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Original Article

Finite-Time Control of Nonlinear Switched Time-Varying Delay System

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Abstract - *In this paper, the boundedness and control of nonlinear switched time-varying delay systems in finite time are studied. By constructing a new Lyapunov-Krasovskii functional (LKF) and based on a new switching rule, sufficient conditions for the boundedness of the system in finite time and the calculation method of the state feedback controller are obtained. Finally, a numerical example is given to prove the feasibility of the proposed method.*

Keywords - *Time-varying delay, Switching system, Lyapunov-Krasovskii function, Finite time boundedness, Control.*

1. Introduction

Nowadays, the switching system plays an important role in the field of control applications [1-6]. However, the time-delay phenomenon often appears in practical engineering applications, and may make the system unstable or even deteriorate its performance [7-11]. At the same time, when the system is nonlinear [12-15], the dynamic behavior is more complicated, and it is widely used in practical engineering applications, Such as mobile robot system [16], multi-modal controller [17], variable structure controller [18-19]. Cheng et al. discussed the stability of linear switched systems [20]. Hadi et al. studied the control problem of nonlinear systems with constant delay [21]. Therefore, the research on nonlinear switched time-varying delay systems is very popular and necessary.

The purpose of this paper is to combine LKF and switching rules to get a new switching law. Finite-time boundedness and state feedback control are also discussed. Primarily, consider the following nonlinear switched system:

$$\begin{cases}\dot{x}(t) = A_{0\sigma(t)}x(t) + A_{1\sigma(t)}x\big(t-h(t)\big) + A_{\mu\sigma(t)}\mu(t) + A_{w\sigma(t)}\omega(t) + F_{\sigma(t)}\big(x(t)\big),\\ x(t_0) = \phi(t_0), \forall t_0 \in [-h_2, 0].\end{cases} \tag{1}$$

where $x(t) \in R^n$ is state variable, $h(t)$ is the considered time-varying delay and satisfies $0 \le h_1 \le h(t) \le h_2, 0 < \dot{h}(t) < 1$, $\mu(t) \in R^n$ is control input, $\omega(t) \in L_2$ is external disturbances, and $F_{\sigma(t)}\big(x(t)\big): R^n \to R^n$ is nonlinear function. Switching signal $\sigma(t): [0,\infty] \to M$ is a piecewise constant function, where $M = \{1,2,\cdots,m\}$. Finally, $A_{0\sigma(t)}, A_{1\sigma(t)}, A_{\mu\sigma(t)}, A_{w\sigma(t)}$ are constant real matrices, and h_1, h_2 are constants. The assumptions and definition useful for proving the above switched system are given below.

Assumption 1.1: [22] The external disturbances $\omega(t)$ satisfy $\int_0^{t_f} \omega^T(t)\omega(t)dt \le \delta, \delta > 0$.

Assumption 1.2: [23] The nonlinear function $F_i\big(x(t)\big)$ satisfies $F_i(0) = 0$, it is Lipschitz condition if

$$\big\|F_i\big(x_1(t)\big) - F_i\big(x_2(t)\big)\big\| \le \rho_i \|x_1(t) - x_2(t)\|, \tag{2}$$

Where $\rho_i \in R$ is Lipschitz constant.

Definition 1.1: [24] The switched system (1) is said to be finite-time bounded, if there exist positive constants $c_1, c_2 (c_2 > c_1 > 0), t_f, \delta$, a switching signal $\sigma(t)$, such that

$$\max_{t_0\in[-h_2,0]}\|x(t_0)\|_2^2 < c_2 \Rightarrow \|x(t)\|_2^2 < c_2,$$

$$\forall t \in [0, t_f], h(t) \in [h_1, h_2], \omega(t): \int_0^{t_f} \omega^T(t)\omega(t)dt \le \delta.$$

The following switch signal $\sigma(t)$ will be abbreviated as i, which also indicates that the ith subsystem is active at time t $(i = 1,2,\cdots,m)$. At the same time as the system switching, here we consider the controller with switching. Therefore, the following state feedback controller is considered:

$$\mu(t) = K_i x(t). \tag{3}$$

Then, system (1) can be converted to the following closed-loop system:

$$\begin{cases}\dot{x}(t) = (A_{0i} + A_{\mu i}K_i)x(t) + A_{1i}x(t-h(t)) + A_{wi}\omega(t) + F_i(x(t)),\\ x(t_0) = \phi(t_0), \forall t_0 \in [-h_2, 0].\end{cases} \tag{4}$$

