, the total integrated profit function for single vendor and z buyers can be calculated as,

$$\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i) = \sum_{i=1}^{z} \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$$

5. Solution procedure

Throughout this section, the necessary and sufficient conditions can be used for proving unique optimum solutions *i.e.,* $(C_{vi}$ & $\gamma_{vi})$

5.1. Necessary condition for optimum solutions

Lemma 5.1.

*When the number of shipments m_i, selling price of the item C_{vi}, shipment size of the item γ_{vi}, and the lead time of i^{th} buyer l_i is constant, then the profit function $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ is **concave** with respect to the selling price of the item for i^{th} buyer C_{vi}.*

Proof.

On taking the first and second order partial derivatives of Equation 6 with respect to C_{vi}, we obtain,

$$\frac{\partial \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial C_{vi}} = \alpha - 2\lambda_{vi}C_{vi} - \mu_{vi}l_i + l_i\lambda_{vi}\left(h_{di}^p + h_{vi}^f\right) + \frac{\lambda_{vi}C_{pr}}{P_r} + h^{pf}\left(\frac{\lambda_{vi}\mu_{vi}}{2P_r}\right) + \left(S_v + m_i^2\left(t_i + \frac{\delta_i\mu_{vi}}{m_i}\right)\right) \times \left(\frac{\lambda_{vi}}{m_i\gamma_{vi}}\right) - \left(h_{bi}^p + h_{vi}^f\right) + \left(A_{bi} + B(l_i)\right)\frac{\lambda_{vi}}{\gamma_{vi}}. \quad (7)$$

and

$$\frac{\partial^2 \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial^2 C_{vi}} = 2\lambda_{vi} < 0. \quad (8)$$

■

Lemma 5.2.

*When the number of shipments m_i, selling price of the item C_{vi}, shipment size of the item γ_{vi} and the lead time of i^{th} buyer l_i is constant, then the profit function $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ is **concave** with respect to the selling price of the item for i^{th} buyer γ_{vi}.*

Proof.

On taking the first and second order partial derivatives of Equation 6 with respect to γ_{vi}, we obtain,

$$\frac{\partial \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial \gamma_{vi}} = \frac{\alpha - \lambda_{vi}C_{vi} - \mu_{vi}l_i}{\gamma_{vi}^2}\left(A_{bi} - \delta_{vi}\gamma_{vi} + \frac{S_v}{m_i} + B(l_i) + m_i\left(t_i + \frac{\delta_{vi}\gamma_{vi}}{m_i}\right) - \frac{h^{pf}\gamma_{vi}^2}{2P_r}\right) - \frac{1}{2}\left(h_{vi}^f + h_{bi}^p\right) \times \left(m_i - \frac{(\alpha - \lambda_{vi}C_{vi} - \mu_{vi}l_i)(m_i - 1)}{P_r}\right). \quad (9)$$

and

$$\frac{\partial^2 \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial^2 \gamma_{vi}} = -\frac{2(\alpha - \lambda_{vi}C_{vi} - \mu_{vi}l_i)}{\gamma_{vi}^3}\left(B(l_i) - \delta_{vi}\gamma_{vi} + A_{bi} + \frac{S_v}{m_i} + m_i\left(t_i + \frac{\delta_{vi}\gamma_{vi}}{m_i}\right)\right) < 0. \quad (10)$$

■

From Lemma 5.1, we obtain the optimal value of C_{vi} by equating Equation 7 to zero and that is $\frac{\partial \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial C_{vi}} = 0$ which maximize the $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$.
Then the optimum solution of C_{vi} is,

$$C_{vi} = \frac{1}{2\lambda_{vi}}\left[\alpha - \mu_{vi}l_i - \lambda_{vi}l_i\left(h_{di}^p + h_{vi}^f\right) + \frac{\lambda_{vi}C_{pr}}{P_r} + \frac{\lambda_{vi}}{\mu_{vi}}\left(A_{bi} + B(l_i)\right) + \frac{\lambda_{vi}}{m_i\gamma_{vi}}\left(S_v + m_i^2\left(t_i + \frac{\delta_i\gamma_{vi}}{2P_r}\right)\right) + \frac{\lambda_{vi}\gamma_{vi}}{2P_r}\left(h^{pf} - (m_i - 1)\left(h_{bi}^p + h_{vi}^f\right)\right)\right]. \quad (11)$$

