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Original Article

B-Spline Collocation Solution for Burgers' equation arising in Longitudinal Dispersion Phenomena in Fluid Flow through Porous Media

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Abstract

This paper investigates B-Spline Collocation Solution for Burgers' equation arising in longitudinal dispersion phenomenon in the fluid flow through porous media. In the porous medium clean water, saltwater or tainted water disperse longitudinal way offers to increase to a non-linear partial differential condition as Burgers' equation. The equation is solved by utilizing the B-Spline Collocation method with suitable initial and boundary conditions. The issue of miscible displacement can be found in the seaside territories, where new water beds are step by step uprooted via ocean water. An unequivocally steady B-spline Collocation method has been utilized to discover the concentration $C(X,T)$ of salty or polluted water dispersion in uni-direction. It is completed, that the concentration $C(X,T)$ reduce as distance X just as time T increments. The tables and figures are created by utilizing MATLAB coding.

Keywords: longitudinal dispersion, Burgers' Equation, B-Spline Collocation Method.

1 Introduction

The present paper discusses the problem of longitudinal dispersion, which occurs during two-phase flow of miscible fluids through porous media.The governing equation non-linear partial differential equation yields into the form of Burger's equation and its solution has been obtained by using B-Spline method.There are different methods to solve the Burgers' equation numerically. In this paper, B-Spline Collocation method is presented to discuss the solution of the problem. According to problem, solution shows that a B-spline collocation method is capable of solving Burger's equation accurately. This method is easy to implement and requires no any inner iteration or corrector to deal with the nonlinear terms of the Burger's equation.

2 Statement of the problem

In porous media, a miscible displacement is a form of double-phase A flow in which two phases are completely soluble in each other. Capillary forces between the fluids, therefore do It didn't come into effect. Longitudinal dispersion of the Contaminated or saline water containing $C(x,t)$ It has been considered to flow in the x-direction, the porous medium is homogeneous and is saturated with fresh water. The Miscible Stream (contaminated or saline and freshwater) Under conditions of absolute miscibility, it could be concluded that Behave, at least locally, as a single-phase fluid that would obey Darcy's law. In turn, the change of concentration Diffusion along the channels of flow which would be caused by Thus the bulk coefficients of diffusion are governed by the There's one fluid in the other. There is no mass transfer between the It is assumed that the solid and liquid phases [[7],[8]].The Miscible flow occurs both longitudinally and transversely, but the spreading caused by dispersion is greater than the transverse direction in the direction of flow.

Describing the growth of the mixed region is the problem, i.e. to find the contaminated water concentration $C(X,T)$ As two miscible fluids, as a function of time t and position x, Flow-through Porous Homogeneous Media. Outside of the field (on either side), the single-fluid equation describes the fluid's motion.

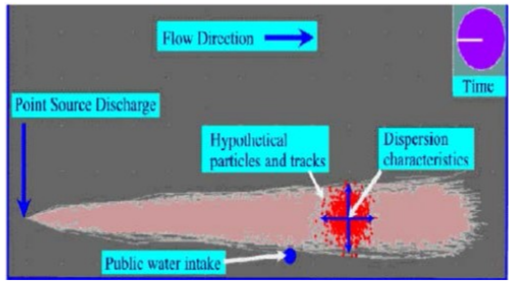

Figure 1: Longitudinal and Transverse dispersion

3 Mathematical formulation of the Problem

In the present problem, the dispersion is considered to be The zone is one-way, i.e. x-direction. The One Dimensional The treatment of dispersion phenomena prohibits circular or radial treatment. Dispersion transverse component. According to Darcy's law, the continuity equation for the In the case of incompressible fluids, the mixture is provided by The [[2],[8]] bear.

$$\frac{\partial \rho}{\partial t} + div \cdot (\rho \bar{V}) = 0 \tag{1}$$

where, ρ is the density for mixture and $\bar{V}$ is the pore seepage velocity vector.
The diffusion equation for the flow of fluid through a Homogeneous porous medium, not growing or increasing The reduction of the dispersing material is supplied by

$$\frac{\partial C}{\partial t} + div \cdot (C\bar{V}) = \nabla \cdot \left[\rho \bar{D} div \left(\frac{C}{\rho}\right)\right] \tag{2}$$