2. Main Results

Theorem 2.1: For any $i, j \in \{1,2,\cdots,m\}$, given parameters $(c_1, c_2, t_f, \delta, \sigma)$, and constants $\alpha > 0, \rho$, if there exist symmetric positive-definite matrices $\bar{P}_i, \bar{Q}_{1i}, \bar{Q}_{2i}, \bar{Q}_{3i} \in R^{n\times n}$ matrices $X_i, \bar{I}_i \in R^{n\times n}$ satisfying the following conditions:

$$\bar{\prod}_i = \begin{bmatrix} \bar{\prod}_{1i} & 0 & 0 & A_{1i}\bar{P}_i & A_{wi} & I \\ * & -e^{\alpha h_1}\bar{Q}_{1i} & 0 & 0 & 0 & 0 \\ * & * & -e^{\alpha h_2}\bar{Q}_{2i} & 0 & 0 & 0 \\ * & * & * & \bar{\prod}_{2i} & 0 & 0 \\ * & * & * & * & -\alpha I & 0 \\ * & * & * & * & * & -I \end{bmatrix}, \tag{5}$$

$$e^{\alpha t_f}\{c_1[\beta_{12} + \beta_2 h_1 e^{\alpha h_1} + (\beta_3 + \beta_4)h_2 e^{\alpha h_2}] + \alpha\delta\} - \beta_{11}c_2 < 0, \tag{6}$$

where $\bar{\prod}_{1i} = A_{0i}\bar{P}_i + \bar{P}_i^T A_{0i}^T + A_{\mu i}X_i + X_i^T A_{\mu i}^T - \alpha\bar{P}_i + \bar{Q}_{1i} + \bar{Q}_{2i} + \bar{Q}_{3i} + \rho_i\bar{I}_i$, $\bar{\prod}_{2i} = -\left(1 - \dot{h}(t)\right)e^{\alpha h_1}\bar{Q}_{3i}$, $\beta_{11} \le \bar{P}_i^{-1} \le \beta_{12}$, $\bar{P}_i^{-1}\bar{Q}_{1i}\bar{P}_i^{-1} \le \beta_2$, $\bar{P}_i^{-1}\bar{Q}_{2i}\bar{P}_i^{-1} \le \beta_3$, $\bar{P}_i^{-1}\bar{Q}_{3i}\bar{P}_i^{-1} \le \beta_4$.

Then, under the switching law

$$\sigma(t) = argmin\, x^T(t)\bar{P}_i^{-1}x(t), \tag{7}$$

and the switched time-delay system (4) is finite-time bounded, the state feedback controller is$\mu(t) = K_i x(t) = X_i\bar{P}_i^{-1}x(t)$.

Proof:

For the systems (4), we consider the following LKF:

$$V_i(x(t)) = x^T(t)P_i x(t) + \int_{t-h_1}^{t} e^{\alpha(t-\theta)}x^T(\theta)Q_1 x(\theta)d\theta + \int_{t-h_2}^{t} e^{\alpha(t-\theta)}x^T(\theta)Q_2 x(\theta)d\theta + \int_{t-h(t)}^{t} e^{\alpha(t-\theta)}x^T(\theta)Q_3 x(\theta)d\theta. \tag{8}$$

Define the following function

$$\begin{aligned} J_1 &= \dot{V}_i(x(t)) - \alpha V_i(x(t)) - \alpha\omega^T(t)\omega(t) \\ &= 2x^T(t)P_i\left[(A_{0i} + A_{\mu i}K_i)x(t) + A_{1i}x(t-h(t)) + A_{\omega i}\omega(t) + F_i(x(t))\right] + x^T(t)Q_1 x(t) \\ &\quad + x^T(t)Q_2 x(t) + x^T(t)Q_3 x(t) - e^{\alpha h_1}x^T(t-h_1)Q_1 x(t-h_1) - e^{\alpha h_2}x^T(t-h_2)Q_2 x(t-h_2) \\ &\quad - \left(1-\dot{h}(t)\right)e^{\alpha h(t)}x^T(t-h(t))Q_3 x(t-h(t)) - \alpha x^T(t)P_i x(t) - \alpha\omega^T(t)\omega(t) \\ &\le x^T(t)[2P_iA_{0i} + 2P_iA_{\mu i}K_i - \alpha P_i + Q_1 + Q_2 + Q_3]x(t) + 2x^T(t)P_iA_{1i}x(t-h(t)) \end{aligned}$$

