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Original Article

A String of Disjoint Job Block in Two Stage Open Shop Model in Fuzzy Environment

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Abstract *- This paper deals with two stage open shop scheduling in which processing time is considered in triangular fuzzy number. The concept of a string of disjoint job blocks in which two distinct job blocks in such a way that first block covers the jobs with fixed route and the second block contains the jobs with arbitrary route is taken into consideration. The objective of the study is to attain an optimal or near optimal schedule through a heuristic approach to minimize the total elapsed time. A numerical illustration is given to justify the proposed algorithm.*

Keywords *- Scheduling, Open Shop, Job Block, String of job block.*

1. Introduction

Scheduling is an imperative process that concerns with the problems of distributing resources to execute a set of actions with the intention to determine the optimum solution while considering an optimization of a function. A good scheduling assists in decreasing hiring costs, enhancing client service and exploiting the possessions optimally. Normally, the scheduling problems are classified into FSSP, JSSP, OSSP and HSSP (hybrid). In the present chapter OSSP models are well thoroughly considered. In OSSP, set 'J' consisting n jobs, comprising each of m operations are processed on a set 'M' consisting m machines in arbitrary order. Hence OSSP is alike the FSSP, with the exception that there are no limitations on the routes of the machines. In the ground of scheduling, both FSSP and OSSP are very admired but the first victorious mathematical model to acquire an optimal solution for two stage FSS problem was presented by Johnson [1] The optimality of Johnson's model draws a major consideration of abundant researchers toward this path. His work was extended by Dannenbring [6], Maggu and Das [5], Singh [10], Maggu and Lal [9], Anup [13], Gupta and Singh [15], Gupta et al. [16,18,19,20] by considering various parameters and different optimality criteria.

Gonzalez and Sahni [4] developed a heuristic algorithm with preemptive jobs for two stage OSSP with the intention of reducing the value of makespan. Further, they revealed that the OSSP with the machines more than two in number and along with non-preemptive jobs is NP-complete.

The literature reveals that the research on open shop scheduling basically centered on around the two-machine or three machine issues with the goal to minimize the makespan. Normally the objective of scheduling is to locate a feasible combination of the routes of machines and occupations (i.e., an achievable timetable) recollecting a definitive target to reduce the makespan in dynamic scheduling. The concept of job block equal to a single job was initiated by Maggu and Das [5] in order to make a congruity between the expense of providing need in help of the client and the expense of providing organization if no need is considered. Anup [13] broadened the research by assigning probabilities with working time of jobs as the time to process the jobs are always not precise. Heydari [14] managed the idea of handling the jobs in a string formed with two distinct job blocks.

2. Assumptions

The above-formulated problem is applicable under the following suppositions:

1. Priority is given to the job 'Ji' over job $J_2, J_3…., J_k$ in job block $(J_1, J_2………, J_k)$.
2. The second job will be processed on a machine when first job is completed.

3. The time of the jobs which can be consumed in transportation from one machine to another is negligible.
4. All the Jobs and Machines are accessible at zero time.

3. Preliminaries

This section provides some fuzzy concepts which are helpful in further considerations.

3.1. Fuzzy Set

Let X be a non-empty set, and $x \in X$ be an element of X, then a fuzzy set $\tilde{F}$ in X is defined by a set of ordered pairs

$$\tilde{F} = \{x, \mu_{\tilde{F}}(x) : x \, \varepsilon \, X\}$$

Where $\mu_{\tilde{F}}(x)$ is called the membership function or grade of membership of x in $\tilde{F}$ which maps X to the membership space N, considered the closed interval [0, 1].