From Lemma 5.2, we obtain the optimal value of γ_{vi} by equating Equation 7 to zero and that is $\frac{\partial \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial \gamma_{vi}} = 0$ which maximize the $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$.
Then the optimum solution of γ_{vi} is,

$$\gamma_{vi} = \frac{\sqrt{2}\sqrt{(\alpha - \lambda_{vi}C_{vi} - \mu_{vi}l_i)P_r m_i(S_v + m_iB(l_i) + m_iA_{bi} + m_i^2 t_i)\Gamma}}{m_i(\alpha - \lambda_{vi}C_{vi} - \mu_{vi}l_i)\left(h_{bi}^p + h^{pf} + h_{vi}^f - m_ih_{bi}^p - m_ih_{vi}^f\right) + m_i^2P_r\left(h_{bi}^p + h_{vi}^f\right)}.$$

Where $\Gamma = \left((\alpha - \lambda_{vi}C_{vi} - \mu_{vi}l_i)\left(h_{bi}^p + h^{pf} + h_{vi}^f - m_ih_{bi}^p - m_ih_{vi}^f\right) + m_iP_rh_{bi}^p + m_iP_rh_{vi}^f\right)$.

5.2. Sufficient condition for optimum solutions

Lemma 5.2.

For fixed values of m_i, $l_i \in [l_{i,f}, l_{i,f-1}]$, then the optimal solutions of C_{vi} and γ_{vi} will maximize the profit function $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ exist, and are unique.

Proof.

For the fixed values of m_i and l_i, the Hessian matrix $\boldsymbol{H}$ is as follows:

$$\boldsymbol{H} = \begin{bmatrix} \frac{\partial^2 \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial^2 C_{vi}} & \frac{\partial^2 \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial C_{vi}\partial \gamma_{vi}} \\ \frac{\partial^2 \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial \gamma_{vi}\partial C_{vi}} & \frac{\partial^2 \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial^2 \gamma_{vi}} \end{bmatrix}$$

The solution will be optimal if the corresponding Hessian matrix $\boldsymbol{H}$ of the profit function $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ is negative definite. Here,

$$\frac{\partial^2 \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial^2 C_{vi}} < 0$$

and

$$\frac{\partial^2 \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial^2 \gamma_{vi}} < 0.$$

Also

$$\frac{\partial^2 \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial C_{vi} \partial \gamma_{vi}} = \frac{\partial^2 \Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)}{\partial \gamma_{vi} \partial C_{vi}}$$

$$= \lambda_{vi}\left(\frac{\delta_i \gamma_{vi} - B(l_i) - A_{bi}\frac{S_v}{m_i} - m_i\left(t_i + \frac{\delta_i \gamma_{vi}}{m_i}\right)}{\gamma_{vi}^2} + \frac{\left(h^{pf} - (m_i - 1)\left(h_{bi}^p + h_{vi}^f\right)\right)}{2P_r}\right).$$

Therefore, the determinant of Hessian matrix $\boldsymbol{H}$ is negative definite, for any positive values of C_{vi} and γ_{vi}. Hence, this completes the proof of Lemma 5.3.

Table 2. Lead time components.

Buyer i	**Lead time components k**	**Normal duration $n_{j,k}$ (year)**	**Minimum duration $m_{j,k}$ (year)**	**Unit crashing cost $e_{j,k}$ ($/year)**
1	1	20/365 = 0.05479	6/365 = 0.01644	0.1 × 365 = 36.5
	2	20/365 = 0.05479	6/365 = 0.01644	1.2 × 365 = 438
	3	16/365 = 0.04383	9/365 = 0.02465	5.0 × 365 = 1825
2	1	20/365 = 0.05479	6/365 = 0.01644	0.5 × 365 =182.5
	2	16/365 = 0.04383	9/365 = 0.02465	1.3 × 365 = 474.5
	3	13/365 = 0.035616	6/365 = 0.01644	5.1 × 365 = 1861.5
3	1	25/365 = 0.06849	11/365 = 0.03013	0.4 × 365 = 146
	2	20/365 = 0.05479	6/365 = 0.01644	2.5 × 365 = 912.5
	3	18/365 = 0.04931	11/365 = 0.03013	5.0 × 365 = 1825

Table 3. Summarized lead time data.