Where, C is the concentration of the fluid A into the other Host fluid B (i.e. C is the mass of fluid A per unit volume of the mixture) and $\bar{D}$ is the dispersion tensor coefficient of having unit $[\text{length}^2 \cdot \text{time}^{-1}]$ with nine components D_{ij} . Flow through a homogeneous porous medium in a laminar flow It can be called stable at a constant temperature. Then

$$div \cdot \bar{V} = 0 \tag{3}$$

Hence equation (2) can be written as

$$\frac{\partial C}{\partial t} + \bar{V} \cdot \nabla C = div\,[\bar{D}.divC] \tag{4}$$

when the seepage velocity $\bar{V}$ is along the x-axis, the non zero components are $D_{11} = D_L = \frac{L}{C_0^2}$ (coefficient of longitudinal dispersion) and $D_{22} = D$ (coefficient of transverse dispersion) and other D_{ij}'s are zero. In this case the equation (4) becomes [2],[7],[10],[11],[13],

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D_L \frac{\partial^2 C}{\partial x^2} \tag{5}$$

Where, u is the component of flow velocity $\bar{V}$ along the X-axis with the dimension $[\text{length} \cdot \text{time}^{-1}]$it is time Based in the non-negative direction along the x-axis, and $D_L = \gamma > 0$ is the cross-sectional flow velocity of porous Medium. It has been observed by Mehta[12, 13, 14] that the seepage u flow velocity is correlated with the concentration of the Material dispersing as material

$$u = \frac{C(x,t)}{C_0}, \text{ for } x > 0 \tag{6}$$

where, the concentration of the contaminated water at $x = 0$ is very high and it is constant $C_0 = 1(Mehta2006)$ By using equation (6) in the (5) , we get,

$$\frac{\partial C}{\partial t} + C\frac{\partial C}{\partial x} = \gamma \frac{\partial^2 C}{\partial x^2} \tag{7}$$

where, γ is the coefficient of longitudinal dispersion. The equation (7) is a non-linear Burgers' equation governing Miscible contaminated water for longitudinal dispersion Passage through porous media. As indicated in the declaration, the Dispersion is the movement of uni-directional displacements by Semi-finite homogeneous porous media, the seepage flow velocity of the contaminated water is believed to be unstable. Here here, The initial dispersion concentration is known to be an The highest constant concentration of contaminants at the input of $x=0$ corresponds to C_0.The porous medium is believed to be nonadsorbent. The governing partial equation of differentials (7) for In a semi finite, longitudinal hydrodynamic dispersion with In a unidirectional flow field, a non-adsorbent porous medium Where γ is the longitudinal dispersion coefficient, C is is the average cross-sectional concentration, u is the unsteady seepage velocity, x is a coordinate parallel to flow and t is time.

4 Numerical Solution of the problem using B-spline method

We set the equation to make the equation (7) dimensionless. Variables dimensionless as $X=\frac{x}{L}$ and $T=\frac{t}{L}$ so that $o\leq X\leq 1, 0\leq T\leq 1$. Hence the equation (7) reduced to

$$\frac{\partial C}{\partial T}+C\frac{\partial C}{\partial X}=\gamma\frac{\partial^2 C}{\partial X^2} \tag{8}$$

The required initial and boundary conditions are taken as

$$\begin{aligned} &C(X,0)=(1-X)^2, &&,0<X\leq 1\\ &C(0,1)=C_0=1. &&,0<T\leq 1\\ &C(1,T)=C_1=0.001, &&,0\leq T<1 \end{aligned} \tag{9}$$

