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Original Article

Neighborhood Sombor Indices

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Abstract *- A molecular graph is a simple graph related to the structure of a chemical compound. In this paper, we introduce the modified neighborhood Sombor index and the modified neighborhood Sombor exponential of a graph. Also we compute the neighborhood Sombor and modified neighborhood Sombor indices and their corresponding exponentials of some important dendrimers. Some properties of the neighborhood Sombor index are obtained.*

Keywords - *Neighborhood Sombor index, Modified neighborhood Sombor index, Dendrimer.*

Mathematics Subject Classification: *05C69, 05C07, 05C35.*

1. Introduction

All graphs considered here are finite, undirected without isolated vertices, loops and multiple edges. For all further notation and terminology we refer the reader to [1].

A molecular graph is simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edge to the bonds between atoms.

Let $S_G(u)$ denote the sum of the degrees of all neighborhood vertices of a vertex u in G.
In [2], Graovac et al. defined the following indices:

$$N_1(G)=\sum_{uv\in E(G)}\left(S_G(u)+S_G(v)\right), \qquad N_2(G)=\sum_{uv\in E(G)}S_G(u)S_G(v).$$

In [3], Kulli defined the neighborhood Sombor index and the neighborhood Sombor exponential of a graph G as

$$NSO(G)=\sum_{uv\in E(G)}\sqrt{S_G(u)^2+S_G(v)^2},$$

$$NSO(G,x)=\sum_{uv\in E(G)}x^{\sqrt{S_G(u)^2+S_G(v)^2}}.$$

Recently, some Sombor indices were studied, for example, in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

We introduce the modified neighborhood Sombor index of a graph G and defined it as

$$^{m}NSO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{S_G(u)^2+S_G(v)^2}}.$$

Considering the modified neighborhood Sombor index, we define the modified neighborhood Sombor exponential of a graph G as

$$^{m}NSO(G,x)=\sum_{ue}x^{\frac{1}{\sqrt{S_G(u)^2+S_G(v)^2}}}.$$

The forgotten topological index was studied by Furtula et al. in [21] and it is defined as

$$F(G)=\sum_{uv\in E(G)}\left[d_G(u)^2+d_G(v)^2\right].$$

The F-neighborhood index of a graph G is defined as

$$NF(G)=\sum_{uv\in E(G)}\left[S(u)^2+S(v)^2\right].$$

In this paper, we determine the neighborhood Sombor index and the modified neighborhood Sombor index for some important dendrimers such as tetrathiafulvalene, POPAM, $NS_2[n]$ and $NS_3[n]$ dendrimers.

2. Results for tetrathiafulvalene dendrimers $TD_2[n]$

In this section, we focus on the molecular graph of a tetrathiafulvalene dendrimer. This family of tetrathiafulvalene dendrimers is denoted by $TD_2[n]$, where n is the steps of growth in this type of dendrimers for n□0. The molecular graph of $TD_2[2]$ is shown in Figure 1.

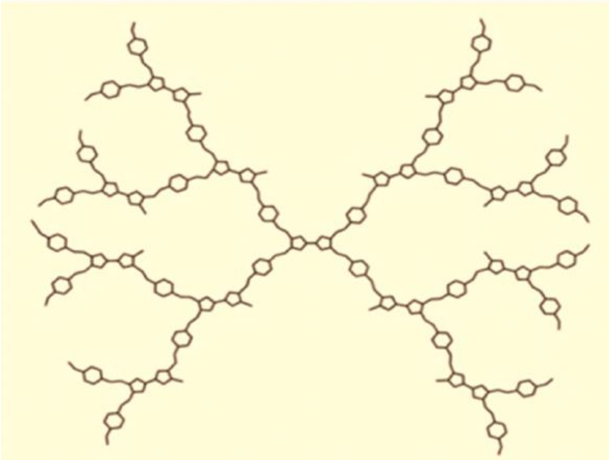

Fig. 1 The molecular graph of $TD_2[2]$

Let G be the molecular graph of tetrathiafulvalene dendrimer $TD_2[n]$. By algebraic method, we obtain that $|V(G)|=31\times2^{n+2} - 74$ and $|E(G)|=35\times2^{n+2} - 85$. Also the edge partition of $TD_2[n]$ based on the degree sum of neighbors of end vertices of each edge is obtained as given in Table 1.

