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*Original Article*

# Optimal Control on Smoking Behavior Spreading Model with Education, Treatment, and Psychological Support

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***Abstract*** *- The spreading phenomena of smoking behavior in the human population can be mathematically modeled. This research discusses the smoking behavior spreading model that consist of six subpopulations. The subpopulation of potential smokers is divided into two types, the first type is potential smokers who have not been educated, the second type is potential smokers who have been educated. The subpopulation of smokers consists of a light smokers subpopulation and a heavy smokers subpopulation. The subpopulation of smokers who quit smoking is divided into two types, the first type quit temporarily and the second type quit permanently. The model describes the rate of change of each subpopulation by being given three control variables, they are education, treatment, and psychological support which aimed to minimize light smokers subpopulation, heavy smokers subpopulation, and control implementation cost. Optimal control completion is using Pontryagin's minimum principle. Then, the simulation is carried out by using the method of forward-backward sweep. The simulation results of numerical indicate that the application of combination oversee strategies education, treatment, and psychological support is effective to control the spread of smoking behavior and its implementation cost.*

***Keywords*** *- Forward-backward sweep method, Numerical simulation, Optimal control, Pontryagin's minimum principle, Smoking behavior model.*

## 1. Introduction

The third country with the biggest number of smokers in the world after China and India is Indonesia, based on WHO data [9]. The Increasing of cigarette consumption affects the raising of diseases and mortality due to smoking. Cigarette contains of 4,800 chemicals with major constituents such as tar, nicotine and carbon monoxide [13]. Nicotine makes smokers addicted to continue smoking, it creates pleasant feeling. More often a person smokes, it is higher the chemical content of cigarettes entering the body. It can cause various diseases, such as types of cancer which is related by respiratory tract through the lungs, bladder disease, pregnancy disorders, vascular diseases such as liver and stroke, etc [14].

Some scientist have been developed more of mathematical models of smoking [1, 3, 6-8, 10]. The mathematical model of smoking behaviour was introduced firstly by [7]. They divided the population into three subpopulation classes, including subpopulations that are prone to smoking (potential smokers), subpopulation of smokers and subpopulations who are doing treatment or quitting smoking. The models from previous studies were developed by [10] to determine smoking frequencies variability by dividing in two types of smokers namely light smokers and heavy smokers, and the public health impacts such diseases caused by smoking. Zainab Alkhudhari et al [15-17] adopted smoking behaviour models by Sharomi and Gumel with four subpopulations, they are potential smokers, active smokers, smokers who quit smoking temporarily, and quit permanently. In the same year, they updated the previous model by considering the occasional smokers effect on potential smokers. A similar model was introduced in 2015by examining the heavy smokers effect on potential smokers [18]. Furthermore, Labzai et al. [5] studied and investigated the optimal control strategies of smoking behaviour models with discrete time and introduce saturated incident rate. Labzai et al used three control strategies namely education [2, 11], treatment, and psychological support to determine the optimal strategy for reducing the amount of light smokers, heavy smokers and smokers who temporarily quit smoking.

Based on previous researches, this research will discuss about optimal control on the model of smoking behaviour by constructing a model based on Alkhudhari et al research by dividing potential smokers subpopulation into two subpopulations namely potential smokers have not been educated and have been educated. The controls used are three controls based on the research of Labzai et al. The purpose of the research was to reduce the subpopulation of light smokers subpopulation, heavy smokers subpopulation, and the implementation costs of education, treatment, and psychological support. The optimum control problem is solved based on the Pontryagin's minimum principle [12]. Numerical simulation uses MATLAB with a forward-backward sweep method. Next, the simulation results were analyzed to find out the most effective control strategies in supervising the increasing amount of smokers.

## 2. Research Method

The smoking behavior spreading model was constructed by involving control education $(u)$, treatment $(v)$, and psychological support $(w)$. Next, an optimal control problem was carried out with the following steps.

