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Original Article

A Snapshot of Journey of Mathematics

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Abstract *- Mathematics has its roots from the period in which humans have their existence. Mathematics was used as the part of their daily life. This paper focuses over the journey of mathematics from simple use of numbers to the present specialized and complex mathematics. The study tried to provide a snapshot of journey of mathematics from the scribes of Mesopotamians to the 20th century's mathematicians. This paper discussed the seven periods in development of mathematics proposed by Ebrahim as Proto-Mathematics, Ancient Mathematics, Classical Mathematics, Mercantile Mathematics, Pre-Modern Mathematics, Modern Mathematics and Post-Modern Mathematics. A brief history of Mathematics with Indian achievements is also covered in the paper. A concise history of contributions made by major mathematicians in various centuries is discussed briefly.*

Keywords *- Specialized, Complex, Proto mathematics, Ancient Mathematics, Classical mathematics, Modern Mathematics.*

1. Introduction

Webster's Dictionary has defined mathematics as "Mathematics is the science of number and their operations inter relations, combinations, generalizations and abstraction and of space configurations and generalizations." Mishra (2008) Mathematics is an exact science of numbers, space, magnitude and measurement. It explains the abstract concepts. Cooke (2005) divided the thought of mathematics into four categories i.e. number, space, symbols and inference. Counting items with cardinal and ordinal numbers is the simplest form to express numbers. In addition to counting, ability to relate spatial and temporal relations develops reasoning. Symbols are provided to the mathematics by a language. "Symbolic ability is the heart of mathematics". Logic behind mathematical inference is necessary to enhance the understanding. Folkerts (2020) mathematics history originally came from the documents written by scribes in Mesopotamia and Egypt. Mesopotamian mathematics, written on clay tablets, was much higher in order than Egyptian mathematics.

Mathematics has its roots in ancient Mesopotamia and Egypt. Egyptians used the decimal numeration system to develop the multiplication through additive property. eg. To multiply with 13, they multiply the number by 1, then by 2, then by 4 and then by 8 and then add the multiplications got by multiplying by 1, 4 and 8 to get the results.

13 X 12

1 X 12 = 12

2 X 12 = 24

4 X 12 = 48

8 X 12 = 96

12 + 48 + 96 = 156

They also worked great on calculus of fractions. Egyptians developed the formula for the value of frustum of a square pyramid as $V = (h/3)(a^2+ab+b^2)$. a and b are the heights of a square and h is the height. Egyptians used a unit by new symbol while Sumerians used symbols but used position for higher unit and gave position value system eg. $343= 3 \text{ X } 10^2 + 4 \text{ X } 10 + 3 = 343$. The next came Babylonian Dynasty under king Hammurabi. They solved linear and quadratic equations in two variables. Arithmetic was done with the help of multiplication tables to list of reciprocals of squares and cubes. $\sqrt{2}$ was given by $1*5/1\sqrt{2}$ i.e. 1.4142 and $1*5/12 = 1.4167$ and $1/\sqrt{2} = 7071$ by $17/24= .7083$. Square roots were calculated as $\sqrt{A} = \sqrt{a2} + \sqrt{h} = a + \frac{h}{2a} = \frac{1}{2}\left(a + \frac{A}{a}\right)$. Babylonian astronomy influenced the Greek astronomy and Babylonian mathematics influenced computational arithmetic. Chinese numeration used decimals and numbers were expressed by symbols made up by bamboo sticks. 𝍠 𝍩 = 𝍩𝍩 which meant 6729. Counting operations were put on boards with blank spaces as zero. Chinese mathematics proved to be better than that of Egyptian and Babylonian mathematics.