$$+2x^T(t)P_iA_{wi}\omega(t)+2x^T(t)P_iF_i\big(x(t)\big)-e^{\alpha h_1}x^T(t-h_1)Q_1x(t-h_1)$$
$$-e^{\alpha h_2}x^T(t-h_2)Q_2x(t-h_2)-\Big(1-\dot{h}(t)\Big)e^{\alpha h_1}x^T\big(t-h(t)\big)Q_3x\big(t-h(t)\big)-\alpha\omega^T(t)\omega(t).$$
(9)

where $\varpi_1(t)=col\big\{x(t),x(t-h_1),x(t-h_2),x\big(t-h(t)\big),\omega(t),F_i\big(x(t)\big)\big\}$.

Since the function $F_i\big(x(t)\big)$ satisfies Lipschitz condition (2), we can get the following inequality

$$\rho_i x^T(t)x(t)-F_i^T\big(x(t)\big)F_i\big(x(t)\big)\ge 0. \tag{10}$$

Substitute (10) into (9), we have:

$$J_1\le\begin{bmatrix}\prod_{1i} & 0 & 0 & P_iA_{1i} & P_iA_{wi} & P_i\\ * & -e^{\alpha h_1}Q_1 & 0 & 0 & 0 & 0\\ * & * & -e^{\alpha h_2}Q_2 & 0 & 0 & 0\\ * & * & * & \prod_{2i} & 0 & 0\\ * & * & * & * & -\alpha I & 0\\ * & * & * & * & * & -I\end{bmatrix}=\varpi_1^T(t)\textstyle\prod_i\varpi_1(t), \tag{11}$$

where $\prod_{1i}=A_{0i}P_i+P_i^TA_{0i}^T+A_{\mu i}X_i+X_i^TA_{\mu i}^T-\alpha P_i+Q_{1i}+Q_{2i}+Q_{3i}+\rho_iI$, $\boldsymbol{\prod_2=-(1-\dot{h}(t))e^{\alpha h_1}Q_3}$.

Next, pre- and post-multiplying (11) by $\mathbf{diag}\{\boldsymbol{P_i^{-1},P_i^{-1},P_i^{-1},P_i^{-1},I,I}\}$. Let $\bar{P}_i=P_i^{-1}$, $\bar{Q}_{1i}=P_i^{-1}Q_1P_i^{-1}$, $\boldsymbol{\bar{Q}_{2i}=P_i^{-1}Q_2P_i^{-1},\bar{Q}_{3i}=P_i^{-1}Q_3P_i^{-1},\bar{I}_i=P_i^{-1}IP_i^{-1},X_i=K_i\bar{P}_i}$, according to the condition (5), we can get $\boldsymbol{\varpi_1^T(t)\bar{\prod}_i\varpi_1(t)<0}$, that is

$$\dot{V}_i\big(x(t)\big)-\alpha V_i\big(x(t)\big)-\alpha\omega^T(t)\omega(t)<0. \tag{12}$$

Multiply both sides of this inequality (12) by $e^{-\alpha t}$, and integrating from t_k to t, we have

$$V_i\big(x(t)\big)<e^{\alpha(t-t_k)}V_i\big(x(t_k)\big)+\alpha\int_{t_k}^{t}e^{\alpha(t-s)}\omega^T(s)\omega(s)ds. \tag{13}$$

On account of the trajectory $\boldsymbol{x(t)}$ is everywhere continuous, we have $\boldsymbol{x(t_k^-)=x(t_k)}$. Thus, according to the conditions switching law (7), we can obtain

$$V_i\big(x(t_k^-)\big)\ge V_i\big(x(t_k)\big). \tag{14}$$

Combining (13) and (14), using the similar iterative method in [25], according to $\boldsymbol{t}\in\boldsymbol{[0,t_f]},\boldsymbol{0<e^{-\alpha s}<1}$ and Assumption 1.1, we have

$$V_i\big(x(t)\big)<e^{\alpha t}V_i\big(x(0)\big)+\alpha\int_0^t e^{\alpha(t-s)}\omega^T(s)\omega(s)ds\le e^{\alpha t_f}V_i\big(x(0)\big)+\alpha e^{\alpha t_f}\delta. \tag{15}$$