- Construct a smoking behavior spreading model with education, treatment, and psychological support as controls.
- Define the objective function to reduce the amount of light smokers subpopulation, heavy smokers subpopulation, and control implementation cost.
- Use Pontryagin's minimum principle to resolve the optimal control problem.
- Determine the state, costate, and optimal conditions.
- Perform numerical simulations using the forward-backward sweep method using MATLAB software.

## 3. Result and Discussion

### *3.1. Construction Model*

In this part, we discuss a smoking behavior spreading model with education, treatment, and psychological support. The subpopulation was divided into six subpopulations: potential smokers subpopulation who have not been educated $(P_1)$, potential smokers subpopulation who have been educated $(P_2)$, light smokers subpopulation $(L)$ who smoked less than 12 cigarettes in a day, heavy smokers subpopulation $(S)$ who smoked more than 12 cigarettes in a day, quit temporarily smokers subpopulation $(Q_t)$ consist of people who temporarily quit smoking, and quit permanently smokers $(Q_p)$ consist of individuals who permanently quit smoking. There are three controls namely $u$, $v$, and $w$. The first control presents the proportion to be used for awareness program in education, thus we record as $uP_1$ is the comparison of the potential smoker people who changed to the potential smoker who have been educated. The second control shows the proportion to be applied for subjected to treatment, we entry $vS$ is the comparison of people who will change from the heavy smoker subpopulation to the people who permanently quit smoking. The third control describes the proportion to get psychological support, the sign $wQ_t$ is the proportion of people who temporarily quit smoking and who will change into people who permanently quit smoking. The model can be written in the form of differential equations:

$$
\begin{aligned}
\frac{dP_1(t)}{dt} &= A - \mu P_1(t) - \beta_1 P_1(t)L(t) - u(t)P_1(t), \\
\frac{dP_2(t)}{dt} &= u(t)P_1(t) - \beta_2 P_2(t)L(t) - \mu P_2(t), \\
\frac{dL(t)}{dt} &= \big(\beta_1 P_1(t) + \beta_2 P_2(t)\big)L(t) - \beta_3 L(t)S(t) - \mu L(t), \qquad (1) \\
\frac{dS(t)}{dt} &= \beta_3 L(t)S(t) + \alpha Q_t(t) - \big(\mu + \gamma + v(t)\big)S(t), \\
\frac{dQ_t(t)}{dt} &= \gamma(1-\sigma)S(t) - \big(\mu + \alpha + w(t)\big)Q_t(t), \\
\frac{dQ_p(t)}{dt} &= \sigma\gamma S(t) + v(t)S(t) + w(t)Q_t(t) - \mu Q_p(t).
\end{aligned}
$$

Based on the form above, the diagram of the model can be described as in Figure 1. The description of the parameters used in the model is presented in Table 1. We used just arbitrary academic data.

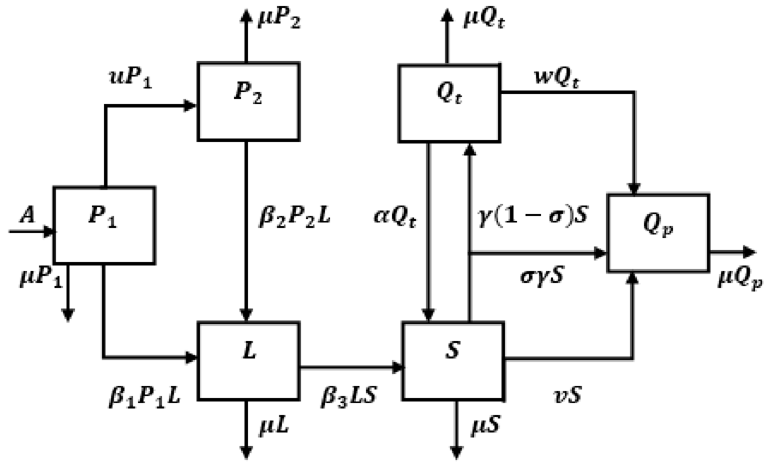



**Fig. 1 Flow diagram of the model**

### *3.2. Objective Function*

The aim of optimal control in this research was to reduce the amount of light smokers subpopulation, heavy smokers subpopulation, and the charge expended in applying the three control strategies, namely