Thales of Miletus considered as the traditional father of Greek mathematics visited Egypt and Babylon leads to the circumstances under which modern mathematics flourished. Goal of early Greek mathematics was "the understanding of man's place in the universe according to a rational scheme". In 450 BCE, ancient Greek numeral system, known as "Attic or Herodianic numerals" were developed. Thales among the "seven sages of Greek" developed Thale's Theorem and intercept theorem. Pythagoras of Greek emerged as a well-known mathematician in history. Pythagoras theorem was named after him. Zeno of Elea in 5th Century gave Dicotomy Paradox related to Achilles and Tortoise. Democritus produced his work named "On Numbers, On Geometrics, On Tangencies, On Mapping and On Irrationals". He observed that a cone is 1/3 the volume of a cylinder with same base and height. "Method of Exhaustion" was developed by Eudoxus of Cnidus. He also developed a theory of proportion applicable to rational and irrational magnitudes. Greek mathematicians worked to create the deductive method for proving or disproving any theorem through assumed axioms. Islamic mathematics from 8th century onwards, fused Indian and Greek mathematics together. They developed all the known symmetry forms those can be depicted on a 2 dimensional surface. Muhammad Al-Khwarizmi strongly supported the Hindu numerical system i.e. 0 and 1-9. Al-Karaji worked on algebra and developed algebraic calculus. He used mathematical induction method to prove his results and declared that if first statement in the infinite sequence of statements is true then the other statements in sequence would be true. He used induction method to prove Binomial Theorem. Nasir-Al-Din Al- Tusi during 13th century depicted trigonometry as a separate discipline than astronomy. He also developed extensive exposition of spherical trigonometry with six case of triangle in spherical trigonometry. He also gave law of sines for plane triangles as $^{a}/_{(\sin A)} = {}^{b}/_{(\sin B)} = {}^{c}/_{(\sin C)}$. Abul Hasan al-Uqlidisi from 10th century gave the positional use of Arabic numerals. Persian Ibn al-Haytham in 11th century stated the Alhazen's proble and depicted the formula for the sum of the fourth powers. Persian Kamal al-Din al-Farisi from 13th century solved optical problems by using theory of conic sections. Moroccan Ibn al-Banna al-Marrakushi during 13th century discovered the new pair of amicable numbers. Super magic square developed by Albercht Durer as "Melencolia". Simon Stevin made popularized the use of decimal fractions and decimal arithmetic. Descartes proposed Cartesian coordinate system that each point on a plane can be represented by two numbers dividing the plane in four quadrants with positive and negative values. He also developed "Rule of Signs" for "determining the number of positive or negative real roots of a polynomial". Pascal is famous for "Pascal's Triangle of binomial coefficients". Newton along with Gottfried Leibniz developed infinitesimal calculus through differentiation and integration. Leibniz's developed a mechanical forerunner for computer and solved linear equations with the use of matrices. During 17th century, Bernoulli with Jacob and Johan developed "Calculus of Variations". braham de Moivre gave de Moivre's formula, $(\cos x + i\sin x)^n = \cos(nx) + i\sin(nx)$, for complex numbers and trigonometry. Lagrange gave Lagrange's Mean Value Theorem for base to 19th century mathematics. During 18th century, Legendre proposed six polynomials as solutions to Legendre's deferential equation. Christian Goldbach proposed the Goldbach Conjecture, which proposed that every even integer > 2 can be expressed as the sum of two primes (e.g. 4 = 2 + 2; 8 = 3 + 5; 14 = 3 + 11 = 7 + 7; etc) or every integer > 5 can be expressed as the sum of 3 primes. Also "**weak**" Goldbach Conjecture, stated that all odd numbers >7 are the sum of 3 odd prime numbers. Goldbach also proved the theorem named Goldbach-Euler Theorem on perfect powers. During the 19th century, Joseph Fourier proposed Fourier series. Jean-Robert Argand, in his paper proposed how "complex numbers (of the form a + bi, where i is $\sqrt{-1}$) could be represented on geometric diagrams and manipulated using trigonometry and vectors". Evariste Galois stated that it is impossible for solving polynomials equations of degree > 4 through algebraic method. Gauss and Riemann worked on non-Euclidean geometry or elliptic geometry. Charles Babbage developed "difference engine" to calculate logarithms and trigonometric functions. George Peacock and George Boole developed symbolic algebra and Boolean algebra respectively. William Hamilton in 1843 developed theory of quaternions. Englishman Arthur Cayley developed octonions with the help of Hamilton's quaternions. He was a pioneer of "higher dimensional geometry, matrix algebra, modern group theory, theory of invariants and the theory of higher singularities. Cauchy proved Cauchy's theorem of group theory in 1845. Carl Jacobi contributed to determinants and matrices, analysis, and theory of periodic functions and elliptic functions and their relation to the elliptic theta function. August Ferdinand Möbius discovered Möbius strip, homogeneous coordinates and discussed geometric and projective transformations. Felix Klein made contributions in non-Euclidean geometry. Marius Sophus lie developed the theory of continuous symmetry. In 1866, Niccolo Paganini discovered smallest pair of amicable numbers i.e. 1, 184 and 1210. Georg Cantor developed the set theory. John Venn developed Venn diagram in 1881. G. H. Hardy and Srinivasa Ramanujan proved themselves to be the pioneers of mathematics of early 20th century. Johann Gustav Hermes constructed a regular polygon with 65,537 sides ($2^{16} + 1$), using a compass and straight edge as Euclid in over ten years. David Hilbert set 23 Hilbert problems out of which 10 are solved, 7 are partially solved, 2 are still to be solved and 4 loosely stated to be open as solved or unsolved. Andrey Kolmogorov developed modern axiomatic foundations of probability theory in the 1930s**.** Alan Turing, Kurt Gödel, Hermann Weyl, Andre weil, John von Neumann, Albert Einstein, , Alexander Grothendieck, , Benoît Mandelbrot, Julia, Claude Shannon and Mandelbrot are some of the pioneers of 20th century.