The region $[0,1]$ is partitioned into uniformly sized finite elements f the length h by knots X_j such $0=X_0<X_1<\cdots<X_N=1$. Let $\Phi_m(X)$ be cubic B-splines with knots at the points $X_m, m=0,\cdots,N$. The set of splines $\{\Phi_{-1},\Phi_0,\Phi_1,\cdots,\Phi_N,\Phi_{N+1}\}$ forms a basis for functions defined over $[0,1]$ .Thus, an approximation $C_N(X,T)$ to the exact solution $C(X,T)$ can be expressed in terms of the cubic B-splines as trial functions:

$$C_N(X,T)=\sum_{m=-1}^{N+1}\delta_m(T)\Phi_m(X), \tag{10}$$

Cubic B-splines Φ_m with the required properties are defined by relationship

$$\Phi_m=\frac{1}{h^3}\begin{cases}(X-X_{m-2})^3 & [X_{m-2},X_{m-1}],\\ h^3+3h^2(X-X_{m-1})+3h(X-X_{m-1})^2-3(X-X_{m-1})^3 & [X_{m-1},X_m],\\ h^3+3h^2(X_{m+1}-X)+3h(X_{m+1}-X)^2-3(X_{m+1}-X)^3 & [X_m,X_{m+1}],\\ (X_{m+2}-X)^3 & [X_{m+1},X_{m+2}],\\ 0 & \text{Otherwise},\end{cases} \tag{11}$$

where $h=X_{m+1}-X_m, m=-1,\cdots,N+1$. The variable of $C_N(X,T)$ over typical element $[X_m,X_{m+1}]$ is given by

$$C_N(X,T)=\sum_{j=m-1}^{m+2}\delta_j(T)\Phi_j(X). \tag{12}$$

Using trial functon (10) and cubic splines (11), the values of C, C', C'' at the knots are determined in terms of the element parameters δ_m by

$$\begin{aligned} C_m&=C(X_m)=\delta_{m-1}+4\delta_m+\delta_{m+1},\\ C'&=C'(X_m)=\frac{3}{h}(\delta_{m+1}-\delta_{m-1})\\ C''&=C''(X_m)=\frac{6}{h^2}(\delta_{m-1}-2\delta_m+\delta_{m+1}) \end{aligned} \tag{13}$$

Where the symbol $'$ and $''$ denote first and second differentiation with respect to X , respectively.
At the knots, an approximate solution C_m for the Burgers' equation

$$C_T + CC_X - \gamma C_{XX} = 0 \tag{14}$$

can be obtained by considering the solution of

$$(C_T)_m^n + (1-\Theta) f_m^n + \Theta f_m^{n+1} = 0 \tag{15}$$

where $(f)_m^n = (CC_X)_m^n - \gamma (C_{XX})_m^n$.
The B-Spline Collocation method to the governing equation (8) with the appropriate conditions of the expression (9) has been employed as under

$$\begin{aligned}
&\delta_{m-1}^{n+1}\left(1+\Theta\Delta TL_2-\frac{3\Theta\Delta TL_1}{h}-\gamma\frac{6\theta\Delta T}{h^2}\right)+\delta_m^{n+1}\left(4+4\Theta\Delta TL_2+\gamma\frac{12\Theta\Delta T}{h^2}\right)\\
&+\delta_{m+1}^{n+1}\left(1+\Theta\Delta TL_2+\frac{3\Theta\Delta tL_1}{h}-\gamma\frac{6\Theta\Delta T}{h^2}\right)\\
&=L_1-(1-\Theta)\Delta t\left[L_1L_4+L_3L_2-L_3L_4-\gamma L_5\right]+\Theta\Delta tL_1L_2,
\end{aligned} \tag{16}$$

Where

$$\begin{aligned}
&L_1=\delta_{m-1}^n+4\delta_m^n+\delta_{m+1}^n, \qquad L_2=\frac{3}{h}\left(\delta_{m+1}^n-\delta_{m-1}^n\right)\\
&L_3=\delta_{m-1}^{n-1}n+4\delta_m^{n-1}+\delta_{m+1}^{n-1},\ L_4=\frac{3}{h}\left(\delta_{m+1}^{n-1}-\delta_{m-1}^{n-1}\right)\\
&L_5=\frac{6}{h^2}\left(\delta_{m-1}^n-2\delta_m^n+\delta_{m+1}^n\right)
\end{aligned}$$