Moerover, based (8), the condition $0\le h_1\le h(t)\le h_2$, and Definition 1.1, we get

$$\begin{aligned}V_i\big(x(0)\big)&\le\beta_{12}x^T(0)x(0)+\beta_2\int_{-h_1}^{0}e^{-\alpha\theta}x^T(\theta)x(\theta)d\theta+\beta_3\int_{-h_2}^{0}e^{-\alpha\theta}x^T(\theta)x(\theta)d\theta\\&\quad+\beta_4\int_{-h(t)}^{0}e^{-\alpha\theta}x^T(\theta)x(\theta)d\theta\\&\le\beta_{12}c_1+\beta_2c_1h_1e^{\alpha h_1}+(\beta_3+\beta_4)c_1h_2e^{\alpha h_2}.\end{aligned} \tag{16}$$

Then,

$$V_i\big(x(t)\big)<e^{\alpha t_f}\{c_1[\beta_{12}+\beta_2h_1e^{\alpha h_1}+(\beta_3+\beta_4)h_2e^{\alpha h_2}]+\alpha\delta\}. \tag{17}$$

On the other hand,

$$V_i(x(t)) \geq \beta_{11} x^T(t)x(t). \tag{18}$$

Therefore, the following result can be obtained

$$x^T(t)x(t) < \frac{e^{\alpha t_f}\{c_1[\beta_{12}+\beta_2 h_1 e^{\alpha h_1}+(\beta_3+\beta_4)h_2 e^{\alpha h_2}]+\alpha\delta\}}{\beta_{11}} < c_2. \tag{19}$$

According to the condition (6) and Definition 1.1 it is easy to get that the nonlinear switched system (4) is finite-time bounded. This completes the proof.

3. Numerical simulation

In this section, a numerical example is given to certify the correctness of the proposed method. Consider the switched system (4) with two subsystems as following:

$$A_{01}=\begin{bmatrix}-1.2 & -0.1\\ 0.2 & -1.5\end{bmatrix},\ A_{02}=\begin{bmatrix}-2.1 & 0.5\\ -1.5 & 0.2\end{bmatrix},\ A_{11}=\begin{bmatrix}1 & 0\\ -0.2 & 0.9\end{bmatrix},\ A_{12}=\begin{bmatrix}-1.2 & 0\\ 0.5 & -0.1\end{bmatrix},\ A_{\mu 1}=\begin{bmatrix}1 & 0\\ -0.2 & 0.8\end{bmatrix},$$

$$A_{\mu 2}=\begin{bmatrix}3 & 0\\ 0.7 & -1\end{bmatrix}, A_{w1}=\begin{bmatrix}-0.9 & 0\\ 0.2 & -1\end{bmatrix}, A_{w2}=\begin{bmatrix}0.5 & -0.2\\ 0 & 0.1\end{bmatrix}, F_1=\begin{bmatrix}-5\cos(x_2)\\ sin(x_1)\end{bmatrix}, F_2=\begin{bmatrix}0\\ 0.1\cos(x_2)\end{bmatrix},$$

$$\omega(t)=\begin{bmatrix}0.05(1+1.5t)\\ \dfrac{-5}{(5+t)}\end{bmatrix}, h(t)=0.15+0.1\,sin(t).$$

Theorem 2.1 shows that the switched system (4) is finite-time bounded when the switching law (7) is satisfied. The corresponding parameter selected are $c_1=0.01, t_f=3, \delta$=0.1, $h_1=0.05, h_2=0.25$, $\dot{h}(t)=0.1, \alpha=0.2, \rho_1=\rho_2=0.5$. Using the LMI toolbox of MATLAB to solve conditions (5) and (6), the following feasible solutions can be obtained. Meanwhile, the state trajectories of the closed-loop system (4) are also given below when the initial condition $x_0=[-1.5,0.5]^T$ in **Figure1**. **Figure 2** shows the value of $x^T(t)x(t)$. Obviously, this is less than the value of c_2 that we get.

