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Original Article

On Some New Continuous Mapping and Compactness in Neutrosophic Topological Spaces

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Abstract *- The intention of this article is to introduce and develop the concept of Neutrosophic almost gpr continuous mappings, Neutrosophic almost gpr closed mappings using NGPRCS and the interrelations between the new mappings and existing mappings are established. Also we extend our study to Neutrosophic gpr compactness in Neutrosophic topological spaces.*

Keywords - *Neutrosophic topology, Neutrosophic Point, NGPRCS, Neutrosophic almost gpr continuous mappings, Neutrosophic almost gpr closed mappings and Neutrosophic gpr compactness.*

AMS Subject Classifications: 54A40, 03E72.

1. Introduction

In 1965, L.A.Zadeh [17] introduced the notion of fuzzy sets [FS]. It shows the degree of membership of the element in a set X. Later, fuzzy topological space was introduced by C.L. Chang [3] in 1968. In 1986, K.Atanassov [2] introduced the notion of intuitionistic fuzzy sets [IFS], where the degree of membership and degree of non-membership are discussed. Later, Intuitionistic fuzzy topological spaces was introduced by Coker [4] in 1997. In 2010, Florentin Smarandache [6] defined the Neutrosophic set on three components, namely Truth (membership), Indeterminacy, Falsehood (non-membership). In 2012, A. A Salama and S. A. Alblowi [14] introduced the concept of Neutrosophic topological space by using Neutrosophic sets. A. A. Salama [15] introduced Neutrosophic closed set and Neutrosophic continuous function in Neutrosophic topological spaces. I.Mohammed Ali Jaffer and K.Ramesh [10] introduced Neutrosophic generalized pre regular closed sets in Neutrosophic topological spaces. Wadei Al-Omeri and Saeid Jafari [16] introduced Generalized Closed Sets, Generalized Pre-Closed, Generalized connectedness and Generalized compactness in Neutrosophic Topological Spaces. Parimala M et al[12] introduced Neutrosophic $\alpha\psi$ Homeomorphism in Neutrosophic Topological Spaces

In this direction, we introduce the concept of Neutrosophic almost gpr continuous mappings, Neutrosophic almost gpr closed mappings and the interrelations among these mappings and existing mappings in Neutrosophic topological spaces are established. Further we extend our study to the concept of Neutrosophic gpr compactness in Neutrosophic topological spaces.

2. Preliminaries

Definition 2.1: [14] Let X be a non-empty fixed set. A Neutrosophic set (NS for short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A.

Remark 2.2: [14] A Neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ can be identified to an ordered triple $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ in non-standard unit interval $]^-0, 1^+[$ on X.

Remark 2.3: [14] For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \nu_A\rangle$ for the Neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$.

Example 2.4: [14] Every IFS A is a non-empty set in X is obviously a NS having the form $A = \{\langle x, \mu_A(x), 1 - (\mu_A(x)+ \nu_A(x)), \nu_A(x)\rangle : x \in X\}$. Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the NS 0_N and 1_N in X as follows:

0_N may be defined as:
1_N may be defined as:

(0_1) $0_N = \{\langle x, 0, 0, 1\rangle: x \in X\}$
(0_2) $0_N = \{\langle x, 0, 1, 1\rangle: x \in X\}$
(0_3) $0_N = \{\langle x, 0, 1, 0\rangle: x \in X\}$
(0_4) $0_N = \{\langle x, 0, 0, 0\rangle: x \in X\}$

(1_1) $1_N = \{\langle x, 1, 0, 0\rangle: x \in X\}$
(1_2) $1_N = \{\langle x, 1, 0, 1\rangle: x \in X\}$
(1_3) $1_N = \{\langle x, 1, 1, 0\rangle: x \in X\}$
(1_4) $1_N = \{\langle x, 1, 1, 1\rangle: x \in X\}$

Definition 2.5: [14] Let $A = \langle \mu_A, \sigma_A, \nu_A\rangle$ be a NS on X, then the complement of the set A [C(A) for short] may be defined as three kind of complements:
(C_1) $C(A) = \{\langle x, 1-\mu_A(x), 1-\sigma_A(x), 1-\nu_A(x)\rangle: x \in X\}$
(C_2) $C(A) = \{\langle x, \nu_A(x), \sigma_A(x), \mu_A(x)\rangle: x \in X\}$
(C_3) $C(A) = \{\langle x, \nu_A(x), 1-\sigma_A(x), \mu_A(x)\rangle: x \in X\}$

Definition 2.6: [14] Let X be a non-empty set and Neutrosophic sets A and B in the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle: x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle: x \in X\}$. Then we may consider two possible definitions for subsets ($A \subseteq B$).
(1) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x)$ and $\mu_A(x) \ge \mu_B(x)\ \forall\ x \in X$
(2) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x)$ and $\mu_A(x) \ge \mu_B(x)\ \forall\ x \in X$

Definition 2.7: [14] Let X be a non-empty set and $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle: x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle: x \in X\}$ are NSs. Then
(1) $A \cap B$ may be defined as:
(I_1) $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\nu_A(x) \vee \nu_B(x)\rangle$
(I_2) $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\nu_A(x) \vee \nu_B(x)\rangle$
(2) $A \cup B$ may be defined as:
(U_1) $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\nu_A(x) \wedge \nu_B(x)\rangle$
(U_2) $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\nu_A(x) \wedge \nu_B(x)\rangle$

We can easily generalize the operations of intersection and union in Definition 2.8., to arbitrary family of NSs as follows:

Proposition 2.8: [14] For all A and B are two Neutrosophic sets then the following conditions are true:
$C(A \cap B) = C(A) \cup C(B)$; $C(A \cup B) = C(A) \cap C(B)$.

Definition 2.9: [14] A Neutrosophic topology [NT for short] on a non-empty set X is a family τ of Neutrosophic subsets in X satisfying the following axioms:
(NT_1) $0_N, 1_N \in \tau$,
(NT_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(NT_3) $\cup\, G_i \in \tau$ for every $\{ G_i : i \in J\} \subseteq \tau$.

Throughout this paper, the pair (X, τ) is called a Neutrosophic topological space (NTS for short). The elements of τ are called Neutrosophic open sets [NOS for short]. A complement C(A) of a NOS A in NTS (X, τ) is called a Neutrosophic closed set [NCS for short] in X.

Definition 2.10: [14] Let (X, τ) be NTS and $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle: x \in X\}$ be a NS in X. Then the Neutrosophic closure and Neutrosophic interior of A are defined by
$NCl(A) = \cap\{K : K \text{ is a NCS in X and } A \subseteq K\}$
$NInt(A) = \cup\{G : G \text{ is a NOS in X and } G \subseteq A\}$
It can be also shown that NCl(A) is NCS and NInt(A) is a NOS in X.
a) A is NOS if and only if A = NInt(A),
b) A is NCS if and only if A = NCl(A).

Definition 2.11: [7] A NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle: x \in X\}$ in a NTS (X, τ) is said to be
(i) Neutrosophic regular closed set (NRCS for short) if A = NCl(NInt(A)),
(ii) Neutrosophic regular open set (NROS for short) if A = NInt(NCl(A)),
(iii) Neutrosophic semi closed set (NSCS for short) if $NInt(NCl(A)) \subseteq A$,
(iv) Neutrosophic semi open set (NSOS for short) if $A \subseteq NCl(NInt(A))$,
(v) Neutrosophic pre closed set (NPCS for short) if $NCl(NInt(A)) \subseteq A$,
(vi) Neutrosophic pre open set (NPOS for short) if $A \subseteq NInt(NCl(A))$.
(vii) Neutrosophic α- closed set (Nα CS for short) if $NCl(NInt(NCl(A))) \subseteq A$,
(viii) Neutrosophic α- open set (Nα OS for short) if $A \subseteq NInt(NCl(NInt(A)))$,

Definition 2.12: [16] Let (X, τ) be NTS and A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} be a NS in X. Then the Neutrosophic pre closure and Neutrosophic pre interior of A are defined by

NPCl(A) = ∩{K : K is a NPCS in X and A ⊆ K},
NPInt(A) = ∪{G : G is a NPOS in X and G ⊆ A}.

