

Original Article

Some Corona Product of Root Square Mean Labeling of Graphs

M. Sivasakthi¹, S. Meena², S. Gangadevi³

¹Department of Mathematics, Krishnasamy college of Science Arts and Management for Women, Cuddalore.

²Department of Mathematics, Government Arts college, C.Mutlur, Chidambaram-608 102

³Research Scholar, Department of Mathematics, Krishnasamy college of science Arts and Management for Women, Cuddalore.

Received: 30 May 2022

Revised: 07 July 2022

Accepted: 10 July 2022

Published: 14 July 2022

Abstract - A graph $G = (V, E)$ with p vertices and q edges is said to be a Root Square Mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, \dots, q+1$ in such a way that when each edge $e = uv$ is labelled with $f(e = uv) = \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$ or $\left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$, then the resulting edge labels are distinct. In this case f is called a Root Square Mean labeling of G . In this paper we prove that some corona product of Root Square Mean labeling of graphs, such as $L_n \odot K_1, Q_n \odot K_1, T_n \odot K_1$ are Root Square Mean labeling of graphs.

Keywords - Graph, Root Square Mean graph, $L_n \odot K_1, Q_n \odot K_1, T_n \odot K_1$

AMS subject classification - 05078.

1. Introduction

The graph considered here will be finite, undirected and simple. The vertex set is denoted by $V(G)$ and the edge set is denoted by $E(G)$. The concept of graph labeling was introduced by Rosa [6]. For all detailed survey of graph labeling we refer to Gallian[1]. For all other standard terminology and notations we follow Harary [2]. The concept of mean labeling has been introduced by S.Somasundaram and R. Ponraj in 2004 [5]. S.S.Sandhya, S.Somasundaram and S.Anusa introduced the concept of Root Square Mean labeling of graphs. The definitions and other informations which are useful for the present investigation are given below. The following definitions are useful for present investigation.

Definition 1.1:

A graph $G = (V, E)$ with p vertices and q edges is said to be a Root Square Mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, \dots, q+1$ in such a way that when each edge $e = uv$ is labelled with $f(e = uv) = \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$ or $\left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$, then the resulting edge labels are distinct. In this case f is called a Root Square Mean labeling of G .

Definition 1.2:

The Corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by one copy of G_1 and $|G_1|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2 .

Definition 1.3:

The product graph $p_2 \times p_n$ is called a ladder and it is denoted by L_n .

Definition 1.4:

A Triangular Snake T_n is obtained from a path $u_1 u_2 \dots u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n-1$. That is every edge of a path is replaced by a triangle C_3 .

Definition 1.5:

A Quadrilateral Snake Q_n is obtained from a path $u_1 u_2 \dots u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and then joining v_i and w_i . That is every edge of a path is replaced by a cycle C_4 .

In this paper we prove that $L_n \odot K_1, Q_n \odot K_1, T_n \odot K_1$ are Root Square mean graph.



2. Main Results

Theorem 2.1:

The Ladder graph $L_n \odot K_1$ is a root square mean graph.

Proof:

Let $u_1 u_2 \dots u_n$ and $v_1 v_2 \dots v_n$ be a two path of length n, join u_i and v_i , then the resultant graph is L_n , for $1 \leq i \leq n$.

Let x_i, y_i be the pendent vertex joined by the path u_i and v_i respectively.

Define a function $f: V(G) \rightarrow \{1, 2, \dots, q+1\}$ by

$$f(u_1) = 3$$

$$f(u_i) = 5i - 4 \quad \text{for } 2 \leq i \leq n$$

$$f(v_1) = 2$$

$$f(v_i) = 5i - 2 \quad \text{for } 2 \leq i \leq n$$

$$f(x_1) = 4$$

$$f(x_i) = 5i - 5 \quad \text{for } 2 \leq i \leq n$$

$$f(y_1) = 1$$

$$f(y_i) = 5i - 1 \quad \text{for } 1 \leq i \leq n$$

Then the resulting edge labels are distinct.

$$f(u_i u_{i+1}) = 5i - 1 \quad \text{for } 1 \leq i \leq n$$

$$f(u_i v_i) = 5i - 3 \quad \text{for } 1 \leq i \leq n$$

$$f(v_1 v_2) = 6$$

$$f(v_i v_{i+1}) = 5i + 1 \quad \text{for } 2 \leq i \leq n$$

$$f(u_1 x_1) = 3$$

$$f(u_i x_i) = 5i - 5 \quad \text{for } 2 \leq i \leq n$$

$$f(v_1 y_1) = 1$$

$$f(v_i y_i) = 5i - 2 \quad \text{for } 2 \leq i \leq n$$

Thus f provides a root square mean labeling of graph G .

Hence G is a root square mean labeling of graphs.