The system (16) consists of $N+1$ linear equations in $N+3$ unknowns $d^n = \left(\delta_{-1}^n, \delta_0^n, \delta_1^n, \delta_2^n, ..., \delta_N^n, \delta_{N+1}^n\right)$. To obtain a unique solution to this system we need two additional constraints. these are obtained from the boundary condition. Imposition of boundary conditions enables us to elimination of parameters $\delta_{-1}, \delta_{N+1}$ from the system (16) with of equation

$$\begin{aligned}
C(X_0)&=\delta_{-1}^{n+1}+4\delta_0^{n+1}+\delta_1^{n+1}=1\Rightarrow\delta_{-1}^{n+1}=1-(4\delta_0^{n+1}+\delta_1^{n+1})\\
C(X_N)&=\delta_{N-1}^{n+1}+4\delta_N^{n+1}+\delta_{N+1}^{n+1}=0.001\Rightarrow\delta_{N+1}^{n+1}=0.001-(\delta_{N-1}^{n+1}+4\delta_N^{n+1})
\end{aligned} \tag{17}$$

so the system (16) is reduced to $(N+1)\times(N+1)$ matrix system, which can be solved by using the Thomas Algorithm.

5 NUMERICAL AND GRAPHICAL REPRESENTATION OF THE SOLUTION

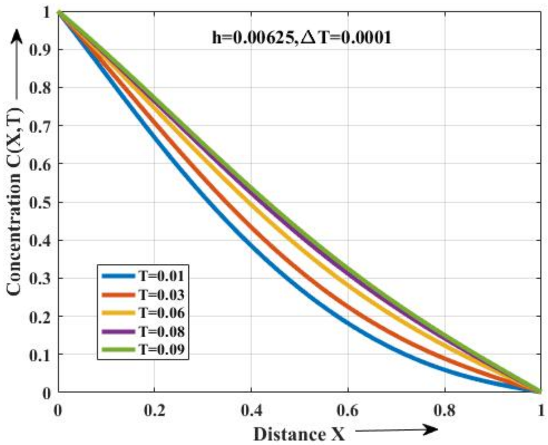

Figure 2: The Concentration $C(X,T)$ vs. distance. X

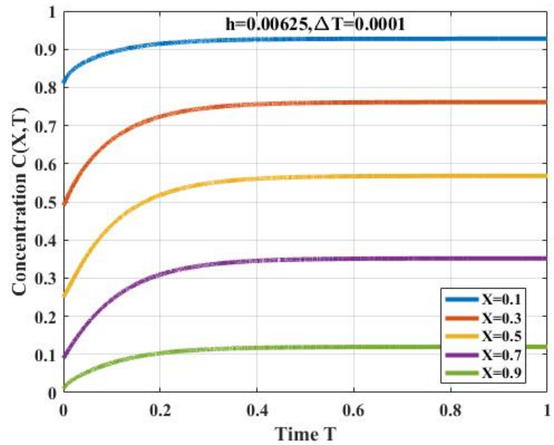

Figure 3: The Concentration $C(X,T)$ vs. Time T

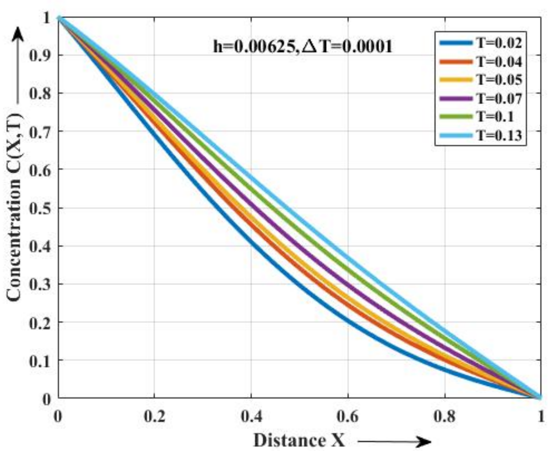

Figure 4: The Concentration $C(X,T)$ vs. distance. X

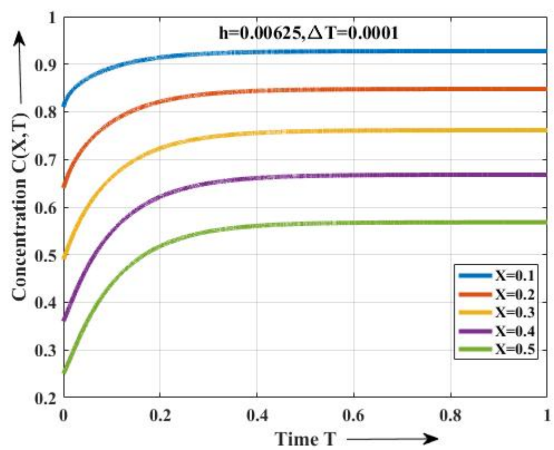

Figure 5: The Concentration $C(X,T)$ vs. Time T

X/T	T=0.02	T=0.04	T=0.05	T=0.07	T=0.1	T=0.13
X=0	1	1	1	1	1	1
0.05	0.92435	0.93294	0.93616	0.94144	0.94739	0.95173
0.1	0.84651	0.86356	0.87003	0.88066	0.89265	0.90143
0.15	0.76802	0.79290	0.80250	0.81833	0.83629	0.84945
0.2	0.69034	0.72200	0.73447	0.75519	0.77881	0.79618
0.25	0.61477	0.65189	0.66686	0.69196	0.72076	0.74201
0.3	0.54234	0.58349	0.60052	0.62934	0.66266	0.68733
0.35	0.47384	0.51763	0.53622	0.56799	0.60501	0.63254
0.4	0.40982	0.45500	0.47462	0.50849	0.54827	0.57799
0.45	0.35064	0.39612	0.41626	0.45133	0.49283	0.52400
0.5	0.29649	0.34136	0.36151	0.39687	0.43903	0.47085
0.55	0.24744	0.29094	0.31062	0.34537	0.38710	0.41874
0.6	0.20349	0.24492	0.26369	0.29698	0.33720	0.36784
0.65	0.16453	0.20322	0.22067	0.25170	0.28939	0.31822
0.7	0.13040	0.16563	0.18138	0.20942	0.24364	0.26991
0.75	0.10078	0.13182	0.14553	0.16994	0.19983	0.22285
0.8	0.07528	0.10136	0.11272	0.13295	0.15777	0.17695
0.85	0.05333	0.07370	0.08246	0.09804	0.11720	0.13204
0.9	0.03422	0.04822	0.05418	0.06476	0.07780	0.08790
0.95	0.01710	0.02423	0.02725	0.03260	0.03919	0.04431
X=1	0.001	0.001	0.001	0.001	0.001	0.001