3.2. Fuzzy Number

A fuzzy subset $\tilde{F}$ in R (real line) is called a fuzzy number if it must possess the following three properties:

i. **Convexity Property:** A fuzzy subset $\tilde{F}$ is said to be convex if $\tilde{F}(\alpha x_1 + (1-\alpha)\, x_2) \geq min\,\{\tilde{F}(x1), \tilde{F}(x2)\}$ for all $x_1, x_2 \in \Re$ and $\alpha \in [0,1]$ i.e., convex property states that the line formed by $\alpha -$ cut is continuous.

ii. **Normality Property:** A fuzzy subset $\tilde{F}$ is said to be normal if $\exists\, x_0 \in \Re$ such that $\tilde{F}(x_0) = 1$ i.e., normalization property states that the maximum value of membership is one.

iii. **$\tilde{F}$ is piecewise continuous.**

3.3. Triangular Fuzzy Number (TFN)

A fuzzy number $\tilde{F} = (u, v, w)$, $u \leq v \leq w$ on a set of real numbers is said to be a TFN number if its membership function $\mu_{\tilde{F}} : \Re \rightarrow [0,1]$ has the following characteristics:

1. If the mapping $\mu_{\tilde{F}} : \Re \rightarrow [0,1]$ is a continuous.
2. $\mu_{\tilde{F}}(x) = 0 \; \forall$ x ε $(-\infty, u] \cup (w, \infty]$.
3. $\mu_{\tilde{F}}(x)$ is strictly increasing and continuous on $[u, v]$.
4. $\mu_{\tilde{F}}(x) = 1 \; \forall$ x = v
5. $\mu_{\tilde{F}}(x)$ is strictly decreasing and continuous on $[v, w]$ shown in figure 1.

$$\mu_{\tilde{F}}(x) = \begin{cases} 0; \; x \leq u, \\ \dfrac{x-u}{v-u}; \; u \leq x < v, \\ 1; \; x = v, \\ \dfrac{w-x}{w-v}; \; v < x \leq w, \\ 0; \; x \geq w \end{cases}$$

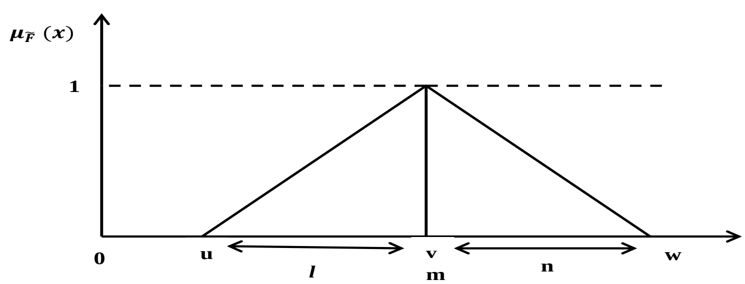

Fig. 1 Shows Triangular fuzzy number $\tilde{F} = (u, v, w) = (l, m, n)$

3.4. Average High Ranking <A.H.R.>

The defuzzified value of the given TFN number $\tilde{F} = (\alpha, \beta, \gamma)$ is called as Average High Ranking (AHR) given by Yager where α in favorable condition, β in normal (mid-value) condition and γ is in worst (bad) condition, calculated by the formula defined as:

$$AHR(\tilde{F}) = \frac{3\beta + \gamma - \alpha}{3}$$

4. Model Notations

The notations and symbols used in this paper are defined as below:

Notations		Description
i	:	Index for jobs.
A_i	:	Triangular fuzzy time to process the i^{th} (i = 1, 2, 3, ..., n) job on machine A.
B_i	:	Triangular fuzzy time to process the i^{th} (i = 1, 2, 3, ..., n) job on machine B.
A_i'	:	Total time to process the i^{th} job on machine A in terms of AHR .
B_i'	:	Total time to process the i^{th} job on machine B in terms of AHR.
$\boldsymbol{S}$	:	Optimum/near optimum string of jobs for machine route A→B.
$\boldsymbol{S'}$	:	Optimum/near optimum string of jobs for machine route B→A.

