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Original Article

α −Contraction Conditions on Fuzzy Metric Spaces and Fixed Points

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Abstract *- In this paper, we using* (α) *−contraction fuctions in complete fuzzy metric space and establish sequential characterization properties of fuzzy metric space. We prove the existence of common fixed point theorems for* (α) *−contractions mapping in fuzzy metric space using the property of Lebesgue fuzzy metric space and give a few models on the side of our outcomes.*

Keywords *- Fuzzy metric space, Contraction mapping, Fixed point, Lebesgue property,* (α) *−contraction functions.*

1. Introduction

Topology is the take a look at of geometric residences that does not rely best on the exact form of the gadgets, however as an alternative it acts on how the factors are related to each different. Infact, topology deals with the ones homes that stay invariant under the continuous transformation of a map. Zadeh (18) offered and tested the concept of a fuzzy set in his essential paper. The investigation of fuzzy units commenced a broad fuzzy of some numerical ideas and has packages to one of kind elements of applied sciences. The concept of fuzzy dimension spaces was offered at the start with the aid of kramasll and Michalek (11). Banach constriction popular is certainly a vintage style aftereffect of contemporary exam. Specifically, Mihet, (14) presented the thoughts of fuzzy ψ-contractive mappings which develop the magnificence of fuzzy compressions in gregori and sapena (6) and severa creator's abbas. Samet and vetro (17) provided the idea of α −contractive mappings and used similar thoughts to make some exciting constant declaration hypotheses in setting of metric spaces.

Thereafter specific fixed point issues for α-contractions in fuzzy sets have been pointed out speedy by specific authors $(see$ [1,2,8,11,12,13,]).

in this paper utilizing the belief of constant point theorem for (α) −contractions mapping in fuzzy metric space we prove a consequences which improves the recent works of Khan et al. [8], kabir et al. [9], and Samet et al. [17],

2. Preliminaries

Definition 2.1. The 3-tuple $(E, d,*)$ is called a fuzzy metric space if X is an arbitrary non-empty set, * is a continuous t-norm and d is a fuzzy metric in $X^2 \times [0,\infty] \to [0,1]$, satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$.

$[\boldsymbol{FM.1}]$ $M(x, y, 0) = 0$

$[\boldsymbol{FM.2}]$ $M(x, y, t) = 1 \; \forall \; t > 0 \; iff \;\; x = y.$

$[\boldsymbol{FM.3}]$ $M(x, y, t) = M(x, y, t)$

$[\boldsymbol{FM.4}]$ $M(x, y, t) * M(x, z, s) \leq M(x, z, t + s)$

$[\boldsymbol{FM.5}]$ $M(x, y, .): [0,\infty] \to [0,1]$, Is left continuous

$[\boldsymbol{FM.6}]$ $\lim_{t\to\infty} M\,(x, y, t) = 1.$

Definition 2.2. Let $(E, d,*)$ be a fuzzy metric space and let a sequence X_n in x is said to be converge to $x \in X$ if $\lim_{n\to\infty} M\,(X_n, x, t) = 1$, for each $t > 0$.

Definition 2.3. Let (E, d) be a metric space and $T: X \to X$ be a given mapping. We say that T is an $\alpha - \psi$ −contractive mapping if there exists two functions $\alpha: X \times X \to [0, +\infty)$ and $\psi \in \Psi$ such that

$$\alpha(x, y)d(Tx, Ty) \leq \psi\big(d(x, y)\big)$$

for all $x, y \in X$.