Definition 2.13: [13] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic generalized closed set (NGCS for short) if NCl(A) ⊆ U whenever A ⊆ U and U is NOS in (X, τ). A NS A of a NTS (X, τ) is called Neutrosophic generalized open set (NGOS for short) if C(A) is a NGCS in (X, τ).

Definition 2.14: [9] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic α- generalized closed set (NαGCS for short) if NαCl(A) ⊆ U whenever A ⊆ U and U is a NOS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic α- generalized open set (NαGOS for short) if C(A) is a NαGCS in (X, τ).

Definition 2.15: [16] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic generalized pre closed set (NGPCS for short) if NPCl(A) ⊆ U whenever A ⊆ U and U is a NOS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic generalized pre open set (NGPOS for short) if C(A) is a NGPCS in (X, τ).

Definition 2.16: [10] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic generalized pre regular closed set (NGPRCS for short) if NPCl(A) ⊆ U whenever A ⊆ U and U is a NROS in (X, τ). The family of all NGPRCSs of a NTS(X, τ) is denoted by NGPRC(X). A NS A of a NTS (X, τ) is called Neutrosophic generalized pre regular open set (NGPROS for short) if C(A) is NGPRCS in (X, τ).

Definition 2.17: [10] A Neutrosophic topological space (X, τ) is called a Neutrosophic pre regular $T_{1/2}$ ($NPRT_{1/2}$ for short) space if every NGPRCS in (X, τ) is NPCS in (X, τ). A Neutrosophic topological space (X, τ) is called a Neutrosophic pre regular $T^*_{1/2}$ ($NPRT^*_{1/2}$ for short) space if every NGPRCS in (X, τ) is NCS in (X, τ).

Definition 2.18: [15] Let (X, τ) and (Y, σ) be two Neutrosophic topological spaces. A mapping f: (X, τ)→(Y, σ) is called Neutrosophic continuous if the inverse image of every NCS in (Y, σ) is a NCS in (X, τ).

Definition 2.19: [1] Let (X, τ) and (Y, τ_1) be two Neutrosophic topological spaces. Then the

(i) mapping f: (X, τ)→(Y, τ_1) is called a Neutrosophic α continuous if the inverse image of every NCS in (Y, σ) is a NαCS in (X, τ).
(ii) mapping f: (X, τ)→(Y, τ_1) is called a Neutrosophic pre continuous if the inverse image of every NCS in (Y, σ) is a NPCS in (X, τ).

Definition 2.20: [5] Let (X, τ) and (Y, τ_1) be two Neutrosophic topological spaces. A mapping f: (X, τ)→(Y, τ_1) is called Neutrosophic generalized continuous mapping if the inverse image of every NCS in (Y, τ_1) is a NGCS in (X, τ).

Definition 2.21: [11] Let (X, τ) and (Y, τ_1) be two Neutrosophic topological spaces. A mapping f: (X, τ)→(Y, τ_1) is called Neutrosophic α generalized continuous mapping if the inverse image of every NCS in (Y, τ_1) is a NαGCS in (X, τ).

Definition 2.22: [8] Let (X, τ) and (Y, τ_1) be two Neutrosophic topological spaces. A mapping f: (X, τ)→(Y, τ_1) is called Neutrosophic regular α generalized continuous mapping if the inverse image of every NCS in (Y, τ_1) is a NRαGCS in (X, τ).

Definition 2.23: [12] Let (X, τ) and (Y, σ) be two NTSs. A mapping f: (X, τ) → (Y, σ) is called Neutrosophic closed mapping (resp. Neutrosophic open mapping) (NCM (resp. NOM) for short) if the image of every NCS (resp. NOS) in (X, τ) is a NCS(resp. NOS) in (Y, σ).

Definition 2.24: [16] A NTS (X, τ) is called neutrosophic compact space iff every neutrosophic open cover of (X, τ) has a finite subcover.

Definition 2.25: [16] A NTS (X, τ) is called NG compact space iff every NG open cover of (X, τ) has a finite subcover.

3. Neutrosophic almost gpr continuous mappings

Definition 3.1. A mapping f: (X, τ)→(Y, τ_1) is said to be a neutrosophic almost generalized pre regular continuous mapping (NaGPR continuous mapping for short) if $f^{-1}(A)$ is a NGPRCS in X for every NRCS A in Y.

Example 3.2: Let X= {a, b} and Y = {u, v}. Then τ = {0_N, U, 1_N} and τ_1 = {0_N, V, 1_N} are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.5, 0.3, 0.5), (0.5, 0.3, 0.5)⟩ and V =⟨y, (0.3, 0.2, 0.6), (0.1, 0.2, 0.7)⟩. Define a mapping f: (X, τ)→(Y, τ_1) by f(a) = u and f(b) = v. Here the Neutrosophic set V^c is a NRCS in Y. Then $f^{-1}(V^c)$ is a NGPRCS in (X, τ) as $f^{-1}(V^c) \subseteq 1_N$ and Npcl($f^{-1}(V^c)$) ⊆ 1_N where 1_N is a NROS in X. Therefore f is a NaGPR continuous mapping.

Theorem 3.3: Every Neutrosophic continuous mapping is a NaGPR continuous mapping but not conversely.
Proof: Let f : (X, τ) → (Y, τ_1) be a Neutrosophic continuous mapping. Let V be a NRCS in Y. Since every NRCS is a NCS, V is a NCS in Y. Then $f^{-1}(V)$ is a NCS in X, by hypothesis. Since every NCS is a NGPRCS, $f^{-1}(V)$ is a NGPRCS in X. Hence f is a NaGPR continuous mapping.

Example 3.4: Let X = {a, b} and Y = {u,v}. Then τ = {0_N, U, 1_N} and τ_1 = {0_N, V, 1_N} are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.6, 0.3, 0.3), (0.7, 0.3, 0.3)⟩ and V = ⟨y, (0.4, 0.3, 0.6), (0.4, 0.3, 0.6)⟩. Define a mapping f: (X, τ)→(Y, τ_1) by f(a) = u and f(b) = v. Here the Neutrosophic set V^c is a NRCS in Y. Then $f^{-1}(V^c)$ is a NGPRCS in X as $f^{-1}(V^c) \subseteq 1_N$ and $Npcl(f^{-1}(V^c)) = f^{-1}(V^c) \subseteq 1_N$ where 1_N is a NROS in X. Hence f is a NaGPR continuous mapping. But f is not a Neutrosophic continuous mapping.

Theorem 3.5: Every Nα continuous mapping is a NaGPR continuous mapping but not conversely.
Proof: Let f : (X, τ) → (Y, τ_1) be a Nα continuous mapping. Let V be a NRCS in Y. Since every NRCS is a NCS, V is a NCS in Y. Then $f^{-1}(V)$ is a NαCS in X. Since every NαCS is a NGPRCS, $f^{-1}(V)$ is a NGPRCS in X. Hence f is a NaGPR continuous mapping.

