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Original Article

On Soft Nano Weakly Generalized Closed Maps

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Abstract *- In this paper we introduce a new class of closed map called soft nano weakly generalized closed map. Some of its properties are discussed. Also, soft nano weakly generalised homeomorphisim and soft nano weakly generalised* homeomorphisim is defined and its properties are studied.*

Keywords – $SNwg-$ *closed map*, $SNwg$ *–homeomorphisim*, $SNwg^{*}$ *– homeomorphisim.*

1. Introduction

Different researchers introduced various forms of generalized closed maps and related topological properties. Generalized closed mappings were introduced and studied by Malghan [5].wg-closed sets, wg-closed maps and rwg-closed maps were introduced and studied by Nagaveni [16,17].

The notion of soft topology is introduced by Shabir and Naz [10]. The theory of nano topology and the weaker forms of nano open sets is given by Lellis Thivagar et al. [11] The concept of soft nano topological spaces, soft nano continuity, weaker forms of soft nano open sets, weaker and generalised forms of soft nano continuous functions and weaker and generalised forms of closed(open) mappings in soft nano topological were introduced and studied by Benchalli et al [2-5]

In this paper, we introduced new class of continuous function called soft nano weakly generalized continuous function and are analysed the strength and weakness of them with other existing continuous functions. For many results the converses do not hold good which are substantiated by counter examples. Some of its properties are discussed. Also $SNwg$ –homeomorphisim and $SNwg^{*}$ – homeomorphisim is defined and its properties are studied.

2. Preliminaries

We recall the following definitions which are useful in the sequel.

Definition 2.1[1]:
Let (F,A)be a soft set, then equivalence class of $F(a)$ denoted by $[F(a)]$ is defined as $[F(a)]=\{F(b): F(b)RF(a)\}$.

Definition 2.2 [3]:
Let U be a non-empty finite set of objects called the universe and E be a set of parameters. Let R be a soft equivalence relation on U. The triplet (U,R,E) is said to be the soft approximation space. Let $X\subseteq U$.
(i) The soft lower approximation of X with respect to R and set of parameters E, is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by
$(L_R(X),E)$. i.e., $(L_R(X),E)=\cup\{R(x): R(x)\subseteq X$, where $R(x)$ denotes the equivalence class determined by $x\in U$.
(ii) The soft upper approximation of X with respect to R and set of parameters E, is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by
$(U_R(X),E)$. i.e., $(U_R(X),E)=\cup\{R(x): R(x)\cap X\neq\emptyset\}$
(iii) The soft boundary region of X with respect to R and set of parameters E, is the set of all objects, which can be classified neither inside X nor as outside X with respect to R and is denoted by $(B_R(X),E)$,That is $(B_R(X),E)=(U_R(X),E)-(L_R(X),E)$.

Definition 2.3 [3]:
Let U be a non-empty universal set and E be a set of parameters. Let R be a soft equivalence relation on U. Let $X\subseteq U$. Let $(\tau_R(X),U,E)=\{U,\emptyset,(L_R(X),E),(U_R(X),E),(B_R(X),E)\}$. Then, $(\tau_R(X),U,E)$ is a soft topology on (U,E), called soft nano topology with respect to X. Elements of the soft nano topology are known as the soft nano open sets and $(\tau_R(X),U,E)$ is called soft nano topological space. The complements of soft nano open sets are called as soft nano closed sets in $(\tau_R(X),U,E)$.

Definition 2.4 [3]:
If $(\tau_R(X), U, E)$ is a soft nano topological space with respect to X and E, where $X \subseteq U$ and if $(A, E) \subseteq (U, E)$, then the soft nano interior of (A, E) is defined as the union of all soft nano open subsets of (A, E) and it is denoted by $NInt(A, E)$. i.e. $NInt(A, E)$ is the largest soft nano open subset of (A, E).
The soft nano closure of (A, E) is defined as the intersection of all soft nano closed sets containing (A, E) and it is denoted by $NCl(A, E). i.e. NCl(A, E)$ is the smallest soft nano closed set containing (A, E).

Definition 2.5:
A subset (A, E) of a soft nano topological space $(U, \tau_R(X))$ is called

(i) soft nano g-closed [4] if $N\,Cl(A, E) \subseteq (V, E)$, whenever $(A, E) \subseteq (V, E)$ and (V, E) is nano open.
(ii) SNwg-closed [19] if $N\,Cl(N\,Int(A, E)) \subseteq (V, E)$, whenever $(A, E) \subseteq (V, E)$ and (V, E) is nano open.