Example 2.2:

Root square mean labeling of G obtained from $L_7 \odot K_1$ is given in fig 1.1

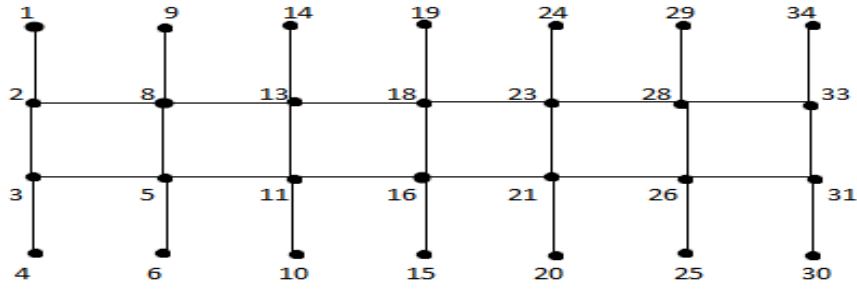


Fig. 1.1

Theorem 2.3:

The Quadrilateral snake $Q_n \odot K_1$ is a root square mean graph.

Proof:

Consider a path $u_1 u_2 \dots u_n$ by joining u_i and u_{i+1} be two new vertices v_i and w_i respectively and then adjacent to v_i and w_i . Then the graph is called Quadrilateral snake graph Q_n .

Let x_i, l_i, m_i be the pendent vertex attached by the point u_i, v_i, m_i respectively.

Define a function $f: V(G) \rightarrow \{1, 2, \dots, q+1\}$ by,

$$\begin{aligned}
 f(u_1) &= 1 \\
 f(u_i) &= 7i - 6 && \text{for } 2 \leq i \leq n \\
 f(v_i) &= 7i - 4 && \text{for } 1 \leq i \leq n \\
 f(w_1) &= 5 \\
 f(w_i) &= 7i - 1 && \text{for } 2 \leq i \leq n \\
 f(x_1) &= 1 \\
 f(x_i) &= 7i - 7 && \text{for } 2 \leq i \leq n \\
 f(l_i) &= 7i - 3 && \text{for } 1 \leq i \leq n \\
 f(m_1) &= 6 \\
 f(m_i) &= 7i - 2 && \text{for } 2 \leq i \leq n
 \end{aligned}$$

Then the resulting edge labels are distinct.

$$\begin{aligned}
 f(u_i v_i) &= 7i - 5 && \text{for } 1 \leq i \leq n - 1 \\
 f(v_i w_i) &= 7i - 3 && \text{for } 1 \leq i \leq n - 1 \\
 f(w_i u_{i+1}) &= 7i && \text{for } 1 \leq i \leq n - 1 \\
 f(u_i u_{i+1}) &= 7i - 2 && \text{for } 1 \leq i \leq n - 1 \\
 f(u_i x_i) &= 7i - 6 && \text{for } 1 \leq i \leq n \\
 f(l_i v_i) &= 7i - 4 && \text{for } 1 \leq i \leq n - 1 \\
 f(m_i w_i) &= 7i - 1 && \text{for } 1 \leq i \leq n - 1
 \end{aligned}$$

Thus f provides root square mean labeling of graph.

Hence G is a root square mean graph.

Example: 2.4:

Root square mean labeling of $Q_6 \odot K_1$ is given in fig 1.2

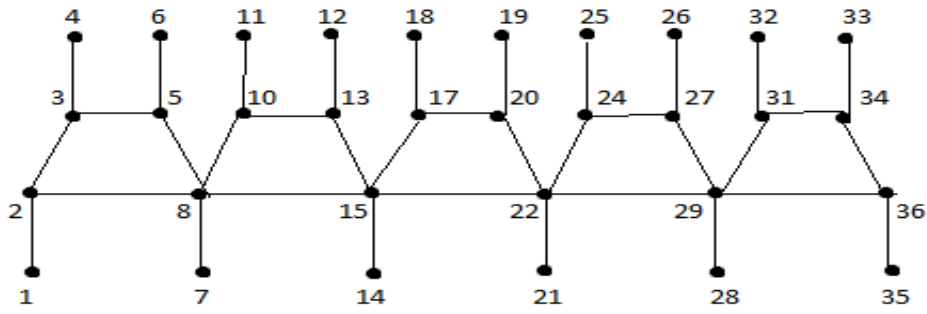


Fig. 1.2

Theorem 2.5:

The triangular snake $T_n \odot K_1$ is a root square mean graph.

Proof:

Let T_n be a triangular snake. Let $V(G) = \{ u_1 u_2 \dots u_n, v_1 v_2 \dots v_n \}$

Let $t_1 t_2 \dots t_n, w_1 w_2 \dots w_n$ be the pendent vertex joined with u_i and v_i respectively.

Define a function $V(G) \rightarrow \{1, 2, \dots, q+1\}$ by

- $f(u_1) = 2$
- $f(u_{i+1}) = 5i$ for $1 \leq i \leq n-1$
- $f(v_1) = 3$
- $f(v_{i+1}) = 5i+4$ for $1 \leq i \leq n-1$
- $f(w_1) = 4$
- $f(w_{i+1}) = 5i+3$ for $1 \leq i \leq n-1$
- $f(t_1) = 1$
- $f(t_{i+1}) = 5i+1$ for $1 \leq i \leq n-1$

Then the resulting edge labels are distinct.