Example 3.6: Let X = {a, b} and Y = {u,v}. Then τ = {0_N, U, 1_N} and τ_1 = {0_N, V, 1_N} are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.5, 0.4, 0.5), (0.4, 0.4, 0.6)⟩ and V = ⟨y, (0.4, 0.3, 0.6), (0.4, 0.3, 0.6)⟩. Define a mapping f: (X, τ)→(Y, τ_1) by f(a) = u and f(b) = v. Here the Neutrosophic set V^c is a NRCS in Y. Then $f^{-1}(V^c)$ is a NGPRCS in X as $f^{-1}(V^c) \subseteq 1_N$ and $Npcl(f^{-1}(V^c)) = f^{-1}(V^c) \subseteq 1_N$ where 1_N is a NROS in X. Hence f is a NaGPR continuous mapping. But f is not a Nα continuous mapping since V^c is Neutrosophic closed set in Y but $f^{-1}(V^c)$ is not a NαCS in X as $Ncl(Nint(Ncl(f^{-1}(V^c)))) = 1_N \nsubseteq f^{-1}(V^c)$.

Theorem 3.7: Every NP continuous mapping is a NaGPR continuous mapping but not conversely.
Proof: Let f : (X, τ) → (Y, τ_1) be a NP continuous mapping. Let V be a NRCS in Y. Since every NRCS is a NCS, V is a NCS in Y. Then $f^{-1}(V)$ is a NPCS in X. Since every NPCS is a NGPRCS, $f^{-1}(V)$ is a NGPRCS in X. Hence f is a NaGPR continuous mapping.

Example 3.8: Let X = {a, b} and Y = {u,v}. Then τ = {0_N, U_1, U_2, 1_N} and τ_1 = {0_N, V, 1_N} are Neutrosophic topologies on X and Y respectively, where U_1 = ⟨x, (0.5, 0.3, 0.5), (0.6, 0.3, 0.3)⟩, U_2 = ⟨x, (0.5, 0.3, 0.5), (0.2, 0.3, 0.8)⟩ and V = ⟨y, (0.2, 0.4, 0.7), (0.3, 0.5, 0.6)⟩. Define a mapping f: (X, τ)→(Y, τ_1) by f(a) = u and f(b) = v. Here the Neutrosophic set V^c = ⟨y, (0.7, 0.6, 0.2), (0.6, 0.5, 0.3)⟩ is a NRCS in Y. Then $f^{-1}(V^c)$ = ⟨x, (0.7, 0.6, 0.2), (0.6, 0.5, 0.3)⟩ is a NGPRCS in X as $f^{-1}(V^c) \subseteq 1_N$ and $Npcl(f^{-1}(V^c)) = f^{-1}(V^c) \subseteq 1_N$ where 1_N is a NROS in X. Hence f is a NaGPR continuous mapping. But f is not a NP continuous mapping since V^c is NCS in Y but $f^{-1}(V^c)$ is not a NPCS in X as $Ncl(Nint(f^{-1}(V^c))) \subseteq U_2^c \nsubseteq f^{-1}(V^c)$.

Theorem 3.9: Every NG continuous mapping is a NaGPR continuous mapping but not conversely.
Proof: Let f : (X, τ) → (Y, τ_1) be a NG continuous mapping. Let V be a NRCS in Y. Since every NRCS is a NCS, V is a NCS in Y. Then $f^{-1}(V)$ is a NGCS in X. Since every NGCS is a NGPRCS, $f^{-1}(V)$ is a NGPRCS in X. Hence f is a NaGPR continuous mapping.

Example 3.10: Let X = {a, b} and Y = {u,v}. Then τ = {0_N, U, 1_N} and τ_1 = {0_N, V, 1_N} are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.6, 0.7, 0.3), (0.7, 0.7, 0.3)⟩ and V = ⟨y, (0.4, 0.4, 0.5), (0.4, 0.5, 0.6)⟩. Define a mapping f: (X, τ)→(Y, τ_1) by f(a) = u and f(b) = v. Here the Neutrosophic set V^c is a NRCS in Y. Then $f^{-1}(V^c)$ is a NGPRCS in X as $f^{-1}(V^c) \subseteq 1_N$ and $Npcl(f^{-1}(V^c)) = f^{-1}(V^c) \subseteq 1_N$ where 1_N is a NROS in X. Hence f is a NaGPR continuous mapping. But f is not a NG continuous mapping since V^c is Neutrosophic closed set in Y but $f^{-1}(V^c)$ is not a NGCS in X as $f^{-1}(V^c) \subseteq U$ and $Ncl(f^{-1}(V^c)) = 1_N \nsubseteq U$ where U is a NOS in X.

Theorem 3.11: Every NGP continuous mapping is an NaGPR continuous mapping but not conversely.
Proof: Let f : (X, τ) → (Y, τ_1) be a NGP continuous mapping. Let V be a NRCS in Y. Since every NRCS is a NCS, V is a NCS in Y. Then $f^{-1}(V)$ is a NGPCS in X. Since every NGPCS is a NGPRCS, $f^{-1}(V)$ is a NGPRCS in X. Hence f is a NaGPR continuous mapping.

Example 3.12: Let X = {a, b} and Y = {u,v}. Then τ = {0_N, U_1, U_2, 1_N} and τ_1 = {0_N, V, 1_N} are Neutrosophic topologies on X and Y respectively, where U_1 = ⟨x, (0.6, 0.5, 0.4), (0.6, 0.5, 0.4)⟩, U_2 = ⟨x, (0.8, 0.5, 0.2), (0.8, 0.5, 0.2)⟩ and V = ⟨y, (0.3, 0.5, 0.7), (0.3, 0.5, 0.7)⟩. Define a mapping f: (X, τ)→(Y, τ_1) by f(a) = u and f(b) = v. Here the Neutrosophic set V^c is a NRCS in

Y. Then $f^{-1}(V^c)$ is a NGPRCS in X as $f^{-1}(V^c) \subseteq 1_N$ and $Npcl(f^{-1}(V^c)) = 1_N \subseteq 1_N$ where 1_N is a NROS in X. Hence f is a NaGPR continuous mapping. But f is not a NGP continuous mapping since V^c is NCS in Y but $f^{-1}(V^c)$ is not a NGPCS in X as as $f^{-1}(V^c) \subseteq U_2$ and $Npcl(f^{-1}(V^c)) = 1_N \nsubseteq U_2$ where U_2 is a NOS in X.

Theorem 3.13: Every NαG continuous mapping is a NaGPR continuous mapping but not conversely.
Proof: Let $f : (X, \tau) \rightarrow (Y, \tau_1)$ be a NαG continuous mapping. Let V be a NRCS in Y. Since every NRCS is a NCS, V is a NCS in Y. Then $f^{-1}(V)$ is a NαGCS in X. Since every NαGCS is a NGPRCS, $f^{-1}(V)$ is a NGPRCS in X. Hence f is a NaGPR continuous mapping.