Definition 2.4. Let $T: X \to X$ and $\alpha: X \times X \times [0, \infty) \to [0,1]$, we say that T is α −admissible if

$x, y \in X, \alpha(X, y, t) \leq 1 \ \Rightarrow \alpha(Tx, Ty, t) \leq 1.$

Definition 2.5. Let (E, d) be a fuzzy metric space and $(T, \varphi) : (E, d) \to (E, d)$ be a given fuzzy mapping. Then we say that (T, φ) is fuzzy (α, β) −Banach contractive mapping, if there exists two fuzzy functions $(\alpha, \psi), (\beta, \alpha) : \mathrm{FC}(E) \to \mathcal{R}(E^*)$ and $0 \leq r < 1$ such that

$$(\alpha, \psi)(f_e)(\beta, \phi)(g_{e'})\big((T, \varphi)f_e, (T, \varphi)f_e, (T, \varphi)g_{e'}\big) \leq r.d(f_e, g_{e'}), \quad \forall f_e, g_{e'} \in FC(E).$$

Definition 2.6. A fuzzy metric space (E, d) is said to have the lebesgue property if given an open cover $\mathcal{G}\ of\ (E, \tau_d)$, there exist $r \in (0,1), t > 0$ such that $\{FC(x, r, t): x \in X\}$ refines $\mathcal{G}$. we call such fuzzy metric spaces lebesgue.

Proposition 2.7. Let (E, d) be a metric space. Then (E, d) is lebesgue if and only if (E, τ_d) is lebesgue.

3. Main Results

Theorem 3.1. Let $(E, d,*)$ be a complete fuzzy metric space. Let (T, φ) be a fuzzy $\alpha - \psi$ −contractive mapping from (E, d) into itself satisfies

$$\alpha(x, y, t)M\big({f_{e_0}}^0, (T, \varphi){f_{e_0}}^0, t\big) \geq 1$$

(T, φ) is a fuzzy α −admissible. There exists ${f_{e_0}}^0 FC(E) \in X$ such that

$$\alpha\big({f_{e_0}}^0, (T, \varphi){f_{e_0}}^0, t\big) \leq 1,$$

(T, φ) is Fuzzy continuous.

Then (T, φ) has a unique fixed point, that is, there exists $f_e \in FC(E)$ such that

$$(T, \varphi)f_e = f_e$$

Proof: Let ${f_{e_0}}^0 \in$ such that $\alpha\big({f_{e_0}}^0, (T, \varphi){f_{e_0}}^0\big) \leq 1$.

Define the sequence $\{{f_{e_n}}^n\}$ in (E, d) by

$${f_{e_{n+1}}}^{n+1} = (T, \varphi){f_{e_n}}^n, \forall n \in \mathbb{N}.$$

If ${f_{e_n}}^n = {f_{e_{n+1}}}^{n+1}$, for some $n \in \mathbb{N}$, then $f_e = {f_{e_{n+1}}}^{n+1}$ is a unique fixed point of (T, φ).

Assume that ${f_{e_n}}^n \neq {f_{e_{n+1}}}^{n+1}$, $\forall\ n \in \mathbb{N}$.

Since T is a fuzzy α −admissible,

we have

$$(\alpha, \emptyset)M\big({f_{e_0}}^0, {f_{e_n}}^n\big) = \alpha\big({f_{e_0}}^0, (T, \varphi){f_{e_0}}^0\big) \leq 1.$$

By induction, we get

$$\alpha\big({f_{e_0}}^0, {f_{e_{n+1}}}^{n+1}\big) \leq 1, \forall\ n \in \mathbb{N}. \qquad (3.1.1)$$

Applying the inequality (3.1.1) with $f_e = {f_{e_{n+1}}}^{n+1}$ and $g_e = {f_{e_n}}^{n}$, and using (3.1.1), we obtain

$$d\left({f_{e_n}}^{n}, {f_{e_{n+1}}}^{n+1}\right) = d\left((T,\varphi){f_{e_{n-1}}}^{n-1}, (T,\varphi){f_{e_{n-2}}}^{n-2}, (T,\varphi){f_{e_n}}^{n}\right)$$

$$\leq \alpha({f_{e_{n-1}}}^{n-1})\,(\alpha,\emptyset)({f_{e_{n-2}}}^{n-2})d\left((T,\varphi){f_{e_{n-1}}}^{n-1}, (T,\varphi){f_{e_n}}^{n}\right)$$