Buyer i	**Lead time (year)**	**B(l_i) ($/shipment)**
1	56/365 = 0.15342	0
	42/365 = 0.11506	1.4
	28/365 = 0.076712	18.2
	21/365 = 0.05753	53.20
2	49/365 = 0.013424	0
	35/365 = 0.098590	7
	28/365 = 0.076712	16.1
	21/365 = 0.057534	51.8
3	63/365 = 0.1726	0
	49/365 = 0.13424	5.6
	35/365 = 0.09589	40.6
	28/365 = 0.076712	75.6

6. Numerical Analysis

In order to illustrate the solution procedure, let us consider an inventory system with the following data considering three buyers for our convenience. For simplicity, the parameters for three buyers are arranged in row matrix. i.e., the demand rate of the ith buyer, $d_{vi} = [d_{v1}, d_{v2}, d_{v3}]$. The numerical data is taken from the work by Karthick and Uthayakumar [15].

Parameters related to vendor:

S_v = 1000 (\$/setup), P_r = 18,000 (\$/year), C_{pr}= 500,000 (\$/year), h^{pf}= 30 (\$/unit/year).

Parameters related to buyer:

$[d_{v1}, d_{v2}, d_{v3}]$ = [200, 200, 200] (\$/unit), $[A_{b1}, A_{b2}, A_{b3}]$= [300, 200, 250] (\$/order),
$[h^f_{b1}, h^f_{b2}, h^f_{b3}]$ = [10, 20, 10] (\$/unit/year), $[h^p_{d1}, h^p_{d2}, h^p_{d3}]$ = [10, 5, 7] (\$/unit/year),
$[t_1, t_2, t_3]$ = [20, 10, 15] (\$/shipment), $[\delta_1, \delta_2, \delta_3]$ = [5, 4, 3] (\$/shipment).

General parameters:

α = 15,000 (units/year), $[\lambda_{v1}, \lambda_{v2}, \lambda_{v3}]$ = [15, 10, 13] (unit^2/\$/year), $[\mu_{v1}, \mu_{v2}, \mu_{v3}]$ = [30, 20, 25] (customer/day).

Table 2 describes the lead time components for the three buyers with normal and minimum duration, and the crashing cost for each component is calculated accordingly. And, the controllable lead time for each buyer and the associated crashing cost are summarized in Table 3.

Table 4. Optimal values

Buyer i	l_i	m_i	C_{vi}	γ_{vi}	Vendor's profit	Buyer's profit	Π_{vbi}	Total profit
1	0.05753	2	566.6340	219.9374	1053197.76	2372001.06	3425198.82	12813885.98
2	0.057534	2	786.0172	214.2746	1172681.03	4173288.89	5345969.92	
3	0.076712	2	616.1231	223.2216	1144274.22	2898443.02	4042717.24	

We obtained the optimal results for different parameters and are shown in Table 4. The graphical representations relating the number of shipments with respect to the selling price C_{vi} and profitability of each buyers are shown in Figure 2 and Figure 3 respectively. The effect of the selling price C_{vi} and shipment size γ_{vi} on the total cost Π_{vbi} is shown in Figure 4 The total profits/gains made by the vendor and buyer through this model is \$ 12813885.98. Comparing the buyer's profit, vendor's profit exhibits the greatest benefit from numerical results.