$$J\big(u(t),v(t),w(t)\big)=\int_0^T\left[L(t)+S(t)+\frac{1}{2}a_1u^2(t)+\frac{1}{2}a_2v^2(t)+\frac{1}{2}a_3w^2(t)\right]dt. \quad (2)$$

Based on equation above, we will look for $u^*$, $v^*$, and $w^*$ which result in minimizing the objective function value that is

$$J\big(u^*(t),v^*(t),w^*(t)\big)=\min\big\{J\big(u(t),v(t),w(t)\big)\big|u,v,w\in U\big\}, \quad (3)$$

with

$$U=\{(u,v,w)|0\le u(t)\le 1, 0\le v(t)\le 1, 0\le w(t)\le 1, t\in[0,T]\}. \quad (4)$$

**Table 1. Parameter description and estimate values**

| Parameter | Description | Value |
|---|---|---|
| $A$ | The birth rate | 0.0004 |
| $\mu$ | The death rate | 0.003 |
| $\alpha$ | The contact rate which $Q_t$ who revert back to $S$ | 0.05 |
| $\beta_1$ | Transmission rate from $P_1$ to $L$ | 0.04 |
| $\beta_2$ | Transmission rate from $P_2$ to $L$ | 0.02 |
| $\beta_3$ | Transmission rate from $L$ to $S$ | 0.01 |
| $\gamma$ | The rate of quitting smoking | 0.02 |
| $\sigma$ | Probability of $L$ can be permanently quit smoking | 0.5 |

### *3.3. Hamiltonian Function*

In purpose to obtain the obligatory condition for the best control, the Hamiltonian function defined by

$$\begin{aligned} H\big(P_1,P_2,L,S,Q_t,Q_p,u,v,w,\lambda\big) = {} & L(t)+S(t)+\frac{1}{2}c_1u^2(t)+\frac{1}{2}c_2v^2(t)+\frac{1}{2}c_3w^2(t)\\ & +\lambda_1\big(A-\mu P_1(t)-\beta_1P_1(t)L(t)-u(t)P_1(t)\big)\\ & +\lambda_2\big(u(t)P_1(t)-\beta_2P_2(t)L(t)-\mu P_2(t)\big)\\ & +\lambda_3\Big(\big(\beta_1P_1(t)+\beta_2P_2(t)\big)L(t)-\beta_3L(t)S(t)-\mu L(t)\Big)\\ & +\lambda_4\Big(\beta_3L(t)S(t)+\alpha Q_t(t)-\big(\mu+\gamma+v(t)\big)S(t)\Big)\\ & +\lambda_5\Big(\gamma(1-\sigma)S(t)-\big(\mu+\alpha+w(t)\big)Q_t(t)\Big)\\ & +\lambda_6\big(\sigma\gamma S(t)+v(t)S(t)+w(t)Q_t(t)-\mu Q_p(t)\big). \end{aligned} \quad (5)$$

### *3.4. Optimal Control Problem*

**Theorem 1:** Provided an optimal control $u^*(t), v^*(t), w^*(t) \in U$ and the solutions $P_1^*(t)$, $P_2^*(t)$, $L^*(t)$, $S^*(t)$, $Q_t^*(t)$, and $Q_p^*(t)$ of the corresponding conditions system (1) and (2), there exist costate functions $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$, $\lambda_5$, and $\lambda_6$ that satisfying