Example 3.14: Let X = {a, b} and Y = {u,v}. Then $\tau = \{0_N, U, 1_N\}$ and $\tau_1 = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.7, 0.5, 0.3), (0.9, 0.5, 0.1)⟩ and V = ⟨y, (0.4, 0.5, 0.6), (0.3, 0.5, 0.7)⟩. Define a mapping f: $(X, \tau)\rightarrow(Y, \tau_1)$ by f(a) = u and f(b) = v. Here the Neutrosophic set V^c is a NRCS in Y. Then $f^{-1}(V^c)$ is a NGPRCS in X as $f^{-1}(V^c) \subseteq 1_N$ and $Npcl(f^{-1}(V^c)) = f^{-1}(V^c) \subseteq 1_N$ where 1_N is a NROS in X. Hence f is a NaGPR continuous mapping. But f is not a NαG continuous mapping, since V^c is NCS in Y but $f^{-1}(V^c)$ is not a NαGCS in X as $f^{-1}(V^c) \subseteq U$ and $N\alpha cl(f^{-1}(V^c)) = 1_N \nsubseteq U$.

Theorem 3.15: Every NRαG continuous mapping is a NaGPR continuous mapping but not conversely.
Proof: Let $f : (X, \tau) \rightarrow (Y, \tau_1)$ be a NRαG continuous mapping. Let V be a NRCS in Y. Since every NRCS is a NCS, V is a NCS in Y. Then $f^{-1}(V)$ is a NRαGCS in X. Since every NRαGCS is a NGPRCS, $f^{-1}(V)$ is a NGPRCS in X. Hence f is a NaGPR continuous mapping.

Example 3.16: Let X = {a, b} and Y = {u,v}. Then $\tau = \{0_N, U_1, U_2, 1_N\}$ and $\tau_1 = \{0_N, V_1, V_2, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where U_1 = ⟨x, (0.5, 0.5, 0.5), (0.6, 0.5, 0.3)⟩, U_2 = ⟨x, (0.4, 0.3, 0.6), (0.2, 0.3, 0.7)⟩, V_1= ⟨y, (0.5, 0.5, 0.5), (0.2, 0.5, 0.8)⟩ and V_2= ⟨y, (0.7, 0.7, 0.3), (0.7, 0.7, 0.2)⟩ . Define a mapping f: $(X, \tau)\rightarrow(Y, \tau_1)$ by f(a) = u and f(b) = v. Here the Neutrosophic set V_1^c is a NRCS in Y. Then $f^{-1}(V_1^c)$ is a NGPRCS in X as $f^{-1}(V_1^c) \subseteq U_1$ and $Npcl(f^{-1}(V_1^c)) = f^{-1}(V_1^c) \subseteq U_1$ where U_1 is a NROS in X. Hence f is a NaGPR continuous mapping. But f is not a NRαG continuous mapping, since V_2^c is NCS in Y but $f^{-1}(V_2^c)$ is not a NRαGCS in X as $f^{-1}(V_2^c) \subseteq U_2$ and $N\alpha cl(f^{-1}(V_2^c)) = U_1^c \nsubseteq U_2$ where U_2 is a NROS in X.

Theorem 3.17: Let $f : (X, \tau) \rightarrow (Y, \tau_1)$ be a mapping where $f^{-1}(V)$ is a NRCS in X for every NCS in Y. Then f is a NaGPR continuous mapping but not conversely.
Proof: Let A be a NRCS in Y. Since every NRCS is a NCS, V is a NCS in Y. Then $f^{-1}(V)$ is a NRCS in X. Since every NRCS is a NGPRCS, $f^{-1}(V)$ is a NGPRCS in X. Hence f is a NaGPR continuous mapping.

Example 3.18: Let X = {a, b} and Y = {u,v}. Then $\tau = \{0_N, U, 1_N\}$ and $\tau_1 = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.5, 0.4, 0.5), (0.4, 0.4, 0.6)⟩ and V = ⟨y, (0.4, 0.3, 0.6), (0.4, 0.3, 0.6)⟩. Define a mapping f: $(X, \tau)\rightarrow(Y, \tau_1)$ by f(a) = u and f(b) = v. Then f is a NaGPR continuous mapping. But not a mapping as defined in Theorem 3.17.

Theorem 3.19: Let $f : (X, \tau) \rightarrow (Y, \tau_1)$ be a mapping. Then the following are equivalent:
(i) f is a NaGPR continuous mapping,
(ii) $f^{-1}(A)$ is a NGPROS in X for every NROS A in Y.

Proof: (i) ⇒ (ii) Let A be a NROS in Y. Then A^c is a NRCS in Y. By hypothesis, $f^{-1}(A^c)$ is a NGPRCS in X. That is $f^{-1}(A)^c$ is a NGPRCS in X. Therefore $f^{-1}(A)$ is a NGPROS in X.
(ii) ⇒ (i) Let A be a NRCS in Y. Then A^c is a NROS in Y. By hypothesis, $f^{-1}(A^c)$ is a NGPROS in X. That is $f^{-1}(A)^c$ is a NGPROS in X. Therefore $f^{-1}(A)$ is a NGPRCS in X. Then f is a NaGPR continuous mapping.

Definition 3.20: A NS A is said to be neutrosophic dense (ND for short) in another NS B in NTS (X, τ) if Ncl(A) = B.

Theorem 3.21: Let $p_{(\alpha, \beta)}$ be a NP in X. A mapping $f : (X, \tau)\rightarrow(Y, \tau_1)$ is a NaGPR continuous mapping if for every NOS A in Y with $f(p_{(\alpha, \beta)}) \in A$, there exists a NOS B in X with $p_{(\alpha, \beta)} \in B$ such that $f^{-1}(A)$ is ND in B.
Proof: Let A be a NROS in Y. Then A is NOS in Y. Let $f(p_{(\alpha, \beta)}) \in A$, then there exists a NOS B in X such that $p_{(\alpha, \beta)} \in B$ and $Ncl(f^{-1}(A)) = B$. Since B is an NOS, $Ncl(f^{-1}(A))$ is also a NOS in X. Therefore $Nint(Ncl(f^{-1}(A))) = Ncl(f^{-1}(A))$. Now $f^{-1}(A) \subseteq Ncl(f^{-1}(A)) = Nint(Ncl(f^{-1}(A))) \subseteq Ncl(Nint(Ncl(f^{-1}(A))))$. This implies $f^{-1}(A)$ is a NPOS in X and hence an NGPROS in X. Thus f is a NaGPR continuous mapping.

Theorem 3.22: Let f : (X, τ) $\rightarrow$ (Y, τ_1) be a mapping where X is an $NPRT_{1/2}$ space. Then the following are equivalent:
(i) f is an NaGPR continuous mapping,
(ii) $f^{-1}(A) \subseteq Npint(f^{-1}(Nint(Ncl(A))))$ for every NPOS A in Y.

Proof:
(i) $\Rightarrow$ (ii) Let A be a NPOS in Y. Then $A \subseteq Nint(Ncl(A))$. Since, Nint(Ncl(A)) is NROS in Y, by hypothesis $f^{-1}(Nint(Ncl(A)))$ is NGPROS in X. Since X is $NPRT_{1/2}$ space, $f^{-1}(Nint(Ncl(A)))$ is a NPOS in X. Therefore $f^{-1}(A) \subseteq f^{-1}(Nint(Ncl(A))) = Npint(f^{-1}(Nint(Ncl(A))))$.
(ii) $\Rightarrow$ (i) Let A be a NROS in Y. Then A is a NPOS in Y. By hypothesis $f^{-1}(A) \subseteq Npint(f^{-1}(Nint(Ncl(A)))) = Npint(f^{-1}(A)) \subseteq f^{-1}(A)$. This implies $f^{-1}(A)$ is a NPOS in X and hence is a NGPROS in X. Therefore f is a NaGPR continuous mapping.