Table 1. Edge partition of $TD_2[n]$ based on $S_G(u)$ and $S_G(v)$

$S_G(u), S_G(v)\backslash uv$□$E(G)$	Number of edges
(2, 4)	2^{n+2}
(3, 6)	$2^{n+2} - 4$
(4, 6)	2^{n+2}
(5, 5)	$7\times2^{n+2} - 16$
(5, 6)	$11\times2^{n+2} - 24$
(5, 7)	$3\times2^{n+2} - 8$
(6, 6)	$2^{n+2} - 4$
(6, 7)	$8\times2^{n+2} - 24$
(7, 7)	$2\times2^{n+2} - 5$

Theorem 1. The neighborhood Sombor index of a tetrathiafulvalene dendrimer $TD_2[n]$ is

$$NSO(G)=\left(5\sqrt{5}+2\sqrt{13}+55\sqrt{2}+11\sqrt{61}+3\sqrt{74}+8\sqrt{85}\right)2^{n+2}$$
$$-\left(5\sqrt{5}+2\sqrt{13}+55\sqrt{2}+11\sqrt{61}+3\sqrt{74}+8\sqrt{85}\right).$$

Proof: From the definition and by using Table 1, we have

$$NSO(G)=\sum_{uv\in E(G)}\sqrt{S_G(u)^2+S_G(v)^2}$$
$$=2^{n+2}\sqrt{2^2+4^2}+\left(2^{n+2}-4\right)\sqrt{3^2+6^2}+2^{n+2}\sqrt{4^2+6^2}$$
$$+\left(7\times2^{n+2}-16\right)\sqrt{5^2+5^2}+\left(11\times2^{n+2}-24\right)\sqrt{5^2+6^2}+\left(3\times2^{n+2}-8\right)\sqrt{5^2+7^2}$$

$$+\left(2^{n+2}-4\right)\sqrt{6^2+6^2}+\left(8\times 2^{n+2}-24\right)\sqrt{6^2+7^2}+\left(2\times 2^{n+2}-5\right)\sqrt{7^2+7^2}.$$

After simplification, we get the desired result.

Theorem 2. The modified neighborhood Sombor index of a tetrathiafulvalene dendrimer $TD_2[n]$ is

$$^{m}NSO(G)=\left(\frac{1}{2\sqrt{5}}+\frac{1}{3\sqrt{5}}+\frac{1}{2\sqrt{13}}+\frac{7}{5\sqrt{2}}+\frac{11}{\sqrt{61}}+\frac{3}{\sqrt{74}}+\frac{1}{6\sqrt{2}}+\frac{8}{\sqrt{85}}+\frac{2}{7\sqrt{2}}\right)2^{n+2}$$
$$-\left(\frac{4}{3\sqrt{5}}+\frac{16}{5\sqrt{2}}+\frac{24}{\sqrt{61}}+\frac{8}{\sqrt{74}}+\frac{4}{6\sqrt{2}}+\frac{24}{\sqrt{85}}+\frac{5}{7\sqrt{2}}\right).$$

Proof: From the definition and Table 1, we have

$$^{m}NSO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{S_G(u)^2+S_G(v)^2}}$$
$$=\frac{2^{n+2}}{\sqrt{2^2+4^2}}+\frac{2^{n+2}-4}{\sqrt{3^2+6^2}}+\frac{2^{n+2}}{\sqrt{4^2+6^2}}+\frac{7\times 2^{n+2}-16}{\sqrt{5^2+5^2}}+\frac{11\times 2^{n+2}-24}{\sqrt{5^2+6^2}}$$
$$+\frac{3\times 2^{n+2}-8}{\sqrt{5^2+7^2}}+\frac{2^{n+2}-4}{\sqrt{6^2+6^2}}+\frac{8\times 2^{n+2}-24}{\sqrt{6^2+7^2}}+\frac{2\times 2^{n+2}-5}{\sqrt{7^2+7^2}}$$

gives the desired result after simplification.