$$\begin{aligned}
\frac{d\lambda_1}{dt} &= \lambda_1\mu + \lambda_1\beta_1 L + \lambda_1 u - \lambda_2 u - \lambda_3\beta_1 L \\
\frac{d\lambda_2}{dt} &= \lambda_2(\beta_2 L + \mu) - \lambda_3\beta_2 L \\
\frac{d\lambda_3}{dt} &= -1 + \lambda_1\beta_1 P_1 + \lambda_2\beta_2 P_2 - \lambda_3(\beta_1 P_1 + \beta_2 P_2 - \beta_3 S - \mu) - \lambda_4\beta_3 S \\
\frac{d\lambda_4}{dt} &= -1 + \lambda_3\beta_3 L - \lambda_4(\beta_3 L - \mu - \gamma - v) - \lambda_5\gamma(1-\sigma) - \lambda_6(\sigma\gamma + v) \\
\frac{d\lambda_5}{dt} &= -\lambda_4\alpha + \lambda_5(\mu + \alpha + w) - \lambda_6 w \\
\frac{d\lambda_6}{dt} &= \lambda_6\mu,
\end{aligned} \tag{6}$$

with the transversality conditions at time $T$

$$\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = \lambda_4(T) = \lambda_5(T) = \lambda_6(T) = 0.$$

Furthermore, the optimal control $u^*(t)$, $v^*(t)$, and $w^*(t)$ are given by

$$\begin{aligned}
u^*(t) &= \min\{max\{0, \bar{u}(t), 1\}\} = \min\left\{max\left\{0, \frac{(\lambda_1-\lambda_2)P_1^*(t)}{c_1}, 1\right\}\right\}, \\
v^*(t) &= \min\{max\{0, \bar{v}(t), 1\}\} = \min\left\{max\left\{0, \frac{(\lambda_4-\lambda_6)S^*(t)}{c_2}, 1\right\}\right\}, \\
w^*(t) &= \min\{max\{0, \bar{w}(t), 1\}\} = \min\left\{max\left\{0, \frac{(\lambda_5-\lambda_6)Q_t^*(t)}{c_3}, 1\right\}\right\}.
\end{aligned} \tag{7}$$

**Proof:**

The optimal control $u, v, w$ are resolved from the optimal control condition, we obtain

$$\frac{\partial H}{\partial u} = c_1 u - \lambda_1 P_1 + \lambda_2 P_1 = 0,$$

$$\frac{\partial H}{\partial v} = c_2 v - \lambda_4 S + \lambda_6 S = 0,$$

$$\frac{\partial H}{\partial w} = c_3 w - \lambda_5 Q_t + \lambda_6 Q_t = 0,$$

which we discover

$$\bar{u}(t) = \frac{(\lambda_1-\lambda_2)P_1^*(t)}{c_1}, \ \bar{v}(t) = \frac{(\lambda_4-\lambda_6)S^*(t)}{c_2}, \bar{w}(t) = \frac{(\lambda_5-\lambda_6)Q_t^*(t)}{c_3}. \tag{8}$$

Given that $u(t), v(t), w(t) \in [0,1]$, thus we get the equation as follows

$$\begin{aligned}
u^*(t) &= \begin{cases} 0, & \bar{u}(t) < 0 \\ \bar{u}(t), & 0 \le \bar{u}(t) \le 1 \\ 1, & \bar{u}(t) > 1 \end{cases}, \\
v^*(t) &= \begin{cases} 0, & \bar{v}(t) < 0 \\ \bar{v}(t), & 0 \le \bar{v}(t) \le 1 \\ 1, & \bar{v}(t) > 1 \end{cases}, \\
w^*(t) &= \begin{cases} 0, & \bar{w}(t) < 0 \\ \bar{w}(t), & 0 \le \bar{w}(t) \le 1 \\ 1, & \bar{w}(t) > 1 \end{cases}.
\end{aligned} \tag{9}$$

By the bound in $U$ of the controls, it is convenient to get $u^*(t)$, $v^*(t)$, and $w^*(t)$ in the form (7).