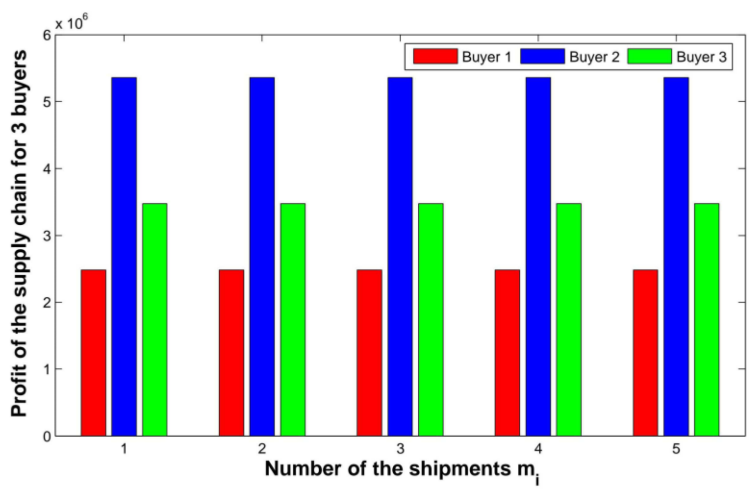

Fig. 2 No. of shipments Vs Profit of the supply chain for 3 buyers

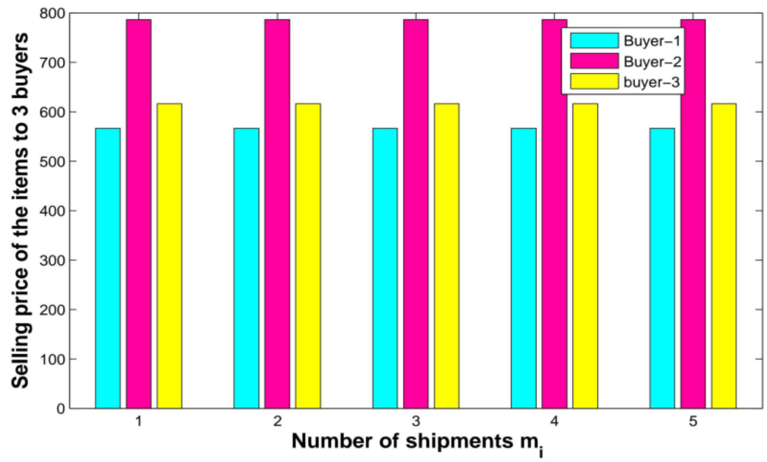

Fig. 3 No. of shipments Vs selling price for three buyers

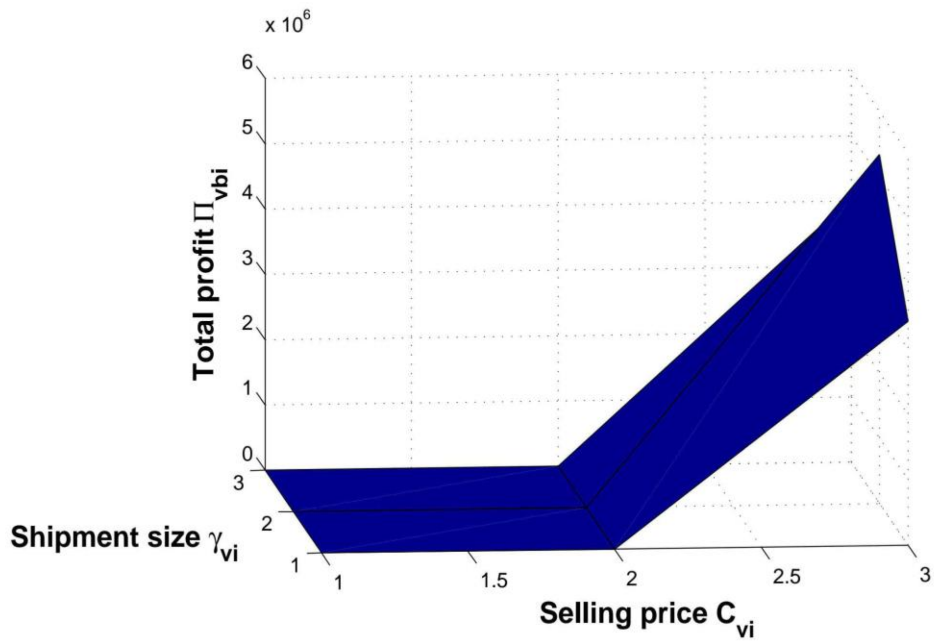

Fig. 4 Effects of C_{vi} , γ_{vi} on total cost

Table 5. Effect of change in the parameters of inventory model of example.