Theorem 3. The neighborhood Sombor exponential of a tetrathiafulvalene dendrimer $TD_2[n]$ is

$$NSO(G,x)=2^{n+2}x^{2\sqrt{5}}+\left(2^{n+2}-4\right)x^{3\sqrt{5}}+2^{n+2}x^{2\sqrt{13}}+\left(7\times 2^{n+2}-16\right)x^{5\sqrt{2}}+\left(11\times 2^{n+2}-24\right)x^{\sqrt{61}}$$
$$+\left(3\times 2^{n+2}-8\right)x^{\sqrt{74}}+\left(2^{n+2}-4\right)x^{6\sqrt{2}}+\left(8\times 2^{n+2}-24\right)x^{\sqrt{85}}+\left(2\times 2^{n+2}-5\right)x^{7\sqrt{2}}.$$

Proof: Using definition and Table 1, we obtain

$$NSO(G,x)=\sum_{uv\in E(G)}x^{\sqrt{S_G(u)^2+S_G(v)^2}}$$
$$=2^{n+2}x^{\sqrt{2^2+4^2}}+\left(2^{n+2}-4\right)x^{\sqrt{3^2+6^2}}+2^{n+2}x^{\sqrt{4^2+6^2}}+\left(7\times 2^{n+2}-16\right)x^{\sqrt{5^2+5^2}}$$
$$+\left(11\times 2^{n+2}-24\right)x^{\sqrt{5^2+6^2}}+\left(3\times 2^{n+2}-8\right)x^{\sqrt{5^2+7^2}}+\left(2^{n+2}-4\right)x^{\sqrt{6^2+6^2}}$$
$$+\left(8\times 2^{n+2}-24\right)x^{\sqrt{6^2+7^2}}+\left(2\times 2^{n+2}-5\right)x^{\sqrt{7^2+7^2}}.$$

After simplification, we obtain the desired result.

Theorem 4. The modified neighborhood Sombor exponential of a tetrathiafulvalene dendrimer $TD_2[n]$ is

$$^{m}NSO(G,x)=2^{n+2}x^{\frac{1}{2\sqrt{5}}}+\left(2^{n+2}-4\right)x^{\frac{1}{3\sqrt{5}}}+2^{n+2}x^{\frac{1}{2\sqrt{13}}}+\left(7\times 2^{n+2}-16\right)x^{\frac{1}{5\sqrt{2}}}+\left(11\times 2^{n+2}-24\right)x^{\frac{1}{\sqrt{61}}}$$
$$+\left(3\times 2^{n+2}-8\right)x^{\frac{1}{\sqrt{74}}}+\left(2^{n+2}-4\right)x^{\frac{1}{6\sqrt{2}}}+\left(8\times 2^{n+2}-24\right)x^{\frac{1}{\sqrt{85}}}+\left(2\times 2^{n+2}-5\right)x^{\frac{1}{7\sqrt{2}}}.$$

Proof: From the definition and by using Table 1, we get

$$^{m}NSO(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{S_G(u)^2+S_G(v)^2}}}$$
$$=2^{n+2}x^{\frac{1}{\sqrt{2^2+4^2}}}+\left(2^{n+2}-4\right)x^{\frac{1}{\sqrt{3^2+6^2}}}+2^{n+2}x^{\frac{1}{\sqrt{4^2+6^2}}}+\left(7\times 2^{n+2}-16\right)x^{\frac{1}{\sqrt{5^2+5^2}}}$$
$$+\left(11\times 2^{n+2}-24\right)x^{\frac{1}{\sqrt{5^2+6^2}}}+\left(3\times 2^{n+2}-8\right)x^{\frac{1}{\sqrt{5^2+7^2}}}+\left(2^{n+2}-4\right)x^{\frac{1}{\sqrt{6^2+6^2}}}$$

$$+\left(8\times 2^{n+2}-24\right)x^{\frac{1}{\sqrt{6^2+7^2}}}+\left(2\times 2^{n+2}-5\right)x^{\frac{1}{\sqrt{7^2+7^2}}}.$$

After simplification, we get the desired result.