$$\leq \alpha({f_{e_{n-1}}}^{n-1})d\left((T,\varphi){f_{e_{n-1}}}^{n-1}, (T,\varphi){f_{e_n}}^{n}\right)$$

$$\leq \alpha d\left((T,\varphi){f_{e_n}}^{n}, (T,\varphi){f_{e_n}}^{n}\right)$$

$$\leq \alpha\psi\left(d({f_{e_{n-1}}}^{n-1}, {f_{e_n}}^{n})\right)$$

By induction, we get

$$d\left({f_{e_n}}^{n}, {f_{e_{n+1}}}^{n+1}\right) \leq \alpha\psi^n\left(d\left({f_{e_0}}^{0}, {f_{e_1}}^{1}\right)\right), \psi^n\left(d\left({f_{e_1}}^{1}, {f_{e_2}}^{2}\right)\right) \ldots, \forall n \in \mathbb{N} \qquad (3.1.2)$$

$$\leq d\left({f_{e_n}}^{n}, {f_{e_{n+1}}}^{n+1}\right) + \sum_{k=n}^{m-1} \xi^d\left({f_{e_k}}^{k}, {f_{e_{k+1}}}^{k+1}\right)$$

$$\leq d\left({f_{e_n}}^{n}, {f_{e_{n+1}}}^{n+1}\right) + \sum_{k=n}^{\infty} \xi^d\left({f_{e_k}}^{k}, {f_{e_{k+1}}}^{k+1}\right)$$

$$\to 0, when\ n \to \infty.$$

From the inequality(3.1.2) and using the triangular inequality and for $n, m \in \mathbb{N}$ with $m > n$,

$$d\left({f_{e_n}}^{n}, {f_{e_m}}^{m}\right) \leq d\left({f_{e_n}}^{n}, {f_{e_{n+1}}}^{n+1}\right) + d\left({f_{e_{n+1}}}^{n+1}, {f_{e_{n+2}}}^{n+2}\right) + \cdots d\left({f_{e_{m-1}}}^{m-1}, {f_{e_m}}^{m}\right)$$

$$= \sum_{k=n}^{m-1} \xi^d(d\left({f_{e_k}}^{k}, {f_{e_{k+1}}}^{k+1}\right))$$

$$\leq \sum_{k=n}^{m-1} \xi^d(d\left({f_{e_k}}^{k}, {f_{e_{k+1}}}^{k+1}\right))$$

$$\leq \sum_{k=n}^{m-1} \xi^d(d\left(\tilde{d}\left({f_{e_0}}^{0}, {f_{e_1}}^{1}\right)\right)$$

$$\leq \sum_{k=n}^{\infty} \xi^d(d\left(\tilde{d}\left({f_{e_0}}^{0}, {f_{e_1}}^{1}\right)\right)$$

Letting $k \to \infty$, we obtain $\{{f_{e_k}}^{k}\}$ is a Cauchy sequence in Fuzzy metric space in (E, d). since (E, d) is complete, there exists ${f_{e_k}}^{k} \in FC(E)\ such\ that\ {f_{e_k}}^{k} \to f_e\ \ as\ n \to \infty$. from the fuzzy continuity of (T, φ),

It follows that

$${f_{e_{k+1}}}^{k+1} = (T,\varphi){f_{e_k}}^{k} \to (T,\varphi)f_e$$

As $n \to \infty$. by the uniqueness of the limit, we get

$$f_e = (T,\varphi)f_e$$

4. Conclusion

In this paper, we presented $\alpha-$contraction functions in complete fuzzy metric space and establish sequential characterization properties of fuzzy metric space. Furthermore, we have provided the significance of the present study and proposed an open problem using the property of Lebesgue fuzzy metric space and give a few models on the side of our outcomes.

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