Para-Meters	% Changes	m_i	C_{vi}	γ_{vi}	Π_v	Π_b	Π_{vbi}	Total profit
S_v	-50%	2 2 2	566.3907 785.6866 615.8059	186.7069 176.2263 187.8546	1060300.37 1179773.51 1151462.33	2370362.46 4172033.16 2896747.09	3430662.83 5351806.67 4048209.42	12830678.92
	-25%	2 2 2	566.4791 785.8604 615.9718	204.0044 196.1783 206.3012	1056537.29 1175983.80 1147648.90	2371280.87 4172762.27 2897695.98	3427818.16 5348746.07 4045344.88	12821909.11
	+25%	2 2 2	566.7775 786.1611 616.2630	234.7835 230.9523 238.9407	1050172.97 1169723.73 1141222.33	2372585.65 4173687.47 2899053.66	3422758.62 5343411.20 4040275.99	12806445.81
	+50%	2 2 2	566.9119 786.2949 616.3938	248.7405 246.4999 253.6825	1047392.25 1167026.23 1138419.32	2373072.37 4173999.41 2899567.02	3420464.62 5341025.64 4037986.34	12799496.60
C_{pr}	-50%	2 2 2	559.6741 779.0642 609.1668	221.6970 215.3155 224.6612	1162000.89 1284316.10 1257548.74	2364240.85 416.3875.26 2886774.34	3526241.74 5448191.36 4144323.08	13118756.18
	-25%	2 2 2	566.1943 785.5878 605.6887	222.5715 215.8341 225.3775	1217490.02 1340857.89 1315128.22	2359816.27 4158806.12 2880468.46	3577306.29 5499664.01 4195596.68	13272566.98
	+25%	2 2 2	570.1140 789.4937 619.6013	219.0523 213.7522 222.4983	999184.05 1117587.93 1088579.20	2375336.56 4177633.25 2903805.81	3375220.61 5295221.18 3992385.01	12662826.80
	+50%	2 2 2	573.5942 792.9702 623.0796	218.1636 213.2285 221.7726	947295.24 1062977.79 1033512.23	2378309.11 4181735.96 2908854.31	3325604.35 5244713.75 3942366.54	12512684.64
P_r	-50%	2 2 2	580.5612 799.9234 630.0364	215.6741 212.1773 220.3140	844428.02 954381.02 924826.10	2383522.98 4190041.76 2918424.86	3227950.99 5144422.78 3843270.96	12215644.70
	-25%	2 2 2	571.7771 790.6525 620.7608	218.4102 213.5778 222.2566	982506.81 1099050.43 1070011.02	2376489.31 4179308.14 2905666.26	3358996.12 5278358.57 3975677.28	12613031.97
	+25%	2 2 2	563.8512 783.2359 613.3406	220.5070 214.6916 223.7985	1096680.55 1217274.37 1189369.77	2368988.05 4169468.34 2893839.16	3465668.60 4169468.34 4083208.93	12935620.23
	+50%	2 2 2	563.8508 781.3819 611.4855	220.5474 214.9691 224.1824	1120921.44 1247175.74 1219657.82	2368935.47 4166834.54 2890657.21	3489856.91 5414010.28 4110315.03	13014182.22

A_{bi}	-50%	2 2 2	566.4465 785.8929 615.9718	200.6650 199.9291 206.3012	1052391.55 1171718.83 1143413.32	2375975.71 4176451.86 2901931.57	3428367.26 5348170.69 4045344.89	12821882.84
	-25%	2 2 2	566.5426 785.9562 616.0491	210.5234 207.2265 214.9288	1052842.18 1172231.89 1143882.66	2373904.19 4174819.31 2900122.38	3426746.37 5347051.20 4044005.04	12817802.61
	+25%	2 2 2	566.7213 786.0761 616.1943	228.9621 221.0973 231.2155	1053478.61 1173076.62 1144602.10	2370236.92 4171846.49 2896873.68	3423715.53 5343411.20 4041475.78	12810114.52
	+50%	2 2 2	566.8050 786.1332 616.2630	237.6418 227.7149 238.9407	1053699.63 1173426.62 1144877.35	2368589.47 1170481.29 2895398.64	3422289.10 5343907.10 4040275.99	12806473.00