4.1. Problem Formulation

(i) Let '$\boldsymbol{n}$' jobs J_1, J_2,….. ,J_n are processed through 2-machines A and B in arbitrary order with no passing allowed.
(ii) Assume $\boldsymbol{A_i}$ & $\boldsymbol{B_i}$ denote total time to process the job J_i; i =1,2,3,.....n on machine A and B separately represented by TFN. Let $\boldsymbol{A_i'}$ & $\boldsymbol{B_i'}$ be the average high ranking (AHR) of the processing times on machines A and B.
(iii) Let $\boldsymbol{S = (J_\alpha, J_\beta)}$ be a string consisting of two disjoint job- blocks. A job-block $\boldsymbol{J_\alpha}$ having 's' jobs with a pre-described order, and another job block $\boldsymbol{J_\beta}$ consists of 'r' jobs with arbitrary order in such a way that $\boldsymbol{J_\alpha} \cap \boldsymbol{J_\beta} = \emptyset$ and $\boldsymbol{r + s = n}$.
(iv) The performance measure is to minimize the total elapsed time.

The mathematical model of above stated problem is described in Table 1.

Table 1. Model formulation

Job	Machine A	Machine B
J_1	$(\alpha_{11}, \beta_{11}, \gamma_{11})$	$(\alpha_{12}, \beta_{12}, \gamma_{12})$
J_2	$(\alpha_{21}, \beta_{21}, \gamma_{21})$	$(\alpha_{12}, \beta_{12}, \gamma_{12})$
J_3	$(\alpha_{31}, \beta_{31}, \gamma_{31})$	$(\alpha_{13}, \beta_{13}, \gamma_{13})$
.	.	.
.	.	.
J_n	$(\alpha_{n1}, \beta_{n1}, \gamma_{n1})$	$(\alpha_{1n}, \beta_{1n}, \gamma_{1n})$

5. Algorithm

The model of OSSP containing two machines with the aim to optimize the makespan is explained as follows.

Step 1: For machine order A→B

Find the average high ranking (AHR) A_i' & B_i' of the processing times for all the jobs on two machines A and B.

Step2: Consider a job-block containing two jobs, say J_k and J_m, with fix order of jobs. Let this job block is equivalent to a single job J_α, i.e., $J_\alpha = (J_k, J_m)$. Now calculate the working time of job J_α on machines A and B as defined below:

(a) $A'_{J\alpha} = A'_{JK} + A'_{Jm} - \min(A'_{Jm}, B'_{Jk})$

(b) $B'_{J\alpha} = B'_{JK} + B'_{Jm} - \min(A'_{Jm}, B'_{Jk})$

Step3: Consider another job block β consisting of $(n - \{J_k, J_m\})$ jobs with an arbitrary route. Apply Johnson's method [1] to obtain the optimum route of jobs in block β. Consider new block is equivalent to γ. Now find the processing time of the block J_γ on machines A and B as defined in step2.

Step4: Convert the given problem into new by substituting the jobs $\{J_k, J_m\}$ by equivalent job J_α and $(n - \{J_k, J_m\})$ jobs by equivalent job J_γ shown in table 2. Then the modified problem can be presented as below:

Table 2. Shows Modified Problem of machine A and B

Jobs (J_i)	**Machine A** A_i'	**Machine B** B_i'
J_α	$A'_{J\alpha}$	$B'_{J\alpha}$
J_γ	$A'_{J\gamma}$	$B'_{J\gamma}$

Step7: Next, obtain another string S' in the same manner as obtained string S by repeating the procedure from step1 to step 6 for machine route B→A

Step8: Construct Flow in-out tables for strings S & S' and calculate the total elapsed time (makespan) for both strings.

Step9: Select a string among the obtained strings S & S' which conquered our objective function.

6. Numerical Illustration

The procedure of the algorithm is illustrated with the following example. Consider a 5×2 OSSP with fuzzy process times as described in Table 3. Assume the block $J_\alpha = (5, 2)$ with fixed order and $J_\beta = (1, 3, 4)$ with arbitrary order run in a string S.