Definition 2.6[4]:
Let $(\tau_R(X), U, E)$ and $(\tau_{R'}(Y), V, K)$be a two soft Nano topological spaces. A mapping $f: (\tau_R(X), U, E) \rightarrow (\tau_{R'}(Y), V, K)$is called

(i) SNWg- continuous on U if the inverse image of every soft nano closed set in V is SNWg-closed in U.
(ii) soft nano generalized closed mapping if the image of each soft nano closed set over U is soft nano generalized closed set over V.

Theorem 2.7[20]
If $f: (\tau_R(X), U, E) \rightarrow (\sigma_R(Y), V, K)$is SNWg- irresolute function. Then f is SNWg-continuous function.

3. Soft Nano Weakly Generalized Maps

Definition 3.1
The map $f: (\tau_R(X), U, E) \rightarrow (\sigma_{R'}(Y), V, K)$ is SNwg- closed map if the image of every soft nano closed (open) set in $(\tau_R(X), U, E)$ is SNWg- closed (open) set in $(\sigma_{R'}(Y), V, K)$.

Example 3.2
Let $U = \{a, b, c, d\}, E = \{h_1, h_2, h_3\}$ and $(G, E) = \{(h_1, \{a\}), (h_2, \{c\}), (h_3, \{b, d\})\}$ be a soft set over U. Let R be a soft equivalence relation on (G, E) defined as follows:
$R = \{ F(h_1) \times F(h_1), F(h_2) \times F(h_2), F(h_3) \times F(h_3), F(h_4) \times F(h_4), F(h_1) \times F(h_2), F(h_2) \times F(h_1)\}$
Then the soft equivalence classes are$[F(h_1)] = \{ F(h_1), F(h_2)\} = [F(h_2)], [F(h_3)] = \{F(h_3)\}$.
Now, let U/R=$\{ F(h_1), F(h_2), F(h_3)\} = \big\{\{a\}, \{c\}, \{b, d\}\big\}$. Let $X = \{a, b\} \subseteq U$. Then soft nano topology is $(\tau_R(X), U, E) = \{U, \emptyset, L_R(X, E), U_R(X, E), B_R(X, E)\}$, were

$(L_R(X), E) = \{(h_1, \{a\}), (h_2, \{a\}), (h_3, \{a\})\}$
$(U_R(X), E) = \{(h_1, \{a, b, d\}), (h_2, \{a, b, d\}), (h_3, \{a, b, d\})\}$
$(B_R(X), E) = \{(h_1, \{b, d\}), (h_2, \{b, d\}), (h_3, \{b, d\})\}$

SNwg-open sets are $U, \emptyset$, $(I_1, E) = \{(h_1, \{a\}), (h_2, \{a\}), (h_3, \{a\})$
$(I_2, E) = \{(h_1, \{b\}), (h_2, \{b\}), (h_3, \{b\})\}$
$(I_3, E) = \{(h_1, \{d\}), (h_2, \{d\}), (h_3, \{d\})\}$
$(I_4, E) = \{(h_1, \{a, b\}), (h_2, \{a, b\}), (h_3, \{a, b\})\}$
$(I_5, E) = \{(h_1, \{b, d\}), (h_2, \{b, d\}), (h_3, \{b, d\})\}$
$(I_6, E) = \{(h_1, \{a, d\}), (h_2, \{a, d\}), (h_3, \{a, d\})\}$
$(I_7, E) = \{(h_1, \{b, c\}), (h_2, \{b, c\}), (h_3, \{b, c\})\}$
$(I_8, E) = \{(h_1, \{c, d\}), (h_2, \{c, d\}), (h_3, \{c, d\})\}$
$(I_9, E) = \{(h_1, \{a, b, c\}), (h_2, \{a, b, c\}), (h_3, \{a, b, c\})\}$
$(I_{10}, E) = \{(h_1, \{a, c, d\}), (h_2, \{a, c, d\}), (h_3, \{a, c, d\})\}$
$(I_{11}, E) = \{(h_1, \{a, b, d\}), (h_2, a, b, d), (h_3, \{a, b, d\})\}$

Let $V = \{w, x, y, z\}, K = \{k_1, k_2, k_3\}$ and
let V/R'=$\{ F(h_1), F(h_2), F(h_3)\} = \big\{\{w\}, \{z\}, \{x, y\}\big\}$. Let $Y = \{w, y\} \subseteq V$. Then the soft nano topology is $(\sigma_{R'}(Y), V, K) = \{V, \emptyset, L_{R'}(Y, K), U_{R'}(Y, K), B_{R'}(Y, K)\}$.