3. Results for POPAM dendrimers $POD_2[n]$

In this section, we focus on the molecular graph of POPAM dendrimers. This family of dendrimers is denoted by $POD_2[n]$, where n is the steps of growth in this type of dendrimers. The molecular graph of $POD_2[2]$ is shown in Figure 2.

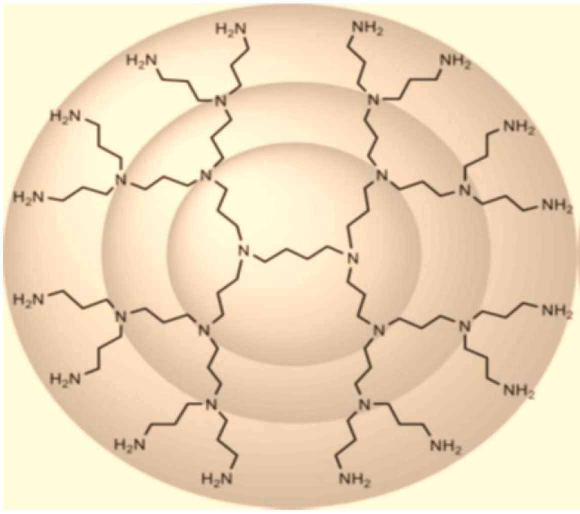

Fig. 2 The molecular graph of $POD_2[n]$

Let G be the molecular graph of POPAM dendrimers $POD_2[n]$. By algebraic method, we obtain that $|V(POD_2[n])|= 2^{n+5} - 10$ and $|E(POD_2[n])|= 2^{n+5} - 11$. The edge partition of $POD_2[n]$ based on the degree sum of neighbors of end vertices of each edge is obtained as given in Table 2.

$S_G(u), S_G(v)\backslash uv\square E(G)$	Number of edges
(2, 3)	2^{n+2}
(3, 4)	2^{n+2}
(4, 4)	1
(4, 5)	$3\times2^n - 6$
(5, 6)	$3\times2^n - 6$

Theorem 5. The neighborhood Sombor index of a POPAM dendrimer $POD_2[n]$ is

$$NSO(G)=2^{n+2}\left(\sqrt{13}+5\right)+4\sqrt{2}+\left(3\times 2^n-6\right)\left(\sqrt{41}+\sqrt{61}\right).$$

Proof: From the definition and by using Table 2, we have

$$NSO(G)=\sum_{uv\in E(G)}\sqrt{S_G(u)^2+S_G(v)^2}$$

$$=2^{n+2}\sqrt{2^2+3^2}+2^{n+2}\sqrt{3^2+4^2}+1\sqrt{4^2+4^2}$$

$$+\left(3\times 2^n-6\right)\sqrt{4^2+5^2}+\left(3\times 2^n-6\right)\sqrt{5^2+6^2}.$$

After simplification, we obtain the desired result.

Theorem 6. The modified neighborhood Sombor index of a POPAM dendrimer $POD_2[n]$ is

$$^{m}NSO(G)=2^{n+2}\left(\frac{1}{\sqrt{13}}+\frac{1}{5}\right)+\frac{1}{4\sqrt{2}}+\left(3\times 2^{n}-6\right)\left(\frac{1}{\sqrt{41}}+\frac{1}{\sqrt{61}}\right).$$

Proof: From the definition and by using Table 2, we have

$$^{m}NSO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{S_G(u)^2+S_G(v)^2}}$$

$$=\frac{2^{n+2}}{\sqrt{2^2+3^2}}+\frac{2^{n+2}}{\sqrt{3^2+4^2}}+\frac{1}{\sqrt{4^2+4^2}}+\frac{3\times 2^n-6}{\sqrt{4^2+5^2}}+\frac{3\times 2^n-6}{\sqrt{5^2+6^2}}.$$

After simplification, we get the desired result.