7. Sensitivity Analysis

Sensitivity analysis is a strategy used to determine how independent variable values under a given assumption affect a particular dependent variable. Here we study the effects of changes in the system parameters S_v , P_r , C_{pr} and A_{bi} on the optimal selling price C_{vi} , the shipment size γ_{vi} , the vendor's profit Π_v , the buyer's profit Π_b, integrated profit of the supply chain $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ and the maximum total profit of the proposed example. A sensitivity analysis is performed by changing each of the parameters by +50%, +25%, -25%, and -50%, taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown in Table 5 and also shown graphically in Figures 5-8. Based on the results of Table 5 and Figures 5-8,
we obtain the following:

(1) Increase in the values of the parameter S_v will result in increase of C_{vi} , γ_{vi} and Π_b but decrease of Π_v , $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ and total profit.

(2) Decrease in the values of the parameters S_v will result in decrease of C_{vi} , γ_{vi} and Π_b but increase of Π_v , $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ and total profit.

(3) Increase in the values of the parameter P_r will result in increase of γ_{vi} , $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ and total profit but decrease of C_{vi} and Π_b.

(4) Decrease in the values of the parameters P_r will result in decrease of γ_{vi} , $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ and total profit but increase of C_{vi} and Π_b.

(5) Increase in the values of the parameter C_{pr} will result in increase of C_{vi} and Π_b but decrease of γ_{vi} , $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ and total profit.

(6) Decrease in the values of the parameter C_{pr} will result in decrease of C_{vi} and Π_b but increase of γ_{vi} , $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ and total profit.

(7) Increase in the values of the parameter A_{bi} will result in increase of C_{vi} , γ_{vi} and Π_v but decrease of Π_b , $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ and total profit.

(8) Decrease in the values of the parameters A_{bi} will result in decrease of C_{vi} , γ_{vi} and $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ but increase of Π_b and $\Pi_{vbi}(m_i, C_{vi}, \gamma_{vi}, l_i)$ and total profit..

In addition, the effects on the model's profitability may be clearly detected only when these parameters values are modified. The sensitivity study reveals that the C_{pr} is very sensitive relative to other parameters. Moreover, the parameters S_v , P_r , A_{bi} are noticed to be marginally sensitive.

8. Managerial insights

This paper considers a supply chain model between the single vendor and multi buyer under a controllable lead time. However, the demand depends upon the selling price and lead time of the i[th] buyer. Furthermore, the managerial insights of the proposed model is given in the following:

(1)The CS agreement policy favours both the vendor and the buyer who can save funds by sharing the cost of holding the goods physically and financially.

(2) Under the CS policy the buyer is not required to pay until the products are sold. Whereas, if the vendor is unable to sell all those products, they can return the products to the vendor, therefore, the vendor has to face the risks and rewards of ownership.

(3)By crashing the lead time period, the vendor can provide streamlined functions/operations, which in turn improves customer satisfaction. In addition, it builds transparency, trust, and cooperation among buyers.

(4) Therefore, the solution of the proposed model can be used to estimate the performance of two-stage supply chain and evaluate the total profit under the controllable lead time which is examined by adding a crashing cost to the buyer.

(5) Furthermore, it is seen that if the production cost could be reduced and the production rate could be increased effectively then the total profit per unit time could be automatically improved.