Theorem 3.23: Let f : (X, τ) $\rightarrow$ (Y, τ_1) be a mapping. If f is NaGPR continuous mapping, then $Ngprcl(f^{-1}(A)) \subseteq f^{-1}(Ncl(A))$ for every NROS A in Y.
Proof: Let A be a NROS in Y. Then Ncl(A) is a NRCS in Y. By hypothesis $f^{-1}(Ncl(A))$ is a NGPRCS in X. Then $Ngprcl(f^{-1}(Ncl(A))) = f^{-1}(Ncl(A))$. Now $Ngprcl(f^{-1}(A)) \subseteq Ngprcl(f^{-1}(Ncl(A))) = f^{-1}(Ncl(A))$. That is $Ngprcl(f^{-1}(A) \subseteq f^{-1}(Ncl(A))$.

Theorem 3.24: Let f : (X, τ) $\rightarrow$ (Y, τ_1) be a mapping. If $f^{-1}(Npint(B)) \subseteq Npint(f^{-1}(B))$ for every NS B in Y, then f is a NaGPR continuous mapping.
Proof: Let $B \subseteq Y$ be a NROS. By hypothesis, $f^{-1}(Npint(B)) \subseteq Npint(f^{-1}(B))$. Since B is a NROS, it is a NPOS in Y. Therefore Npint(B) = B. Hence $f^{-1}(B) = f^{-1}(Npint(B)) \subseteq Npint(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is a NPOS and hence a NGPROS in X. Thus f is a NaGPR continuous mapping.

Remark 3.25: The converse of the above theorem is true if $B \subseteq Y$ is a NROS and X is a $NPRT_{1/2}$ space.

Proof: Let f be a NaGPR continuous mapping. Let B be a NROS in Y. Then $f^{-1}(B)$ is a NGPROS in X. Since X is a $NPRT_{1/2}$ space, $f^{-1}(B)$ is a NPOS in X. Therefore $f^{-1}(Npint(B)) \subseteq f^{-1}(B) = Npint(f^{-1}(B))$. That is $f^{-1}(Npint(B)) \subseteq Npint(f^{-1}(B))$.

Theorem 3.26: Let f : (X, τ) $\rightarrow$ (Y, τ_1) be a mapping. If $Npcl(f^{-1}(B)) \subseteq f^{-1}(Npcl(B))$ for every NS B in Y, then f is a NaGPR continuous mapping.

Proof: Let $B \subseteq Y$ be a NRCS. By hypothesis, $Npcl(f^{-1}(B)) \subseteq f^{-1}(Npcl(B))$. Since B is a NRCS, it is a NPCS in Y. Therefore Npcl(B) = B. Hence $f^{-1}(B) = f^{-1}(Npcl(B)) \supseteq Npcl(f^{-1}(B)) \supseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is a NPCS and hence a NGPRCS in X. Thus f is a NaGPR continuous mapping.

Remark 3.27: The converse of the above theorem is true if $B \subseteq Y$ is a NRCS and X is a $NPRT_{1/2}$ space.

Proof: Let f be a NaGPR continuous mapping. Let B be a NRCS in Y. Then $f^{-1}(B)$ is a NGPRCS in X. Since X is a $NPRT_{1/2}$ space, $f^{-1}(B)$ is a NPCS in X. Therefore $Npcl(f^{-1}(B)) = f^{-1}(B)) \subseteq f^{-1}(Npcl(B))$.

4. Neutrosophic almost gpr closed mappings

Definition 4.1: A mapping f : (X, τ) $\rightarrow$ (Y, τ_1) is called a neutrosophic almost generalized pre regular closed mapping (NaGPRC mapping for short) if f(A) is a NGPRCS in Y for each NRCS A in X.

Example 4.2: Let X= {a, b} and Y = {u, v}. Then $\tau = \{0_N, U, 1_N\}$ and $\tau_1 = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.3, 0.2, 0.6), (0.1, 0.2, 0.7)⟩ and V =⟨y, (0.5, 0.3, 0.5), (0.5, 0.3, 0.5)⟩. Define a mapping f: (X, τ)$\rightarrow$(Y, τ_1) by f(a) = u and f(b) = v. Then f is a NaGPRC mapping.

Theorem 4.3: Every NC mapping is a NaGPRC mapping but not conversely.
Proof: Assume that f: (X, τ) $\rightarrow$ (Y, τ_1) is a NC mapping. Let A be a NRCS in X. Since every NRCS is a NCS, A is a NCS in X. Then f(A) is a NCS in Y. Since every NCS is a NGPRCS, f(A) is a NGPRCS in Y. Hence f is a NaGPRC mapping.

Example 4.4: Let X = {a, b} and Y = {u,v}. Then $\tau = \{0_N, U, 1_N\}$ and $\tau_1 = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.4, 0.3, 0.6), (0.4, 0.3, 0.6)⟩ and V = ⟨y, (0.6, 0.3, 0.3), (0.7, 0.3, 0.3)⟩. Define a mapping f: (X, τ)$\rightarrow$(Y, τ_1) by f(a) = u and f(b) = v. Hence f is a NaGPRC mapping. But f is not a NC mapping.

Theorem 4.5: Every NαC mapping is a NaGPRC mapping but not conversely.
Proof: Let f : (X, τ) → (Y, τ_1) be a NαC mapping . Let A be a NRCS in X. Since every NRCS is a NCS, A is a NCS in X. Then f(A) is a NαCS in Y. Since every NαCS is a NGPRCS, f(A) is a NGPRCS in Y. Hence f is a NaGPRC mapping.

Example 4.6: Let X = {a, b} and Y = {u,v}. Then τ = {0_N, U, 1_N} and τ_1 = {0_N, V, 1_N} are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.4, 0.3, 0.6), (0.4, 0.3, 0.6)⟩ and V = ⟨y, (0.5, 0.4, 0.5), (0.4, 0.4, 0.6)⟩. Define a mapping f: (X, τ)→(Y, τ_1) by f(a) = u and f(b) = v. Hence f is a NaGPRC mapping. But f is not a NαC mapping.

Theorem 4.7: Every NPC mapping is a NaGPRC mapping but not conversely.
Proof: Assume that f : (X, τ) → (Y, τ_1) be a NPC mapping. Let A be a NRCS in X. Since every NRCS is a NCS, A is a NCS in X. Then f(A) is a NPCS in Y. Since every NPCS is a NGPRCS, f(A) is a NGPRCS in Y. Hence f is a NaGPRC mapping.

Example 4.8: Let X = {a, b} and Y = {u,v}. Then τ = {0_N, U, 1_N} and τ_1 = {0_N, V_1, V_2, 1_N} are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.2, 0.4, 0.7), (0.3, 0.5, 0.6)⟩, V_1 = ⟨y, (0.5, 0.3, 0.5), (0.6, 0.3, 0.3)⟩ and V_2 = ⟨y, (0.5, 0.3, 0.5), (0.2, 0.3, 0.8)⟩. Define a mapping f: (X, τ)→(Y, τ_1) by f(a) = u and f(b) = v. Hence f is a NaGPRC mapping. But f is not a NPC mapping.

Theorem 4.9: Every NGC mapping is a NaGPRC mapping but not conversely.
Proof: Assume that f : (X, τ) → (Y, τ_1) be a NGC mapping. Let A be a NRCS in X. Since every NRCS is a NCS, A is a NCS in X. Then f(A) is a NGCS in Y. Since every NGCS is a NGPRCS, f(A) is a NGPRCS in Y. Hence f is a NaGPRC mapping.