Theorem 7. The neighborhood Sombor exponential of a POPAM dendrimer $POD_2[n]$ is

$$NSO(G,x)=2^{n+2}x^{\sqrt{13}}+2^{n+2}x^{5}+1x^{4\sqrt{2}}+\left(3\times 2^{n}-6\right)x^{\sqrt{41}}+\left(3\times 2^{n}-6\right)x^{\sqrt{61}}.$$

Proof: Using the definition and Table 2, we obtain

$$NSO(G,x)=\sum_{uv\in E(G)}x^{\sqrt{S_G(u)^2+S_G(v)^2}}$$

$$=2^{n+2}x^{\sqrt{2^2+3^2}}+2^{n+2}x^{\sqrt{3^2+4^2}}+1x^{\sqrt{4^2+4^2}}+\left(3\times 2^{n}-6\right)x^{\sqrt{4^2+5^2}}+\left(3\times 2^{n}-6\right)x^{\sqrt{5^2+6^2}}.$$

After simplification, we obtain the desired result.

Theorem 8. The modified neighborhood Sombor exponential of a POPAM dendrimer $POD_2[n]$ is

$$^{m}NSO(G,x)=2^{n+2}x^{\frac{1}{\sqrt{13}}}+2^{n+2}x^{\frac{1}{5}}+1x^{\frac{1}{4\sqrt{2}}}+\left(3\times 2^{n}-6\right)x^{\frac{1}{\sqrt{41}}}+\left(3\times 2^{n}-6\right)x^{\frac{1}{\sqrt{61}}}.$$

Proof: From the definition and by using Table 2, we obtain

$$^{m}NSO(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{S_G(u)^2+S_G(v)^2}}}$$

$$=2^{n+2}x^{\frac{1}{\sqrt{2^2+3^2}}}+2^{n+2}x^{\frac{1}{\sqrt{3^2+4^2}}}+1x^{\frac{1}{\sqrt{4^2+4^2}}}$$

$$+\left(3\times 2^{n}-6\right)x^{\frac{1}{\sqrt{4^2+5^2}}}+\left(3\times 2^{n}-6\right)x^{\frac{1}{\sqrt{5^2+6^2}}}.$$

After simplification, we obtain the desired result.

4. Results for $NS_2[n]$ dendrimers

In this section, we focus on the class of $NS_2[n]$ dendrimers with n□1. The graph of $NS_2[3]$ is shown in Figure 3.

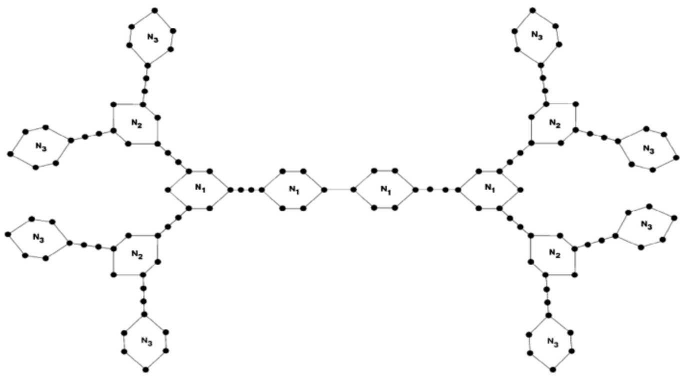

Fig. 3 The graph of NS_2[3]

Let G be the graph of $NS_2[n]$. By calculation, G has $16\times2^n - 4$ vertices and $18\times2^n - 5$ edges. Also by calculation, we obtain that G has seven types of edges based on $S_G(u)$, $S_G(v)$ the degrees of end vertices of each edge as given in Table 3.

Table 3. Edge partition of $NS_2[n]$ based on $S_G(u)$ and $S_G(v)$

$S_G(u), S_G(v)\backslash uv\ E(G)$	Number of edges
(4, 4)	2×2^n
(5, 4)	2×2^n
(5, 5)	$2\times2^n+2$
(5, 6)	6×2^n
(7, 7)	1
(5, 7)	4
(6, 6)	$6\times2^n-12$

Theorem 9. The neighborhood Sombor index of a dendrimer $NS_2[n]$ is

$$NSO(G) = \left(54\sqrt{2} + 2\sqrt{41} + 6\sqrt{61}\right)2^n - 55\sqrt{2} + 4\sqrt{74}.$$

Proof: From the definition and by using Table 3, we have

$$NSO(G) = \sum_{uv\in E(G)} \sqrt{S_G(u)^2 + S_G(v)^2}$$

$$= 2\times2^n\sqrt{4^2+4^2} + 2\times2^n\sqrt{5^2+4^2} + \left(2\times2^n+2\right)\sqrt{5^2+5^2} + 6\times2^n\sqrt{5^2+6^2}$$

$$+1\sqrt{7^2+7^2} + 4\sqrt{5^2+7^2} + \left(6\times2^n-12\right)\sqrt{6^2+6^2}$$

gives the desired result after simplification.