Ebrahim (2010) has identified seven periods in development of mathematics as Proto-Mathematics, Ancient Mathematics, Classical Mathematics, Mercantile Mathematics, Pre-Modern Mathematics, Modern Mathematics and Post-Modern Mathematics. Proto-Mathematics lies between 30000- 2000 BCE. It includes basic and empirical mathematics for counting, shape and symmetry, building and agriculture. Ancient Mathematics Period lies between 2000-800 BCE. It

includes highly sophisticated science by Babylonians and the Egyptians. Mathematics was now used in astronomy, land cultivation, administration, time estimation, calculation of areas and volumes, engineering of monuments, planning and logistics etc. Classical Mathematics Period ranges from 800 BCE to 1500 CE. Now the mathematical notions shifted from empirical to logical under Greeks. Calculus solved the problems of curvilinear and solid geometry. Greeks discovered rational numbers to describe the length. Algebra, developed by Babylonians and Egyptians, flourished in Islamic era under Central Asian, Indian and Arabic mathematicians. Mercantile Mathematics lies from 1400-1500 CE. It leads to development in numeration and algebra equations. Symbolic mathematics was developed. Solution up to 5 degree of polynomials was tackled here. The notion of function was developed by Galileo and its application in geometry by Descartes and Fermat. It further expanded to analytic functions of trigonometry, exponential functions and logarithms. It further leads to development of differential calculus and integral calculus. The Pre-Modern Period lies from 1500-1700 CE. This era leads to improvement in analytic geometrical mathematics and symbolic algebra. Modern Mathematics Period ranges from 1700-1950 CE. This period characterized by generalization, unification and synthesis of all mathematics. It includes modern algebra. Galios theory solved the unsolved problems of classical algebra, classical geometry and theory of equations. Major mathematicians of this era were Weierstrass, Bolzano, Dedekind, Cauchy, Reimann, Hilbert, Cantor Poisson, Laplace, Fourier, Gauss and Lagrange. Post-Modern Period lies from 1950 CE to present. It includes the modern algebra, modern geometry, axiomatized modern abstractions.