$(L_{R'}(Y), K) = \{(k_1, \{w\}), (k_2, \{w\}), (k_3, \{w\})\}$
$(U_{R'}(Y), K) = \{(k_1, \{w, x, y\}), (k_2, \{w, x, y\}), (k_3, \{w, x, y\})\}$
$(B_{R'}(Y), K) = \{(k_1, \{x, y\}), (k_2, \{x, y\}), (k_3, \{x, y\})\}$

SNwg-open sets are $V, \emptyset$

$$(J_1, K) = \{(k_1, \{x\}), (k_2, \{x\}), (k_3, \{x\})\}$$
$$(J_2, K) = \{(k_1, \{y\}), (k_2, \{y\}), (k_3, \{y\})\}$$
$$(J_3, K) = \{(k_1, \{w\}), (k_2, \{w\}), (k_3, \{w\})\}$$
$$(J_4, K) = \{(k_1, \{x, w\}), (k_2, \{x, w\}), (k_3, \{x, w\})\}$$
$$(J_5, K) = \{(k_1, \{x, z\}), (k_2, \{x, z\}), (k_3, \{x, z\})\}$$
$$(J_6, K) = \{(k_1, \{y, w\}), (k_2, \{y, w\}), (k_3, \{y, w\})\}$$
$$(J_7, K) = \{(k_1, \{y, z\}), (k_2, \{y, z\}), (k_3, \{y, z\})\}$$
$$(J_8, K) = \{(k_1, \{x, y\}), (k_2, \{x, y\}), (k_3, \{x, y\})\}$$
$$(J_9, K) = \{(k_1, \{w, x, y\}), (k_2, \{w, x, y\}), (k_3, \{w, x, y\})\}$$
$$(J_{10}, K) = \{(k_1, \{x, w, z\}), (k_2, \{x, w, z\}), (k_3, \{x, w, z\})\}$$
$$(J_{11}, K) = \{(k_1, \{y, w, z\}), (k_2, \{y, w, z\}), (k_3, \{y, w, z\})\}$$

Let $f: (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, K)$, let us consider$f: U \to$V and $p: E \to K$ by $f(a) = x, f(b) = y, f(c) = z$, $f(d) = w$, and $p(h_1) = k_1, p(h_2) = k_2, p(h_3) = k_3$, $f(U) = V$, as $f(\emptyset) = \emptyset, f\big(L_R(X, E)\big) = (J_1, K)$, $f\big(B_R(X, E)\big) = (J_6, K), f\big(U_R(X, E)\big) = (J_6, K)$, Then f is SNwg- open map

Remark 3.3

Composition of two SNwg- closed(open) map need not be a SNwg- closed(open) map.

Example 3.4

Let $U = V = W = \{a, b, c, d\}, E = H = K = \{h_1, h_2, h_3\}$ with $(\tau_R(X), U, E) = \{U, \emptyset, L_R(X, E), U_R(X, E), B_R(X, E)\}$, Where $L_R(X, E) = \{(h_1, \{a\}), (h_2, \{a\}), (h_3, \{a\})\}$, $U_R(X, E)$ =$\{(h_1, \{a, b, d\}), (h_2, \{a, b, d\}), (h_3, \{a, b, d\})\}$, $B_R(X, E) = \{(h_1, \{b, d\}), (h_2, \{b, d\}), (h_3, \{b, d\})\}$,

$(\tau_{R'}(Y), V, H) = \{U, \emptyset, L_{R'}(X, E), U_{R'}(X, E), B_{R'}(X, E)\}$, Where $L_{R'}(Y, H) = \{(h_1, \{b\}), (h_2, \{b\}), (h_3, \{b\})\}$

$U_{R'}(Y, H) = \{(h_1, \{a, b, c\}), (h_2, \{a, b, c\}), (h_3, \{a, b, da, b, c\})\}$,

$$B_{R'}(Y, H) = \{(h_1, \{a, c\}), (h_2, \{a, c\}), (h_3, \{b, d\})\}$$

$(\tau_{R''}(Z), W, K) = \{U, \emptyset, L_{R''}(X, E), U_{R''}(X, E), B_{R''}(X, E)\}$, Where

$$L_{R''}(Z, K) = \{(h_1, \{c\}), (h_2, \{c\}), (h_3, \{c\})\}$$
$$U_{R''}(Z, K) = \{(h_1, \{a, b, c\}), (h_2, \{a, b, c\}), (h_3, \{a, b, c\})\}$$

$B_{R''}(Z, K) = \{(h_1, \{a, b\}), (h_2, \{a, b\ \}), (h_3, \{a, b\})\}$

Let $f: (\tau_R(X), U, E) \to (\tau_{R'}(Y), V, H)$ by let us consider$f: U \to$V and $p: E \to K$ by $f(a) = b, f(b) = c, f(c) = a, f(d) = d$ and $p(h_1) = k_1, p(h_2) = k_2, p(h_3) = k_3$.

Let $g: (\tau_{R'}(Y), V, K) \to (\tau_{R''}(Z), W, K)$ be the identity map. Function f and g are SNwg- closed map but their composition $g \circ f: (\tau_R(X), U, E) \to (\tau_{R''}(Z), W, K)$ is not SNwg- closed map.