Theorem 10. The modified neighborhood Sombor index of a dendrimer $NS_2[n]$ is

$$^{m}NSO(G) = \frac{2\times2^n}{4\sqrt{2}} + \frac{2\times2^n}{\sqrt{41}} + \frac{2\times2^n+2}{5\sqrt{2}} + \frac{6\times2^n}{\sqrt{61}} + \frac{1}{7\sqrt{2}} + \frac{4}{\sqrt{74}} + \frac{6\times2^n-12}{6\sqrt{2}}.$$

Proof: From the definition and Table 3, we obtain

$$^{m}NSO(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{S_G(u)^2 + S_G(v)^2}}$$

$$= \frac{2\times2^n}{\sqrt{4^2+4^2}} + \frac{2\times2^n}{\sqrt{5^2+4^2}} + \frac{2\times2^n+2}{\sqrt{5^2+5^2}} + \frac{6\times2^n}{\sqrt{5^2+6^2}}$$

$$+ \frac{1}{\sqrt{7^2+7^2}} + \frac{4}{\sqrt{5^2+7^2}} + \frac{6\times2^n-12}{\sqrt{6^2+6^2}}.$$

After simplification, we obtain the desired result.

Theorem 11. The neighborhood Sombor exponential of a dendrimer $NS_2[n]$ is

$$NSO(G,x) = 2\times2^n x^{4\sqrt{2}} + 2\times2^n x^{\sqrt{41}} + \left(2\times2^n+2\right)x^{5\sqrt{2}} + 6\times2^n x^{\sqrt{61}}$$

$$+1x^{7\sqrt{2}} + 4x^{\sqrt{74}} + \left(6\times2^n-12\right)x^{6\sqrt{2}}.$$

Proof: Using definition and Table 3, we obtain

$$NSO(G,x)=\sum_{uv\in E(G)} x^{\sqrt{S_G(u)^2+S_G(v)^2}}$$
$$=2\times 2^n x^{\sqrt{4^2+4^2}}+2\times 2^n x^{\sqrt{5^2+4^2}}+\left(2\times 2^n+2\right)x^{\sqrt{5^2+5^2}}+6\times 2^n x^{\sqrt{5^2+6^2}}$$
$$+1x^{\sqrt{7^2+7^2}}+4x^{\sqrt{5^2+7^2}}+\left(6\times 2^n-12\right)x^{\sqrt{6^2+6^2}}$$

gives the desired result after simplification.

Theorem 12. The modified neighborhood Sombor exponential of a dendrimer $NS_2[n]$ is

$$^m NSO(G,x)=2\times 2^n x^{\frac{1}{4\sqrt{2}}}+2\times 2^n x^{\frac{1}{\sqrt{41}}}+\left(2\times 2^n+2\right)x^{\frac{1}{5\sqrt{2}}}+6\times 2^n x^{\frac{1}{\sqrt{61}}}$$
$$+1x^{\frac{1}{7\sqrt{2}}}+4x^{\frac{1}{\sqrt{74}}}+\left(6\times 2^n-12\right)x^{\frac{1}{6\sqrt{2}}}.$$

Proof: From the definition and by using Table 3, we get

$$^m NSO(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{S_G(u)^2+S_G(v)^2}}}$$
$$=2\times 2^n x^{\frac{1}{\sqrt{4^2+4^2}}}+2\times 2^n x^{\frac{1}{\sqrt{5^2+4^2}}}+\left(2\times 2^n+2\right)x^{\frac{1}{\sqrt{5^2+5^2}}}+6\times 2^n x^{\frac{1}{\sqrt{5^2+6^2}}}$$
$$+1x^{\frac{1}{\sqrt{7^2+7^2}}}+4x^{\frac{1}{\sqrt{5^2+7^2}}}+\left(6\times 2^n-12\right)x^{\frac{1}{\sqrt{6^2+6^2}}}.$$

After simplification, we obtain the desired result.