$$\bar{P}_1=\begin{bmatrix}14.2456 & 0.2993\\ 0.2993 & 14.6538\end{bmatrix}, \bar{Q}_{11}=\begin{bmatrix}28.2684 & -0.0000\\ -0.0000 & 28.2684\end{bmatrix}, \bar{Q}_{21}=\begin{bmatrix}27.6653 & -0.0000\\ -0.0000 & 27.6653\end{bmatrix},$$

$$\bar{Q}_{31}=\begin{bmatrix}25.6779 & -0.9632\\ -0.9632 & 24.4462\end{bmatrix}, X_1=\begin{bmatrix}-37.1076 & 0.2929\\ 0.2929 & -31.4437\end{bmatrix}, \bar{I}_1=\begin{bmatrix}-6.4873 & -7.3235\\ -7.3235 & -27.2802\end{bmatrix},$$

$$\bar{P}_2=\begin{bmatrix}0.6280 & 0.0251\\ 0.0251 & 1.3963\end{bmatrix}, \bar{Q}_{12}=\begin{bmatrix}1.4102 & -0.0000\\ -0.0000 & 1.4102\end{bmatrix}, \bar{Q}_{22}=\begin{bmatrix}1.3819 & -0.0000\\ -0.0000 & 1.3819\end{bmatrix},$$

$$\bar{Q}_{32}=\begin{bmatrix}1.6003 & -0.0049\\ -0.0049 & 1.4862\end{bmatrix}, X_2=\begin{bmatrix}-8.4760 & 0.6372\\ 0.6372 & 18.5956\end{bmatrix}, \bar{I}_2=\begin{bmatrix}92.3011 & 10.2407\\ 10.2407 & 58.7988\end{bmatrix},$$

$$\beta_{11}=0.0675, \beta_{12}=1.5945, \beta_2=3.5854, \beta_3=3.5133, \beta_4=4.0691, c_2=12.2765.$$

And the controller gain: $K_1=\begin{bmatrix}-2.6064 & 0.0732\\ 0.0657 & -2.1471\end{bmatrix}, K_2=\begin{bmatrix}-13.5252 & 0.6992\\ 0.4834 & 13.3088\end{bmatrix}$.

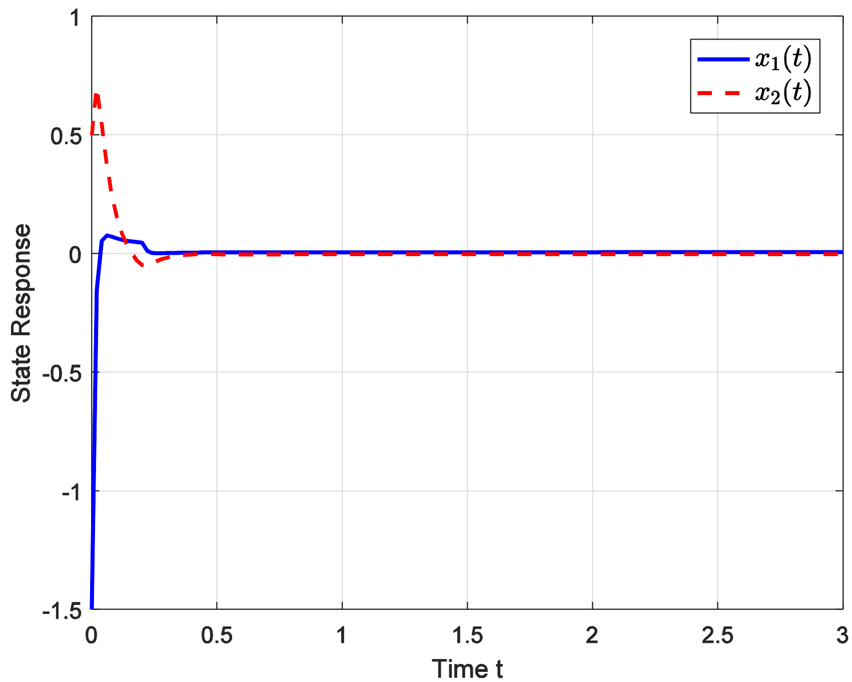

Fig. 1 The time response of the state variables.