Example 4.10: Let X = {a, b} and Y = {u,v}. Then τ = {0_N, U, 1_N} and τ_1 = {0_N, V, 1_N} are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.4, 0.4, 0.5), (0.4, 0.5, 0.6)⟩ and V = ⟨y, (0.6, 0.7, 0.3), (0.7, 0.7, 0.3)⟩. Define a mapping f: (X, τ)→(Y, τ_1) by f(a) = u and f(b) = v. Hence f is a NaGPRC mapping. But f is not a NGC mapping.

Theorem 4.11: Every NGPC mapping is a NaGPRC mapping but not conversely.
Proof: Assume that f : (X, τ) → (Y, τ_1) be a NGPC mapping. Let A be a NRCS in X. Since every NRCS is a NCS, A is a NCS in X. Then f(A) is a NGPCS in Y. Since every NGPCS is a NGPRCS, f(A) is a NGPRCS in Y. Hence f is a NaGPRC mapping.
Example 4.12: Let X = {a, b} and Y = {u,v}. Then τ = {0_N, U, 1_N} and τ_1 = {0_N, V_1, V_2, 1_N} are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.3, 0.5, 0.7), (0.3, 0.5, 0.7) ⟩, V_1 = ⟨y, (0.6, 0.5, 0.4), (0.6, 0.5, 0.4)⟩ and V_2 = ⟨y, (0.8, 0.5, 0.2), (0.8, 0.5, 0.2)⟩. Define a mapping f: (X, τ)→(Y, τ_1) by f(a) = u and f(b) = v. Hence f is a NaGPRC mapping. But f is not a NGPC mapping.

Theorem 4.13: Every NiGPRC mapping is a NaGPRC mapping but not conversely.
Proof: Assume that f : (X, τ) → (Y, τ_1) be a NiGPRC mapping . Let A be a NRCS in X. Then A is a NGPRCS in X. By hypothesis f (A) is a NGPRCS in Y. Therefore f is a NaGPRC mapping.

Example 4.14: Let X= {a, b} and Y = {u, v}. Then τ = {0_N, U, 1_N} and τ_1 = {0_N, V, 1_N} are Neutrosophic topologies on X and Y respectively, where U = ⟨x, (0.2, 0.5, 0.7), (0.3, 0.5, 0.6)⟩ and V = ⟨y, (0.3, 0.5, 0.6), (0.4, 0.5, 0.5)⟩. Define a mapping f: (X, τ) → (Y, τ_1) by f(a) = u and f(b) = v. Here the Neutrosophic set U^c = ⟨x, (0.7, 0.5, 0.2), (0.6, 0.5, 0.3)⟩ is a NRCS in X. Then f(U^c) = ⟨y, (0.7, 0.5, 0.2), (0.6, 0.5, 0.3)⟩ is a NGPRCS in (Y, σ) as f(U^c) $\subseteq 1_N$ implies Npcl(f(U^c)) = f(U^c) $\subseteq 1_N$ where 1_N is a NROS in Y. Therefore f is a NaGPRC mapping. But f is not a NiGPRC mapping since W = ⟨x, (0.3, 0.5, 0.6), (0.4, 0.5, 0.5)⟩ is NGPRCS in X but f(W) is not a NGPRCS in Y as f(W) ⊆ V implies Npcl(f(W)) = $V^c \not\subseteq$ V where V is a NROS in Y. Therefore f is not a Nigpr closed mapping.

Definition 4.15: A mapping f : (X, τ) → (Y, τ_1) is called an neutrosophic almost generalized pre regular open mapping (NaGPRO mapping for short) if f(A) is a NGPROS in Y for each NROS A in X.

Theorem 4.16: Let f : (X, τ) → (Y, τ_1) be a bijective mapping. Then the following statements are equivalent:
(i) f is a NaGPRO mapping,
(ii) f is a NaGPRC mapping.
Proof: Straightforward.

Theorem 4.17: Let $p_{(\alpha, \beta)}$ be a NP in X. A mapping $f : (X, \tau) \rightarrow (Y, \tau_1)$ is a NaGPRO mapping if for every NOS A in X with $f^{-1}(p_{(\alpha, \beta)}) \in A$, then there exists a NOS B in Y with $p_{(\alpha, \beta)} \in B$ such that f (A) is ND in B.
Proof: Let A be a NROS in X. Then A is a NOS in X. Let $f^{-1}(p_{(\alpha, \beta)}) \in A$, then there exists a NOS B in Y such that $p_{(\alpha, \beta)} \in B$ and Ncl(f(A)) = B. Since B is a NOS, Ncl(f(A)) = B is also a NOS in Y. Therefore Nint(Ncl(f(A))) = Ncl(f(A)). Now f(A) $\subseteq$ Ncl(f(A)) = Nint(Ncl(f(A))) $\subseteq$ Ncl(Nint(Ncl(f(A)))). This implies f(A) is a NPOS in Y and hence a NGPROS in Y. Thus f is a NaGPRO mapping.

Theorem 4.18: Let $f : (X, \tau) \rightarrow (Y, \tau_1)$ be a mapping where Y is a $NPRT_{1/2}$ space. Then the following statements are equivalent:
(i) f is a NaGPRC mapping,
(ii) f (A) $\subseteq$ Npint(f (Nint(Ncl(A)))) for every NPOS A in X.
Proof:
(i) $\Rightarrow$ (ii) Let A be a NPOS in X. Then A $\subseteq$ Nint(Ncl(A)). Since Nint(Ncl(A)) is a NROS in X, by hypothesis, f(Nint(Ncl(A))) is an NGPROS in Y. Since Y is a $NPRT_{1/2}$ space, f(Nint(Ncl(A))) is a NPOS in Y. Therefore f(A) $\subseteq$ f(Nint(Ncl(A))) $\subseteq$ Npint(f(Nint(Ncl(A)))).
(ii) $\Rightarrow$ (i) Let A be a NROS in X. Then A is a NPOS in X. By hypothesis, f(A) $\subseteq$ Npint(f(Nint(Ncl(A)))) = Npint(f(A)) $\subseteq$ f(A). This implies f(A) is a NPOS in Y and hence is a NGPROS in Y. Therefore f is a NaGPRC mapping.

Theorem 4.19: Let $f : (X, \tau) \rightarrow (Y, \tau_1)$ be a mapping. If f is a NaGPRC mapping, then Ngprcl(f(A)) $\subseteq$ f(Ncl(A)) for every NPOS A in X.

Proof: Let A be a NPOS in X. Then Ncl(A) is a NRCS in X. By hypothesis f(Ncl(A)) is a NGPRCS in Y. Then Ngprcl(f(Ncl(A)) = f(Ncl(A)). Now Ngprcl(f(A)) $\subseteq$ Ngprcl(f(Ncl(A))) = f(Ncl(A)). That is Ngprcl(f(A)) $\subseteq$ f(Ncl(A)).

Theorem 4.20: Let $f : (X, \tau) \rightarrow (Y, \tau_1)$ be a mapping. If f is a NaGPRC mapping, then Ngprcl(f(A)) $\subseteq$ f(Ncl(Npint(A))) for every NPOS A in X.

Proof: Let A be a NPOS in X. Then Ncl(A) is a NRCS in X. By hypothesis, f(Ncl(A)) is a NGPRCS in Y. Then Ngprcl(f(A)) $\subseteq$ Ngprcl(f(Ncl(A))) = f(Ncl(A)) $\subseteq$ f(Ncl(Npint(A))), since Npint(A) = A.

Theorem 4.21: Let $f : (X, \tau) \rightarrow (Y, \tau_1)$ be a mapping. If f(Npint(B)) $\subseteq$ Npint(f(B)) for every NS B in X, then f is a NaGPRO mapping.