2. Mathematics in India

In India, Indus valley civilization is considered to be the oldest civilization between 3300 BC-1700 BC and peaked between 2600-1900 BC. Evidences have been found to support that Indus valley civilization used binary and decimal ratios. Mohanjo-Daro also affirms the use of mathematics. Vedas also described some information about mathematics in form of addition, subtraction, fraction, multiplication, squares, roots and cubes. In 2nd or 3rd BCE, five types of infinites were given by Jain mathematicians as "infinite in one direction, in two directions, in area, infinite everywhere and perpetually infinite". Buddhist Literature depicts three types of numbers as countable numbers, uncountable numbers and infinite. Decimal place value number system developed in India before 3rd BCE. A report of 4th century BCE listed numbers up to 10^{53} along with 6 different number systems describing numbers up to 10^{421}. Somewhere in 3rd century in Bakhshali Manuscript place value system was recognized. Shatapatha Brahmana enumerates the knowledge of geometry and observational astronomy. In 8th century BCE, "Sulba Sutra", a text enumerating Pythagorean theorem in simplified form for sides of square and rectangle was found. It also contains geometric solutions of quadratic and linear equations for one unknown unit along with square root of 2 as 1.4142156. Indian mathematicians focused on applied mathematics in astronomy predictions and philosophical indirect arguments. Sulba-Sutra describes the dimensions of bricks and altars. The standard units used were "angula", "elbow length" and the "man height". Angula depicts fourteen millet grains, elbow length depicts twenty-four digits while man height describes as five cubits. Baudhayana sulba sutra describes the creation of right angled corners of a square and a rectangle. It further recognizes the Pythagorean theorem as "the square on the diagonal of a given square contains twice the original area and squares on the width and length of any rectangle add up to the square on its diagonal". Complete mathematics and negative numbers are also found in Baudhayana sulba sutra. Pulverizer is also present in all the Indian manuscripts which is an extension of Euclidean algorithm. Other major work was Brahamgupta's formula for quadrilateral inscribed in a circle. He also discovered multiplicative property for the solutions of c1 and c2 in Pell's equation. Aryabhata worked on approximation of π and gave that the circumference of a circle will be 62832 whose diameter is 20000. Bhaskara I in 7th century contributed the numbers in Hindu decimal system. He gave approximation formula for sin x in Mahabhaskariya. Bhaskara II in 12th century contributed the solutions of cubic, quadratic and quartic indeterminate equations in "Lilavati". In Lilavati he wrote about mansuration, trigonometry, negative numbers, rule of three, surds and properties of zero. In Bijganita, Bhaskara II wrote firstly about positive and negative square root of numbers. In Ganitadhyaya, he expanded the work of Aryabhata and Brahamgupta. He described the length of year a minute off its actual value. In Goladhyaya, Bhaskara II wrote about spheres and its similar shapes. The book contained a sin table along with relations between various trigonometric functions. He gave the formula for solving the quadratic equation as $X = -b/a \pm \text{sqrt}(b^2-4ac)/a$. He also gave formula for finding the square root of the sum of a number and a square root as $\text{Sqrt}(a \pm \text{sqrt}(b)) = \text{sqrt}(½(a + \text{sqrt}(a^2 – b)) \pm \text{sqrt}(½(a – \text{sqrt}(a^2 – b))$. Today's famous formulas "sin(a + b) = sin(a) cos(b) + cos(a) sin(b) and sin(a – b). sin(a) cos(b) – cos(a) sin(b)" got their roots in Goladhyaya. He noticed that dividing a nuber by smaller fraction gives a larger number eg $1 \div \frac{1}{2} = 2$ and $1 \div 0 = \infty$. Indian astronomers used trigonometric tables to find the distance between earth and sun and earth and moon and their sizes. Madhava of Sangamagrama in 14th century developed infinite series approximations for trigonometric functions. He founded Kerala School of Astronomy. He founded the solutions of transcendental equations through iteration and found solutions of transcendental numbers by continued fractions. Bakhshali manuscript provides valuable record of ancient mathematical records. It is estimated to be written 400 years ago. It was found in 1881 by a local farmer. It contains decimal system, zeros, decimal calculations and square roots. Indian gave a number of pioneer and remarkable mathematicians to the ages who served as the catalysts in the development of mathematics round the globe.

3. Conclusion

Mathematics today we see has covered a long journey early from ancient times to the modern era. Ancient Mathematics covered Mesopotamia, Egypt and Greek mathematics. Medieval Mathematics includes mathematics from China, India, Islam, and Europe and from America, Africa & the Pacific. Early Modern Mathematics as developed 16th and Modern Mathematics exists somewhere from 18th century till present. Ancient mathematics was as simple as arithmetic and numbers. With the passing era more concepts and notion added up to form it more complex. Mathematics is a subject which is directly or indirectly associated with other subjects like physics, Biology, chemistry, computer Science, Psychology, etc. Therefore, the advancement in mathematics leads to development in other disciplines also. Mathematicians round the globe worked in their respective domains and contribute in such remarkable achievements in this field.

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