Table 1: Concentration $C(X,T)$ $h = 0.00625, \Delta T = 0.0001$

6 CONCLUSION

The graphical and numerical solutions of Burger's equation using the B-spline method have been obtained to Predict the possible level of contaminated water in the Flow of unstable unidirectional seepage, by semi-finite, Subject to the source, homogeneous isotropic porous media The concentration varies according to distance X and time $T > 0$. It is inferred from the tabular values and graphs that as Distance X and time T increase the concentration of Contaminant water decreases. The concentration $C(X,T)$ of the contaminated water decreases as the distance X increases for the given time $T > 0$.Here the initial concentration of contaminated water at $X = 0$ is highest and it decreases as distance X increases for given time $T > 0$. Physically, it is a fact that the concentration at the source is Contaminated water is the maximum at all times and decreases And from the source, dispersing.It is also inferred from the Figure (3,5) of contaminated water concentration Time T for the specified distance X, the concentration of verses Contaminated water increases for small time T and then it As time T increases for given $X = 0.1, 0.2, 0.3, 0.4, 0.5$.The numerical outcomes have appeared for the particular estimations of dT uncovers that the B-Spline collocation gives the solution for Burgers' equation arising in Longitudinal Dispersion Phenomena in Fluid Flow through Porous Media. The numerical results are found in good agreement using B-spline collocation method. The numerical solution using the B-spline Collocation method obtained here is immense. Useful to monitor saline water intrusion until it is Contaminates the aquifer system with fresh water; it is also helpful in making a quantitative prediction on the possible Contamination of the sources of groundwater arising from the Movement of groundwater by buried wastes. The outcome is Consistent with the physical longitudinal phenomenon Miscible fluid dispersion flows through porous media.

7 NOMENCLATURE

C_0=Initial Concentration of concentration of contaminant
C=Concentration of solute in liquid phase
ρ=density of the fluid
$\bar{V}$=Pore seepage velocity vector.
D_L=Longitudinal dispersion coefficient γ based on u
t=Time(s)
x=Linear distance coordinate (m)

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