Proof: Let B $\subseteq$ X be a NROS. By hypothesis, f(Npint(B)) $\subseteq$ Npint(f(B)). Since B is an NROS, it is a NPOS in X. Therefore Npint(B) = B. Hence f(B) = f(Npint(B)) $\subseteq$ Npint(f(B)) $\subseteq$ f(B). This implies f(B) is a NPOS and hence a NGPROS in Y. Thus f is a NaGPRO mapping.

Theorem 4.22: Let $f : (X, \tau) \rightarrow (Y, \tau_1)$ be a mapping. If Npcl(f(B)) $\subseteq$ f(Npcl(B)) for every NS B in X, then f is a NaGPRC mapping.

Proof: Let B $\subseteq$ X be a NRCS. By hypothesis, Npcl(f(B)) $\subseteq$ f(Npcl(B)). Since B is a NRCS, it is a NPCS in X. Therefore Npcl(B) = B. Hence f(B) = f(Npcl(B)) $\supseteq$ Npcl(f(B)) $\supseteq$ f(B). This implies f(B) is a NPCS and hence a NGPRCS in Y. Thus f is a NaGPRC mapping.

Theorem 4.23: The following statements are equivalent for a mapping $f : (X, \tau) \rightarrow (Y, \tau_1)$, where Y is an $NPRT_{1/2}$ space:
(i) f is a NaGPRC mapping,
(ii) f(A) $\subseteq$ Npint(f(Nscl(A))) for every NPOS A in X.
Proof: (i) $\Rightarrow$ (ii) Let A be an NPOS in X. Then A $\subseteq$ Nint(Ncl(A)). Since Nint(Ncl(A)) is an NROS in X, by hypothesis, f(Nint(Ncl(A))) is a NGPROS in Y. Since Y is an $NPRT_{1/2}$ space, f(Nint(Ncl(A))) is an NPOS in Y. Therefore f(A) $\subseteq$ f(Nint(Ncl(A))) $\subseteq$ Npint(f(Nint(Ncl(A)))) $\subseteq$ Npint(f(A $\cup$ Nint(Ncl(A)))) = Npint(f(Nscl(A))). That is f(A) $\subseteq$ Npint(f(Nscl(A))).
(ii) $\Rightarrow$ (i) Let A be an NROS in X. Then A is a NPOS in X. By hypothesis, f(A) $\subseteq$ Npint(f(Nscl(A))). This implies f(A) $\subseteq$ Npint(f(A $\cup$ Nint(Ncl(A)))) $\subseteq$ Npint(f(A $\cup$ A)) = Npint(f(A)) $\subseteq$ f(A). Therefore f(A) is a NGPROS in Y and hence it is a NGPROS in Y. Thus f is a NaGPRC mapping.

Theorem 4.24: Let f : (X, τ) → (Y, τ_1) be a mapping where Y is a $NPRT_{1/2}$ space. If f is a NaGPRC mapping, then Nint(Ncl(Nint(f(B)))) ⊆ f(Npcl(B)) for every NRCS B in X.

Proof: Let B ⊆ X be a NRCS. By hypothesis, f(B) is a NGPRCS in Y. Since Y is a $NPRT_{1/2}$ space, f(B) is a NPCS in Y. Therefore Npcl(f(B)) = f(B). Now Nint(Ncl(Nint(f(B)))) ⊆ f(B) = f(Npcl(B)), since B = Npcl(B). Hence Nint(Ncl(Nint(f(B)))) ⊆ f(Npcl(B)).

Theorem 4.25: Let f : (X, τ) → (Y, τ_1) be a bijective mapping. Then the following statements are equivalent:
(i) f is a NaGPRO mapping,
(ii) f is a NaGPRC mapping,
(iii) f^{-1} is a NaGPR continuous mapping.
Proof: (i) ⇔ (ii) is obvious from the Theorem 4.16.
(ii) ⇒ (iii) Let A ⊆ X be a NRCS. Then by hypothesis, f(A) is a NGPRCS in Y. That is $(f^{-1})^{-1}(A)$ is a NGPRCS in Y. This implies f^{-1} is a NaGPR continuous mapping.
(iii) ⇒ (ii) Let A ⊆ X be a NRCS. Then by hypothesis $(f^{-1})^{-1}(A)$ is a NGPRCS in Y. That is f(A) is a NGPRCS in Y. Hence f is a NaGPRC mapping.

5. Neutrosophic gpr Compactness

Definition 5.1: Let (Z, τ) be a NTS. If a family {⟨z, $\mu_{Ui}(z)$, $\sigma_{Ui}(z)$, $\nu_{Ui}(z)$⟩: i ∈ Λ⟩} of NGPR open sets in (Z, τ) satisfies the condition ∪{⟨z, $\mu_{Ui}(z)$, $\sigma_{Ui}(z)$, $\nu_{Ui}(z)$⟩: i ∈ Λ⟩} = 1_N, then it is called a NGPR open cover of (Z, τ).

A finite subfamily of a NGPR open cover {⟨z, $\mu_{Ui}(z)$, $\sigma_{Ui}(z)$, $\nu_{Ui}(z)$⟩: i ∈ J⟩} of (Z, τ), which is also a NGPR open cover of (Z, τ) is called a finite subcover of {⟨z, $\mu_{Ui}(z)$, $\sigma_{Ui}(z)$, $\nu_{Ui}(z)$⟩: i ∈ Λ⟩}.

Definition 5.2: A NTS (Z, τ) is called neutrosophic gpr compact (NGPR compact for short) if and only if every NGPR open cover of (Z, τ) has a finite subcover.

Definition 5.3: Let (Z, τ) be a NTS and A be a NS in (Z, τ). If a family {⟨z, $\mu_{Ui}(z)$, $\sigma_{Ui}(z)$, $\nu_{Ui}(z)$⟩: i ∈ Λ⟩} of NGPR open sets in (Z, τ) satisfies the condition A ⊆ ∪{⟨z, $\mu_{Ui}(z)$, $\sigma_{Ui}(z)$, $\nu_{Ui}(z)$⟩: i ∈ Λ⟩} = 1_N, then it is called a NGPR open cover of A.

A finite subfamily of a NGPR open cover {⟨z, $\mu_{Ui}(z)$, $\sigma_{Ui}(z)$, $\nu_{Ui}(z)$⟩: i ∈ J⟩} of A, which is also a NGPR open cover of A is called a finite subcover of {⟨z, $\mu_{Ui}(z)$, $\sigma_{Ui}(z)$, $\nu_{Ui}(z)$⟩: i ∈ Λ⟩}.

Definition 5.4: A NS A in a NTS (Z, τ) is called NGPR compact relative to Z iff every NGPR open cover of A has a finite sub cover.

Theorem 5.5: Every NGPR compact space is neutrosophic compact space.
Proof: Let (Z, τ) be a NGPR compact space. Let $\{U_i : i \in \wedge\}$ be a neutrosophic open cover of (Z, τ) by NOSs in (Z, τ). From [10], $\{U_i : i \in \wedge\}$ is a NGPR open cover of (Z, τ) by NGPROSs in (Z, τ). Since (Z, τ) be a NGPR compact space, the NGPR open cover $\{U_i : i \in \wedge\}$ has a finite subcover say $\{U_i : i = 1,2,3\ldots\ldots n\}$ of (Z, τ). Hence (Z, τ) is neutrosophic compact space.