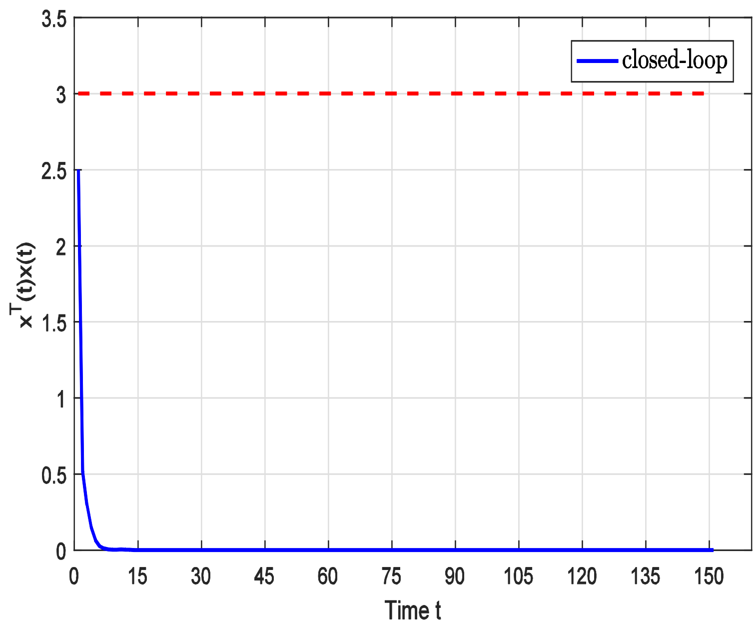

Fig. 2 The time response of $x^T(t)x(t)$ for the closed-loop system (4).

4. Conclusion

In this paper, the boundedness and control of nonlinear switched time-varying delay systems in finite time are studied. By combining LKF with switching rules, a new switching law is obtained, and the boundedness and controller gain of the system in finite time are obtained. Finally, a numerical example is given to prove the feasibility of the proposed method.

References

[1] F. Zhu and P. J. Antsaklis, "Optimal Control of Hybrid Switched Systems: A Brief Survey," *Discrete Event Dynamic Systems*, vol. 25, no. 3, pp. 345-364, 2015.

[2] F. Xiao and L. Wang, "State Consensus for Multi-Agent Systems with Switching Topologies and Time-Varying Delays," *International Journal of Control*, vol. 79, no. 10, pp. 1277-1284, 2006.

[3] J. Yu and L. Wang, "Group Consensus in Multi-Agent Systems with Switching Topologies and Communication Delays," *Systems and Control Letters*, vol. 59, no. 6, pp. 340-348, 2010.

[4] W. Ni and D. Cheng, "Leader-Following Consensus of Multi-Agent Systems Under Fixed and Switching Topologies," *Systems and Control Letters*, vol. 59, no. 3-4, pp. 209-217, 2010.

[5] D. Zheng, H. Zhang, J. Andrew Zhang, et al, "Consensus of the Second-Order Multi-Agent Systems Under Asynchronous Switching with a Controller Fault," *International Journal of Control, Automation and Systems*, vol. 17, no. 1, pp. 136-144, 2019.

[6] Zhendong Sun, "Stabilization and Optimization of Switched Linear Systems," *Automatica*., vol. 42, no. 5, pp. 783-788, 2006.

[7] Gholami. Hadi andBinazadeh, Tahereh, "Sliding-Mode Observer Design and Finite-Time Control of One-Sided Lipschitz Nonlinear Systems with Time-Delay," *Soft Computing*, vol. 23, no. 15, pp. 6429-6440, 2018.

[8] Zhao. JM, Zhang. LJ and Qi. X, "A Necessary and Sufficient Condition for Stabilization of Switched Descriptor Time-Delay Systems Under Arbitrary Switching,"*Asian Journal of Control*, vol. 18, no. 1, pp. 266-272, 2016.

[9] Xiangze. Lin, Haibo. Du andShihua. Li, "Finite-time Boundedness and L2-Gain Analysis for Switched Delay Systems with Norm-Bounded Disturbance," *Applied Mathematics and Computation*, vol. 217, no. 12, pp. 5982-5993, 2011.