5. Results for $NS_3[n]$ dendrimers

In this section, we focus on another type of dendrimers $NS_3[n]$ with n□1. The molecular structure of $NS_3[2]$ is presented in Figure 4.

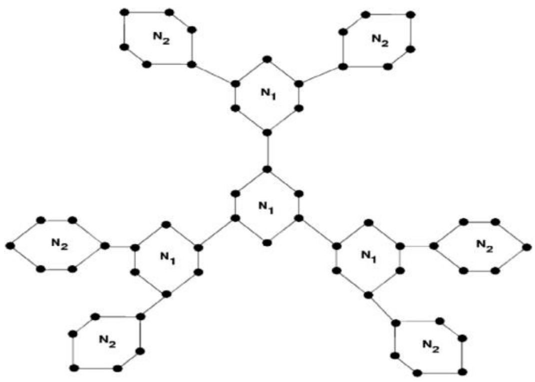

Fig. 4 The structure of NS_3[2]

Let G be the molecular graph of $NS_3[n]$. By calculation, we obtain that G has $18\times2^n - 12$ vertices and $21\times2^n - 15$ edges. Also by calculation, we get that G has five types of edges based on $S_G(u)$ and $S_G(v)$ the degrees of end vertices of each edge as given in Table 4.

Table 4. Edge partition of $NS_3[n]$ based on $S_G(u)$ and $S_G(v)$

$S_G(u), S_G(v)\backslash uv$□$E(G)$	Number of edges
(4, 4)	3×2^n
(5, 4)	3×2^n
(5, 7)	3×2^n
(6, 7)	$9\times2^n - 12$
(7, 7)	$3\times2^n - 3$

Theorem 13. The neighborhood Sombor index of a dendrimer $NS_3[n]$ is

$$NSO(G)=3\times 2^n\left(4\sqrt{2}+\sqrt{41}+\sqrt{74}\right)+\left(9\times 2^n-12\right)\sqrt{85}+\left(3\times 2^n-3\right)7\sqrt{2}.$$

Proof: From the definition and by using Table 4, we have

$$\begin{aligned}NSO(G)&=\sum_{uv\in E(G)}\sqrt{S_G(u)^2+S_G(v)^2}\\&=3\times 2^n\sqrt{4^2+4^2}+3\times 2^n\sqrt{5^2+4^2}+3\times 2^n\sqrt{5^2+7^2}\\&+\left(9\times 2^n-12\right)\sqrt{6^2+7^2}+\left(3\times 2^n-3\right)\sqrt{7^2+7^2}.\end{aligned}$$

After simplification, we get the desired result.

Theorem 14. The modified neighborhood Sombor index of a dendrimer $NS_3[n]$ is

$$^{m}NSO(G)=3\times 2^n\left(\frac{1}{4\sqrt{2}}+\frac{1}{\sqrt{41}}+\frac{1}{\sqrt{74}}\right)+\frac{9\times 2^n-12}{\sqrt{85}}+\frac{3\times 2^n-3}{7\sqrt{2}}.$$

Proof: From the definition and by using Table 4, we have

$$\begin{aligned}^{m}NSO(G)&=\sum_{uv\in E(G)}\frac{1}{\sqrt{S_G(u)^2+S_G(v)^2}}\\&=\frac{3\times 2^n}{\sqrt{4^2+4^2}}+\frac{3\times 2^n}{\sqrt{5^2+4^2}}+\frac{3\times 2^n}{\sqrt{5^2+7^2}}+\frac{9\times 2^n-12}{\sqrt{6^2+7^2}}+\frac{3\times 2^n-3}{\sqrt{7^2+7^2}}.\end{aligned}$$

gives the desired result.