- $f(u_1 u_2) = 4$
- $f(u_{i+1} u_{i+2}) = 5i+3$ for $1 \leq i \leq n-2$
- $f(u_1 v_1) = 2$
- $f(u_{i+1} v_{i+1}) = 5i+2$ for $1 \leq i \leq n-1$
- $f(v_i u_{i+1}) = 5i$ for $1 \leq i \leq n-1$
- $f(u_1 v_1) = 1$
- $f(u_{i+1} t_{i+1}) = 5i+1$ for $1 \leq i \leq n-1$
- $f(w_1 v_1) = 3$
- $f(w_{i+1} v_{i+1}) = 5i+4$ for $1 \leq i \leq n-1$

Thus f provides a root square mean graph G .

Hence G is a root square mean graph.

Example:2.6

A root square mean labeling of triangular snake $T_8 \odot K_1$ is given in fig 1.3

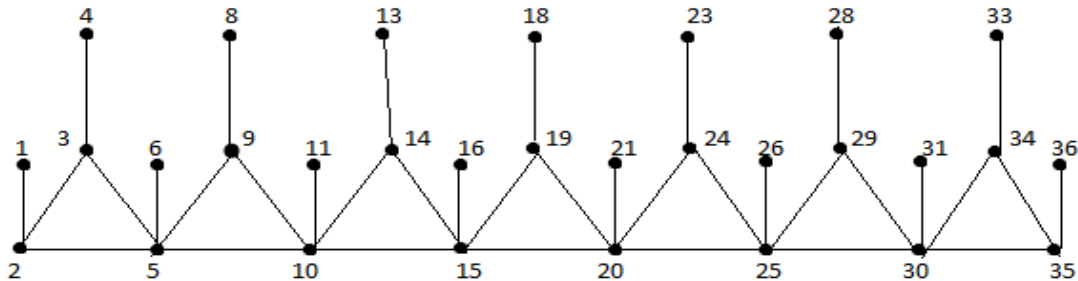


Fig. 1.3

Conclusion

We have proved three results on Root Square Mean labeling of graphs such as $L_n \odot K_1, Q_n \odot K_1, T_n \odot K_1$ are Root Square Mean Labeling of graphs. Similar work can be carried out for other families and in the context of different types of graph labeling techniques.

References

- [1] Gallian J.A, 2010, A dynamic survey of graph labeling. The electronic Journal of Combinatorics 17#DS6.
- [2] Gayathri.B and Gopi.R , Necessary condition for mean labeling, International Journal of Engineering Sciences, Advanced computing and Bio-Technology, 4(3)(2013),43-52.
- [3] Gayathri.B and Gopi.R, cycle related mean graph, Elixir International Journal of Applied Sciences, 71(2014), 25116-25124.
- [4] Gopi.R, Super root square mean labeling of some more graphs, Journal of Discrete Mathematical Sciences & cryptography(communicated).
- [5] Harary .F, 1988, Graph Theory, Narosa Publishing House Reading, New Delhi.
- [6] Meena .S, Sivasakthi.M “ New results on harmonic mean graphs” Malaya Journal of Mathematic, 482-486, 2020.
- [7] Meena.S, Sivasakthi.M “ Harmonic Mean Labeling of Zig- Zag Triangular Graphs” International of Mathematics Trends and Technology, April 66(4), 17-24,2020.
- [8] Ponraj.R and Somasundaram.S (2003), Mean labeling of graphs, National Academy of Science Letter vol.26, p210-213.
- [9] Rosa.A , “on certain valuation of the vertices of a graph “, Theory of graphs intermet symposium, Rome, July(1966) Gardom and Breach N.Y and Dunod Paris (1967) 349-355.
- [10] Somasundaram.S and Ponraj.R 2003, Mean labeling of graph, National Academy of Science Letters vol.26, p210-213.
- [11] Sandhya.S.S, Somasundaram.S, Anusa.S, “Root Square Mean Labeling of Graphs” International Journal of Contemporary Mathematical Sciences, Vol.9, 2014, no.14, 667-676.
- [12] Sandhya .S.S, Somasundaram.S, Anusa.S, “Some New Results on Root Square Mean Labeling” International Journal of Mathematics Archive -5(12), 2014, 130-135.
- [13] Sandhya.S.S, Somasundaram.S, Anusa.S, Root Square Mean Labeling of Subdivision of some Graphs” Global Journal of Theoretical and Applied Mathematics Sciences, Volume 5, Number 1, (2015) pp.1-11.
- [14] Sandhya.S.S, Somasundaram.S, Anusa.S, “Root Square Mean Labeling of Some new Disconnected Graphs” International Journal of Mathematics Trends and Technology, volume 15, number 2,2014. Page no:85-92.
- [15] Thiruganasambandam.K and Venkatesan.K, Super Root Square Mean Labeling of Graph, Internal Journal of Mathematics and soft computing, 5(2)(2015), 189-195.