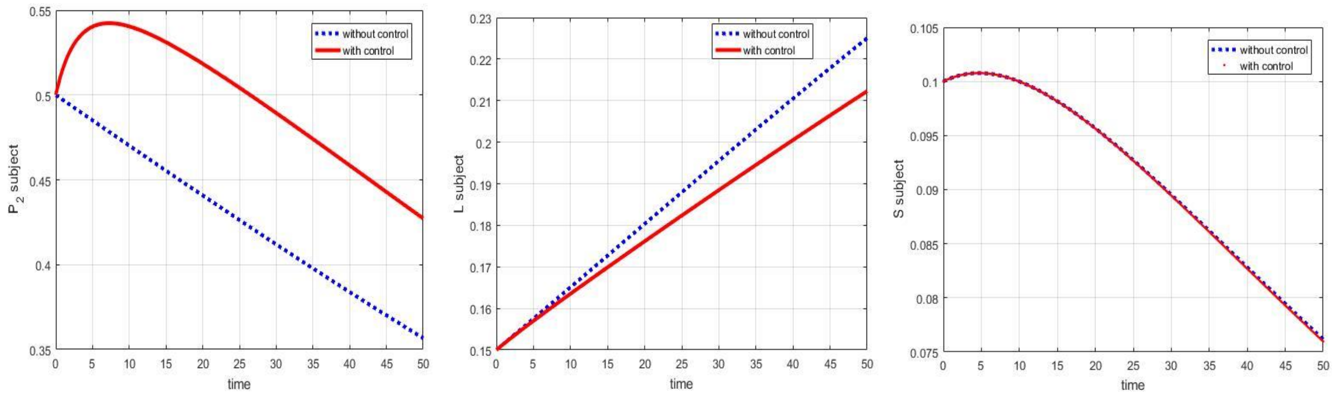
### *3.5. Numerical Method and Simulations*

In this part, we discuss and analyze numerically the optimal control strategies effect like education, treatment, and psychological support. The initial value is assumed as presented in Table 2.

**Table 2. Initial value**

| $P_{1_0}$ | $P_{2_0}$ | $L_0$ | $S_0$ | $Q_{t_0}$ | $Q_{p_0}$ |
|---|---|---|---|---|---|
| 0.1 | 0.5 | 0.15 | 0.1 | 0.05 | 0.07 |

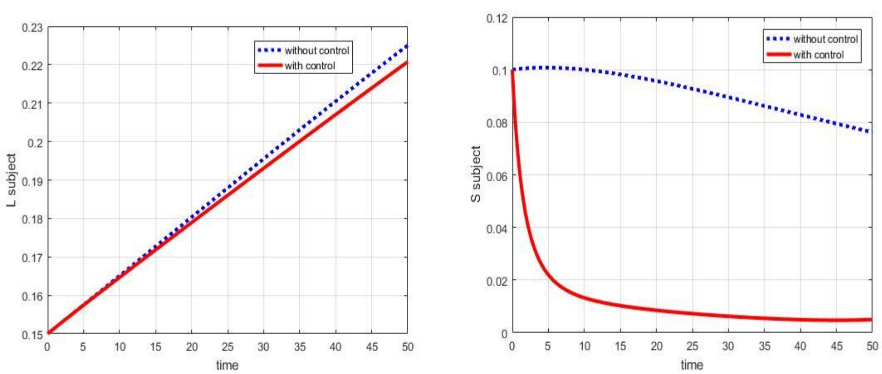
**Strategy A:** Only education control $u \neq 0$. Figure 2(b) opposes the development of light smokers subpopulation with and without control $u$ which the proposed control effect is verified to be positive in decreasing the amount of light smokers subpopulation. But, in Figure 2(c) the number of heavy smokers subpopulation shows no difference when it is given only education control and no control. There is a prominent increase in the amount of potential smokers subpopulation who have been educated in Figure 2(a).