Theorem 15. The neighborhood Sombor exponential of a dendrimer $NS_3[n]$ is

$$\begin{aligned}NSO(G,x)&=3\times 2^n x^{4\sqrt{2}}+3\times 2^n x^{\sqrt{41}}+3\times 2^n x^{\sqrt{74}}\\&+\left(3\times 2^n-6\right)x^{\sqrt{85}}+\left(3\times 2^n-6\right)x^{7\sqrt{2}}.\end{aligned}$$

Proof: Using definition and Table 4, we obtain

$$\begin{aligned}NSO(G,x)&=\sum_{uv\in E(G)}x^{\sqrt{S_G(u)^2+S_G(v)^2}}\\&=3\times 2^n x^{\sqrt{4^2+4^2}}+3\times 2^n x^{\sqrt{5^2+4^2}}+3\times 2^n x^{\sqrt{5^2+7^2}}\\&+\left(9\times 2^n-12\right)x^{\sqrt{6^2+7^2}}+\left(3\times 2^n-3\right)x^{\sqrt{7^2+7^2}}\end{aligned}$$

After simplification, we obtain the desired result.

Theorem 16. The modified neighborhood Sombor exponential of a dendrimer $NS_3[n]$ is

$$\begin{aligned}^{m}NSO(G,x)&=3\times 2^n x^{\frac{1}{4\sqrt{2}}}+3\times 2^n x^{\frac{1}{\sqrt{41}}}+3\times 2^n x^{\frac{1}{\sqrt{74}}}\\&+\left(9\times 2^n-12\right)x^{\frac{1}{\sqrt{85}}}+\left(3\times 2^n-3\right)x^{\frac{1}{7\sqrt{2}}}.\end{aligned}$$

Proof: Using definition and Table 4, we obtain

$$\begin{aligned}^{m}NSO(G,x)&=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{S_G(u)^2+S_G(v)^2}}}\\&=3\times 2^n x^{\frac{1}{\sqrt{4^2+4^2}}}+3\times 2^n x^{\frac{1}{\sqrt{5^2+4^2}}}+3\times 2^n x^{\frac{1}{\sqrt{5^2+7^2}}}\\&+\left(9\times 2^n-12\right)x^{\frac{1}{\sqrt{6^2+7^2}}}+\left(3\times 2^n-3\right)x^{\frac{1}{\sqrt{7^2+7^2}}}.\end{aligned}$$

After simplification, we get the desired result.

6. Properties of neighborhood Sombor index

Theorem 17. Let *G* be a connected graph with *m* edges. Then

$$\frac{1}{\sqrt{2}} N_1(G) \le NSO(G) \le N_1(G).$$

Proof: For any two positive numbers *a* and *b*,

$$\frac{1}{\sqrt{2}}(a+b) \le \sqrt{(a^2+b^2)} \le a+b.$$

For $a=S_G(u)$ and $b=S_G(v)$, the above inequalities transform into

$$\frac{1}{\sqrt{2}}\left(S_G(u)+S_G(v)\right) \le \sqrt{(S_G(u)^2+S_G(v)^2)} \le S_G(u)+S_G(v)$$

Now, we obtain

$$\frac{1}{\sqrt{2}} \sum_{uv\in E(G)} \left(S_G(u)+S_G(v)\right) \le \sum_{uv\in E(G)} \sqrt{(S_G(u)^2+S_G(v)^2} \le \sum_{uv\in E(G)} \left(S_G(u)+S_G(v)\right)$$

with the help of definitions, we arrive the desired result.

Theorem 18. Let *G* be a connected graph with *m* edges. Then

$$NSO(G) \le \sqrt{m\,NF(G)}\ .$$

Proof: Using the Cauchy-Schwarz inequality, we obtain

$$\left(\sum_{uv\in E(G)} \sqrt{S(u)^2+S(v)^2}\right)^2 \le \sum_{uv\in E(G)} 1 \sum_{uv\in E(G)} \left(S(u)^2+S(v)^2\right).$$

$$= mNF(G).$$

Thus $$NSO(G) \le \sqrt{mNF(G)}.$$

7. Conclusion

In this study, the neighborhood Sombor index and the modified neighborhood Sombor index for the tetrathiafulvalene, POPAM, $NS_2[n]$ and $NS_3[n]$ dendrimers are computed. We also established some properties of the neighborhood Sombor index.

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