[10] Zhang. Xu, Wang. Jianfeng, Wu. Meixi andXu. Zhe, "Robust Quasi-Time Dependent Control for Switched Time-Delay Systems with Performance Guarantee," *Optimal Control Applications and Methods*, vol. 41, no. 6, pp. 1831-1843, 2020.

[11] H. Gholami and M. H. Shafiei, "Finite-Time Boundedness and Stabilization of Switched Nonlinear Systems using Auxiliary Matrices and Average Dwell Time Method," *Trans. Inst. Meas. Control*, vol. 42, no. 7, pp. 1406-1416, 2019.

[12] X. Zhao, Y. Yin, B. Niu and X. Zheng, "Stabilization for a Class of Switched Nonlinear Systems With Novel Average Dwell Time Switching by T-S Fuzzy Modeling," *IEEE Transactions on Cybernetics*, vol. 46, no. 8, pp. 1952-1957, 2015.

[13] Zheng. Qunxian and Zhang. Hongbin, "Robust Stabilization of Continuous-Time Nonlinear Switched Systems without Stable Subsystems via Maximum Average Dwell Time," *Circuits, Systems, and Signal Processing*, vol. 36, no. 4, pp. 1654-1670, 2017.

[14] Liu. Xingwen, Zhong. Shouming, Zhao. Qianchuan, "Dynamics of Delayed Switched Nonlinear Systems with Applications to Cascade Systems," *Automatica*, vol. 87, pp. 251-257, 2018.

[15] Xingwen. Liu, "Stability Analysis of a Class of Nonlinear Positive Switched Systems with Delays," *Nonlinear Analysis: Hybrid Systems*, vol. 16, pp. 1-12, 2015.

[16] T. C. Lee and Z. P. Jiang, "Uniform Asymptotic Stability of Nonlinear Switched Systems with an Application to Mobile Robots," *IEEE Transactions on Automatic Control*, vol. 53, no. 5, pp. 1235-1252, 2008.

[17] B. D. O. Anderson, T. Brinsmead, D. Liberzon, et al, "Multiple Model Adaptive Control with Safe Switching," *International Journal of Adaptive Control and Signal Processing*, vol. 15, no. 5, pp. 445-470, 2001.

[18] W. Gao andJ. C. Hung, "Variable Structure Control of Nonlinear Systems: A New Approach," *IEEE Transactions on Industrial Electronics*, vol. 40, no. 1, pp. 45-55, 1993.

[19] Lijun. Gao andYuqiang. Wu, "A Design Scheme of Variable Structure -Infinity Control for Uncertain Singular Markov Switched Systems Based on Linear Matrix Inequality Method," *Nonlinear Analysis-Hybrid Systems*, vol. 1, no. 3, pp. 306-316, 2007.

[20] He. S and Liu. F, "Finite-Time Fuzzy Control of Nonlinear Jump Systems with Time Delays via Dynamic Observer-Based State Feedback," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 4, pp. 605-614, 2012.

[21] Daizhan. Cheng, Lei. Guo, Yuandan. Lin and Yuan. Wang, "Stabilization of Switched Linear Systems," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 661-666, 2005.

[22] Gholami. Hadi and Shafiei. Mohammad Hossein, "Finite-Time H-Infinity Static and Dynamic Output Feedback Control for a Class of Switched Nonlinear Time-Delay Systems," *Applied Mathematics and Computation*, vol. 389, pp. 125557, 2021.

[23] Gholami. H and Binazadeh. T, "Design Finite-Time Output Feed Back Controller for Nonlinear Discrete-Time Systems with Time-Delay and Exogenous Disturbances," *Systems Science and Control Engineering*, vol. 6, no. 1, pp. 20-27, 2018.

[24] J. Song, Y. Niu and Y. Zou, "Finite-Time Stabilization via Sliding Mode Control," *IEEE Trans. Autom. Control*, vol. 62, no. 3, pp. 1478-1483, 2017.

[25] H. Gholami and M. H. Shafiei, "Finite-Time Boundedness and Stabilization of Switched Nonlinear Systems using Auxiliary Matrices and Average Dwell Time Method," *Trans. Inst. Meas. Control*, vol. 42, no. 7, pp. 1406-1416, 2019.