**(a)** **(b)** **(c)**

**Fig. 2 Evolution of different subpopulations with control and without control $u$**

**Strategy B:** Combine education and treatment control strategy. In Figure 3(b), there is an important decrease in the amount of heavy smokers subpopulation when it is given two controls. The cause of this significant decrease is confirmed by the validity of heavy smokers decide to quit smoking after got treatment program.




**(a)** **(b)**

**Fig. 3 Evolution of different subpopulations with control and without control $u$ and $v$**

**Strategy C:** Combine education and psychological support control strategy. The result is an tangible significant decrease in the amount of light and heavy smokers subpopulation from Figure 4(a&b). But, in Figure 4(c) the amount of smokers who permanently quit smoking subpopulation shows the significant increase. The cause of this significant advance is confirmed by the reality that smokers who temporarily quit smoking subpopulation decide to quit smoking permanently after got psychological support program.

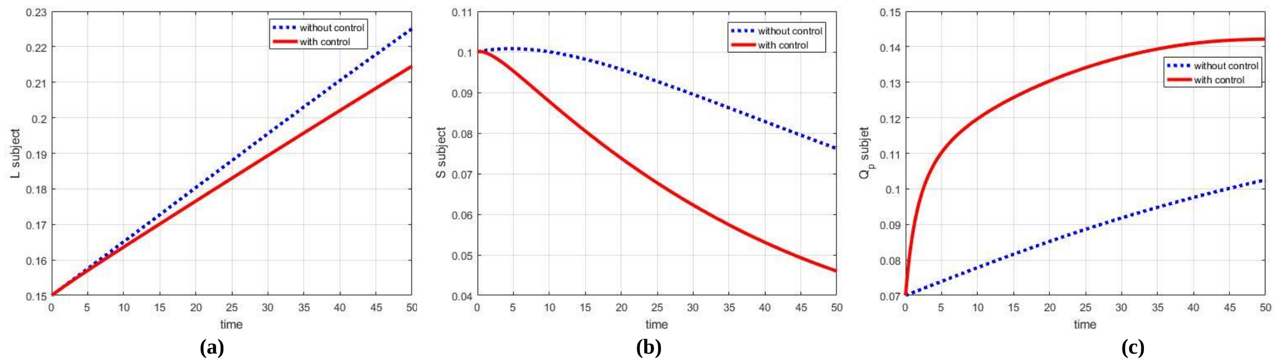



**Fig. 4 Evolution of different subpopulations with control and without control $u$ and $w$**

**Strategy D:** Combine education, treatment, and psychological support control strategy. In this strategy, combining the three controls to receive better result. In Figure 5(c&d), the numbers of light and heavy smokers subpopulation was decreased. In addition, there are significant increase in the amount of potential smokers subpopulation who have been educated and subpopulation of smokers who permanently quit smoking which leads to satisfactory result.