Table 3. Processing time on machines A and B in fuzzy environment

Jobs (J_i)	**Machine A** A_i	**Machine B** B_i
J_1	(9,10,11)	(9,10,11)
J_2	(12,13,14)	(11,12,13)
J_3	(13,14,15)	(10,11,12)
J_4	(15,16,17)	(14,15,16)
J_5	(17,18,19)	(13,14,15)

The aim of the above-illustrated problem is to minimize the total elapsed time by obtaining an optimum string of job-blocks (say) J_α & J_β

Solution: Defuzzify the processing time of all the jobs by applying the formula as described in 3.4 is shown in table 4.

Table 4. Explains Defuzzified processing times of machine A and B

Jobs	**Machine A**	**Machine B**
(J_i)	A'_i	B'_i
J_1	32/3	32/3
J_2	41/3	38/3
J_3	44/3	35/3
J_4	50/3	47/3
J_5	56/3	44/3

As per step 2, 3 & 4 the process time for single job J_α = (5,2) and J_β = (1,3,4) which is equivalent to job block J_γ = (1,4,3) on machines A and B are given in following table 5.

Table 5. Explains Processing times of equivalent jobs for route A to B

Jobs (J_i)	**Machine A**	**Machine B**
	A'_i	**B'_i**
J_α	56/3	41/3
J_γ	50/3	38/3

Now by following step 6, the string ***S*** = {J_α, J_γ} = {J_5, J_2, J_1, J_4, J_3} is optimal for machine route A→ B and the total elapsed time (C_{max}) for string ***S*** is calculated in Table 6.

Table 6. Flow in - Flow out table for route A→B

Jobs	**Machine A**	**Machine B**
(J_i)	**In – Out**	**In – Out**
J_5	(0,0,0) – (17,18,19)	(17,18,19) -(30,32,34)
J_2	(17,18,19) -(29,31,33)	(30,32,34) – (41,44,47)
J_1	(29,31,33) –(38,41,44)	(41,44,47) – (50,54,58)
J_4	(38,41,44) – (53,57,61)	(53,57,61) – (67,72,77)
J_3	(53,57,61) – (66,71,76)	(67,72,77) – (77,83,89)

Now, as per step 7, we have obtained another string ***S'*** = {$J_{\gamma'}$, J_α} = {J_1, J_3, J_4, J_5, J_2} for machine route B→ A, and its flow table is described in Table 7.

Table 7. Flow in - Flow out table for machine route B→A

Jobs	**Machine B**	**Machine A**
(J_i)	In - Out	In - Out
J_1	(0,0,0) – (9,10,11)	(9,10,11) – (18,20,22)
J_3	(9,10,11) – (19,21,23)	(19,21,23) – (32,35,38)
J_4	(19,21,23) – (33,36,39)	(33,36,39) – (48,52,56)
J_5	(33,36,39) – (46,50,54)	(48,52,56) – (65,70,75)
J_2	(46,50,54) – (57,62,67)	(65,70,75) – (77,83,89)

Therefore, the calculated results obtained for different machine routes are described in Table 8.

Table 8. Description of results

Machine Route	String/Sequence	Total Elapsed Time/Makespan
A→B	***S*** = {J_5, J_2, J_1, J_4, J_3}	C_{max} = 87 units of time.
B→A	***S′*** = {J_1, J_3, J_4, J_5, J_2}	C_{max} = 87 units of time.

Hence from the above table, we conclude that the string ***S* and *S′*** both provide the minimum total elapsed time.

Therefore, ***S*** = {J_1, J_3, J_4, J_5, J_2} & ***S′*** = {J_1, J_3, J_4, J_5, J_2} are the desired optimal string of jobs having minimum elapsed time.

7. Discussion and Results

This paper provides an efficient heuristic to the decision-maker to find the optimal result for the problems in that environment where jobs run in a string of disjoint job-block. The goal of the study made in this paper is to acquire an optimum/near optimum string *S* and *S′* both provide the minimum total elapsed time. The future work could be extended with different parameters like breakdown interval, due dates, the setup time of machines, availability of single transportation etc. The study may additionally be stretched out by introducing the processing time of machines by trapezoidal fuzzy numbers.

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