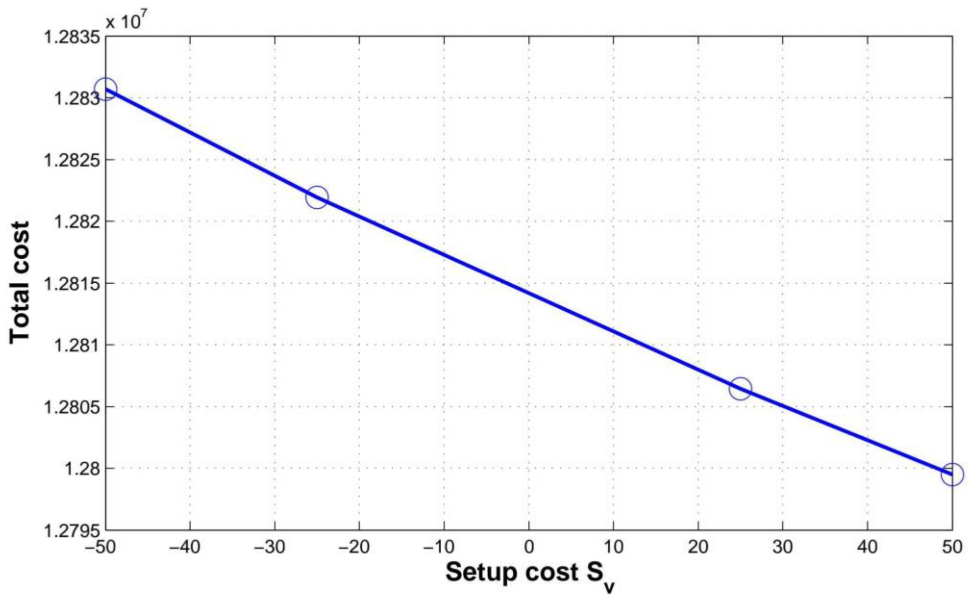

Fig. 5 Effects of S_v on total cost

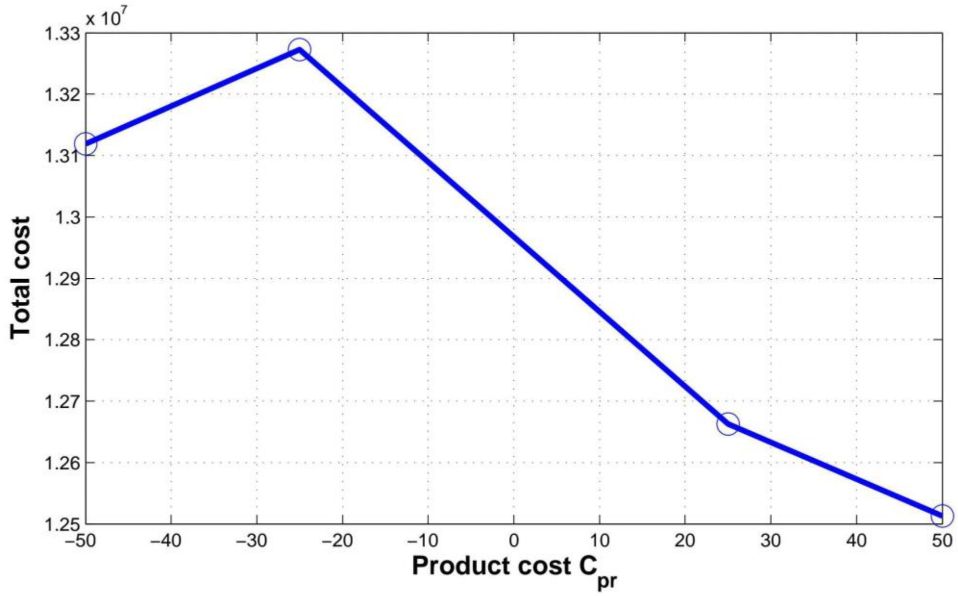

Fig. 6 Effects of C_{pr} on total cost

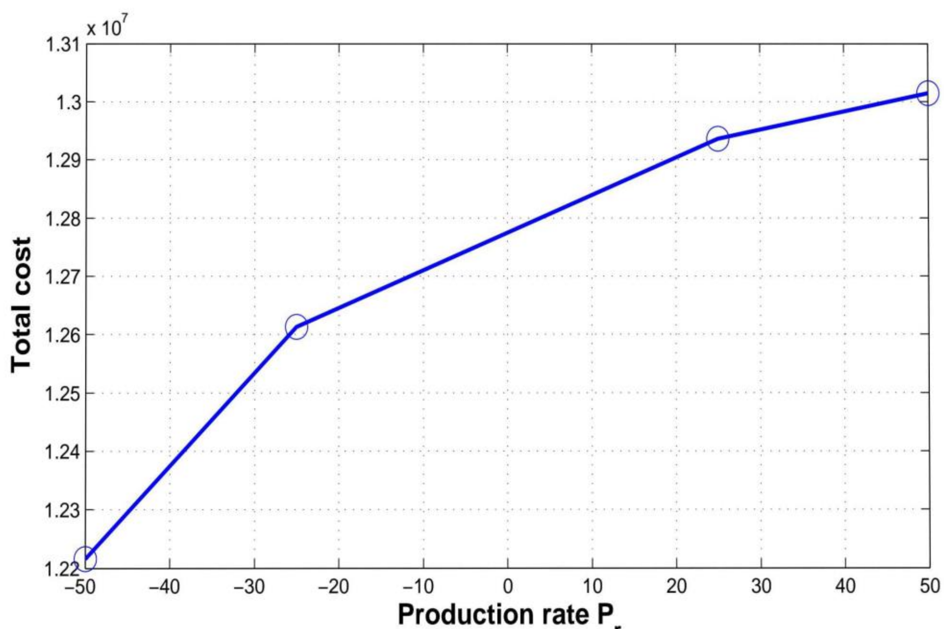

Fig. 7 Effects of P_r on total cost

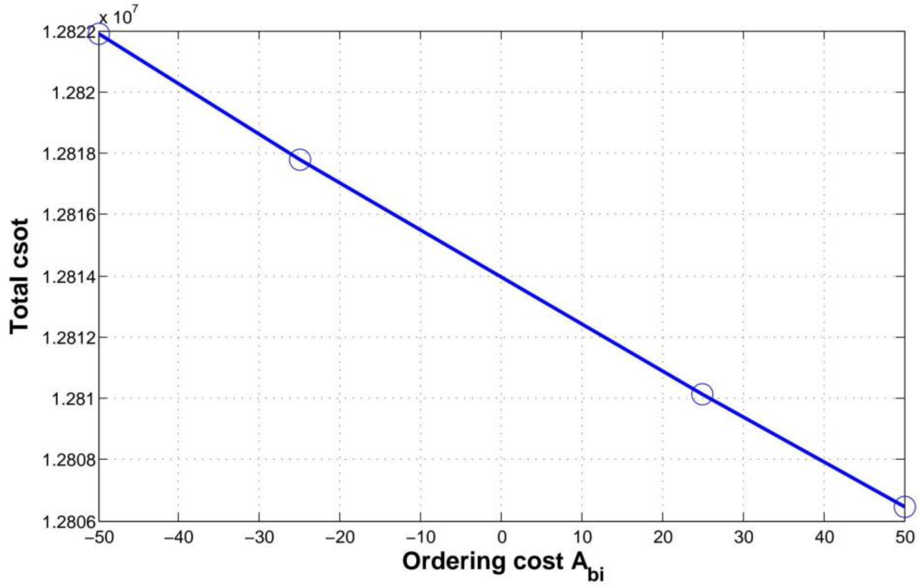

Fig. 7 Effects of A_{bi} on total cost

9. Conclusion

This conclusion section should clearly explain the main findings and implications of the work, highlighting its importance and relevance. In this paper, we formulated an integrated supply chain inventory model with single vendor and multi buyer under controllable lead time. The linear demand rate is assumed as a function of selling price and lead time. The ultimate purpose of this study is to optimize total supply chain profit between vendor and buyer. Therefore, this article will pave the best path for supply to maximize profit in the supply chain. In aspect of theory, the necessary and sufficient conditions of the existence and uniqueness of this optimal solution are proved. Numerical example is provided to demonstrate the applicability of the proposed model. The sensitivity analysis of the optimal solution with respect to the parameters is also included.

This model can be extended in several ways. This work can be extended by considering the various demand rate. Also, this can be extended by integrating new strategies for recovering and repairing damaged products, Considering the payment and transmitting delays would be a useful development of this model. By considering the carbon emissions from transport services would be another extension of this model. Taking all costs in fuzzy environment is also a good extension of this model.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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