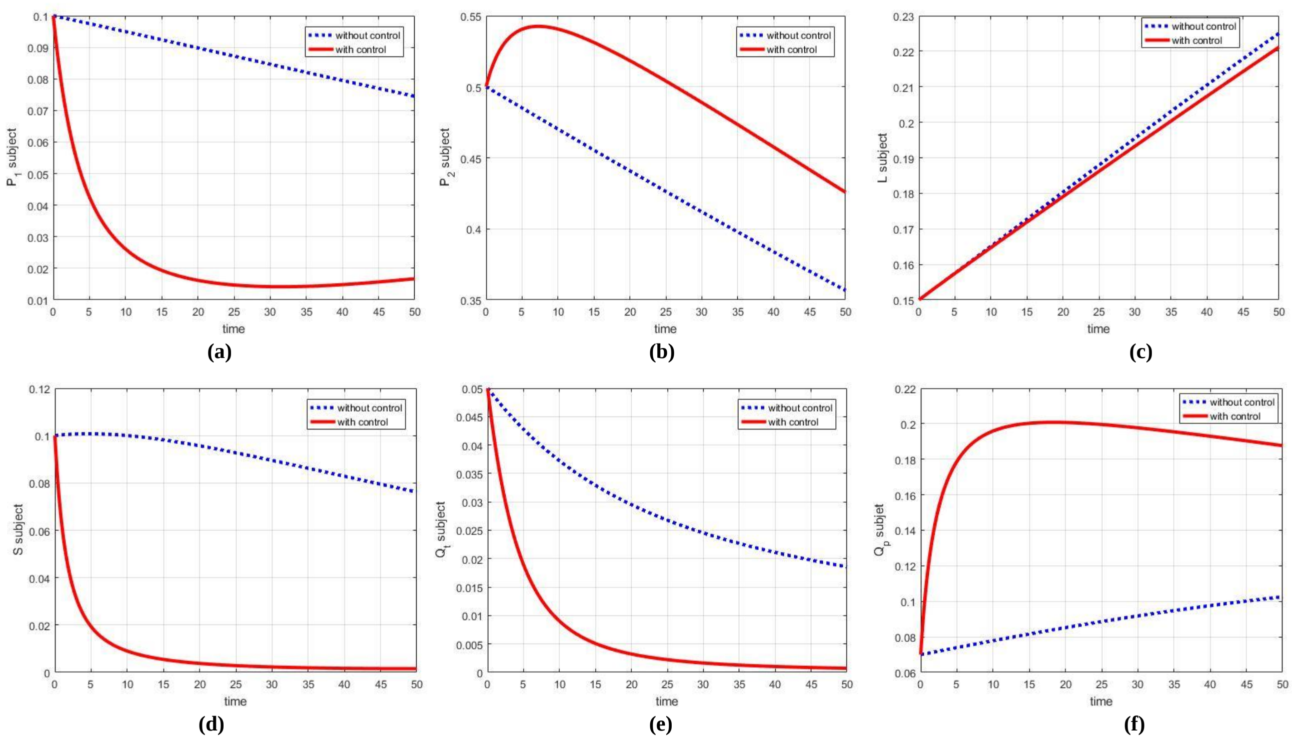



**Fig. 5 Evolution of different subpopulations with control and without control $u$, $v$, and $w$**

The purpose of applying optimal control in this research to minimize light smokers subpopulation, heavy smokers subpopulation, and control implementation cost found in the objective function $J$ in form (2). The value of the objective function is affected by the increase and decrease of the number of light smokers subpopulation, heavy smokers subpopulation, and controls. The minimum objective function value is equal to 22.965 from Table 2, when combination of three control strategies applied for controlling the spread of smoking behavior.

## 4. Conclusion

In this paper, an optimal control problem for the model of smoking behavior spreading has been investigated. Three controls function in this model $u$, $v$, and $w$ represent the percentage of education, treatment, and psychological support. Based on the result of this research, the practices of three controls showed the effectiveness in reducing the amount of light smokers subpopulation $L$, heavy smokers subpopulation $S$, and its implementation cost. In addition, the administration of this control also leads to increased subpopulations of potential smokers who have been educated and smokers who quit smoking permanently.

**Table 2. The objective function vaule for $t = 50$**

| Strategy | Objective Function |
|---|---|
| with control $\boldsymbol{u(t)}$ ($\boldsymbol{c_1 = 0.5}$) | 26.313 |
| with control $\boldsymbol{u(t)}$ and $\boldsymbol{v(t)}$ ($\boldsymbol{c_1 = c_2 = 0.5}$) | 23.142 |
| with control $\boldsymbol{u(t)}$ and w$\boldsymbol{(t)}$ ($\boldsymbol{c_1 = c_3 = 0.5}$) | 25.637 |
| with control $\boldsymbol{u(t), v(t)}$, and $\boldsymbol{w(t)}$ ($\boldsymbol{c_1 = c_2 = c_3 = 0.5}$) | 22.965 |

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