Theorem 5.6: Every NGPR compact space is NG compact space.
Proof: Let (Z, τ) be a NGPR compact space. Let $\{U_i : i \in \wedge\}$ be a NG open cover of (Z, τ) by NGOSs in (Z, τ). From [10], $\{U_i : i \in \wedge\}$ is a NGPR open cover of (Z, τ) by NGPROSs in (Z, τ). Since (Z, τ) be a NGPR compact space, the NGPR open cover $\{U_i : i \in \wedge\}$ has a finite subcover say $\{U_i : i = 1,2,3\ldots\ldots n\}$ of . Hence (Z, τ) is NG compact space.

Theorem 5.7: If (Y, τ) is a neutrosophic compact space and $NPRT^*_{1/2}$ space then (Y, τ) is NGPR compact space.
Proof: Let (Y, τ) be a neutrosophic compact space and $NPRT^*_{1/2}$ space. Assume that $\{U_i : i \in \wedge\}$ be a NGPR open cover of (Y, τ). Here (Y, τ) is $NPRT^*_{1/2}$ space, by hypothesis every NGPROS is NOS. Therefore $\{U_i : i \in \wedge\}$ is a neutrosophic open cover in (Y, τ) and it has a finit subcover say $\{U_i : i = 1,2,3\ldots\ldots n\}$ of (Y, τ). Hence (Y, τ) is NGPR compact space.

Theorem 5.8: Let (Y, τ) and (Z, τ_1) be any two NTSs and g: (Y, τ) → (Z, τ_1) be NGPR continuous surjection. If (Y, τ) is NGPR compact then (Z, τ_1) is neutrosophic compact.

Proof: Let $U_i = \{\langle z, \mu_{Ui}(z), \sigma_{Ui}(z), \nu_{Ui}(z)\rangle : i \in \Lambda\rangle\}$ be a neutrosophic open cover in (Z, τ_1) with $\cup\{\langle z, \mu_{Ui}(z), \sigma_{Ui}(z), \nu_{Ui}(z)\rangle : i \in \Lambda\rangle\} = \bigcup_{i\in\wedge} U_i = 1_N$. Since g is NGPR continuous, $\{g^{-1}(U_i) : i \in \wedge\}$ is a NGPR open cover of Y. Now, $\bigcup_{i\in\wedge} g^{-1}(U_i) = g^{-1}(\bigcup_{i\in\wedge} U_i) = 1_N$. Since, (Y, τ) is NGPR compact, there exist a finite subcover JεΛ, such that $\bigcup_{i\in J} g^{-1}(U_i) = 1_N$. Hence, $g\left(\bigcup_{i\in J} g^{-1}(U_i)\right) = 1_N$. That is $\bigcup_{i\in J} U_i = 1_N$. Since g is surjective, $\{U_i : i\varepsilon J\}$ is a neutrosophic open cover of (Z, τ_1) and hence (Z, τ_1) is neutrosophic compact.

Theorem 5.9: Let (Y, τ) and (Z, τ_1) be any two NTSs and g: (Y, τ) → (Z, τ_1) be NGPR continuous mapping. If A is NGPR compact in (Y, τ) then g(A) is neutrosophic compact in (Z, τ_1).

Proof: Let $U_i = \{\langle z, \mu_{Ui}(z), \sigma_{Ui}(z), \nu_{Ui}(z)\rangle : i \in \Lambda\rangle\}$ be a neutrosophic open cover of g(A) in (Z, τ_1). That is $g(A) \subseteq \bigcup_{i\in\wedge} U_i$. Since g is NGPR continuous, $\{g^{-1}(U_i) : i \in \wedge\}$ is a NGPR open cover of A in (Y, τ). Now, $A \subseteq \bigcup_{i\in\wedge} g^{-1}(U_i)$. Since, A in (Y, τ) is NGPR compact, there exist a finite subcover JεΛ, such that $A \subseteq \bigcup_{i\in J} g^{-1}(U_i) = 1_N$. Hence, g(A) ⊆ $g\left(\bigcup_{i\in J} g^{-1}(U_i)\right) = \bigcup_{i\in J} U_i = 1_N$. Therefore g(A) is neutrosophic compact in (Z, τ_1).

Theorem 5.10: Let g: (Y, τ) → (Z, τ_1) be NGPR irresolute mapping and if A is NGPR compact relative to (Y, τ) then g(A) is NGPR compact relative (Z, τ_1).

Proof: Let $\{U_i : i \in \wedge\}$ be a NGPR open cover of Z such that $g(A) \subseteq \bigcup_{i\in\wedge} U_i$. Then $A \subseteq \bigcup_{i\in\wedge} g^{-1}(U_i)$ where $\{g^{-1}(U_i) : i \in \wedge\}$ is NGPR open cover in Y for each $i \in \wedge$. Since A is NGPR compact relative to Y, there exist a finite sub collection $\{U_i : i\varepsilon J\}$ such that $A \subseteq \bigcup_{i\in J} g^{-1}(U_i)$. That is $g(A) \subseteq \bigcup_{i\in J} U_i$. Hence g(A) is NGPR compact relative (Z, τ_1).

Theorem 5.11: Let g: (Y, τ) → (Z, τ_1) be NGPR irresolute mapping. If (Y, τ) is NGPR compact space then (Z, τ_1) is also a NGPR compact space.

Proof: Let g: (Y, τ) → (Z, τ_1) be NGPR irresolute mapping form NGPR compact space (Y, τ) onto a NTS (Z, τ_1). Let $\{U_i : i \in \wedge\}$ be a NGPR open cover of Z then $\{g^{-1}(U_i) : i \in \wedge\}$ is a NGPR open cover of Y. Since Y is a NGPR compact space, there exist a finite subfamily $\{g^{-1}(U_{i1}), g^{-1}(U_{i2}), g^{-1}(U_{i3}) \ldots\ldots g^{-1}(U_{in})\}$ of $\{g^{-1}(U_i) : i \in \wedge\}$ such that $\bigcup_{j=1}^{n} U_{ij} = 1_N$. Since g is onto, $g(1_N) = 1_N$ and $g\left(\bigcup_{j=1}^{n} g^{-1}(U_{ij})\right) = \bigcup_{j=1}^{n} g\left(g^{-1}(U_{ij})\right) = \bigcup_{j=1}^{n}(U_{ij})$. It follow that $\bigcup_{j=1}^{n}(U_{ij}) = 1_N$ and the family $\{U_{i1}, U_{i2}, \ldots\ldots U_{in}\}$ is a neutrosophic finite subcover of $\{U_i : i \in \wedge\}$. Hence (Z, τ_1) is a NGPR compact space.

Theorem 5.12: A NTS (Y, τ) is NGPR compact space if and only if Y is finite.

Proof: Let (Y, τ) is NGPR compact space. Since $\{U_i : i \in \wedge\}$ is NGPR open cover of (Y, τ). Since (Y, τ) is NGPR compact space, there exist a finite subcover $Y_1 = \{U_i : i = 1,2,3\ldots\ldots n\}$ of Y such that $Y \subseteq \bigcup \{U_i : i = 1,2,3\ldots\ldots n\} = Y_1 \subseteq Y$. Hence $Y = Y_1$. Which is finite. Converse is obvious.

6. Conclusion

The concept of Neutrosophic almost gpr continuous mappings, Neutrosophic almost gpr closed mappings and the interrelations among these mappings and existing mappings in Neutrosophic topological spaces have been introduced and studied. Further we extended our study to the concept of Neutrosophic gpr compactness in Neutrosophic topological spaces.

The above concepts are used to develop theory in the field of Neutrosophic fuzzy topological spaces and Neutrosophic soft topological spaces.

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