Remark 3.5

Image of SNwg- closed set need not be a SNwg- closed set under SNwg- closed map.

Example 3.6

In example 3.4 g is SNwg- closed map. Consider the set $\{(h_1, \{a, b\}), (h_2, \{a, b\ \}), (h_3, \{a, b\})\}$,which is SNwg- closed set in $(\tau_{R'}(Y), V, K)$. But not in $(\tau_{R''}(Z), W, K)$.

Theorem 3.7

Let $f: (\tau_R(X), U, E) \to (\sigma_{R'}(Y), V, K)$ be a SN- closed map and $g: (\tau_{R'}(X), V, K) \to (\tau_{R''}(Z), W, K)$ SNwg- closed map then their composition is SNwg- closed map.

Proof

Let (A, E) be any SN closed set in $(\tau_R(X), U, E)$. Then $f((A, E))$is soft nano closed in $(\sigma_{R'}(Y), V, K)$ and $(g \circ f)(A, E) = g(f(A, E))$ is SNwg closed since g is SNwg closed map. Hence the composition is SNwg- closed map. Hence the composition is SNwg closed map.

Remark 3.8

If f is SNwg- closed map, g is SN closed map then their composition need not be SNwg- closed map.

Theorem 3.9

Let $f: (\tau_R(X), U, E) \to (\sigma_{R'}(Y), V, K)$ be a SNwg- closed map if and only if for each subset (F, K) of $(\sigma_R(Y), V, K)$ and for each nano open set (G, K) containing $f^{-1}((F, K))$ there is a SNwg- open set (B, K) of (V, K) such that $(F, K) \subset (V, K)$ and $f^{-1}((V, K)) \subset (F, K)$

Proof

Suppose f is SNwg- closed. Let (A, K) be a subset of (V, K) and (F, E) is an open set of (U, E) such that $f^{-1}((\mathrm{V}, \mathrm{K})) \subset (F, K)$. Then $(B, K) = (V, K) - f((U, E) - (F, E))$ is a SNwg- open set containing (F, K) such that $f^{-1}((V, K)) \subset (F, K)$.

Conversely, suppose that (C, E) is also closed set of (U, E). Then $f^{-1}\big((V, K) - f((C, E))\big) \subset (U, E) - (C, E)$ and $(U, E) - (C, E)$ is soft nano open. By hypothesis there is SNwg- open set (B, K) of (V, K) such that $(\mathrm{V}, \mathrm{K}) - F((C, E)) \subset (B, K)$ and $f^{-1}((B, K)) \subset (U, E) - (C, E)$. Therefore $(C, E) \subset (U, E) - f^{-1}((B, K))$. Hence$(V, K) - (B, K) \subset f((C, E)) \subseteq f\big((U, E) - f^{-1}((B, K))\big) \subset (V, K) - (B, K)$ which implies that $((C, E)) = (V, K) - (B, K)$. Since$(V, K) - (B, K)$ is SNwg- closed set, $f((C, E))$is SNwg- closed. Hence f is SNwg- closed.

Theorem 3.10

Let $f: (\tau_R(X), U, E) \to (\sigma_{R'}(Y), V, K)$ be any function and if $\tau_R(X) \neq \{U, \emptyset\}$ and $\sigma_R(Y) = \{V, \emptyset\}$. Then

(i) f is not nano closed map
(ii) f is SNg- closed map
(iii) f is SNwg- closed map

Proof

(i)Since the only soft nano closed set in $(\sigma_R(Y), V, K)$ are $\{V, \emptyset\}$ the image of nano closed set are not nano closed in $(\sigma_{R'}(Y), V, K)$. Hence f is not nano closed map.

(ii) Let (A, E)be any nano closed set in $(\tau_R(X), U, E)$. The soft nano open set containing $f\big((A, E)\big)$ is V. Hence$Ncl\big(f(A, E)\big) \subset V$. That is $f(A, E)$is SNg- closed. Therefore $f(A, E)$is SNg- closed map.

(iii)Let (A, E)be any nano closed set in $(\tau_R(X), U, E)$. The soft nano open set containing $f(A)$ is V. Hence$Ncl\Big(Nint\big(f(A, E)\big)\Big) \subset V$. That is $f(A, E)$is SNwg- closed. Therefore $f\big((A, E)\big)$is SNwg- closed map.

Theorem 3.11

Let $f: (\tau_R(X), U, E) \to (\sigma_{R'}(Y), V, K)$ be any function. Let (A, E)be any soft nano closed set in $(\tau_R(X), U, E)$. Suppose if (V, K) is an only soft nano open sent containing $f\big((A, E)\big)$, then f is SNwg- closed map

Proof

By assumption $f\big((A, E)\big) \subset (V, K)$ where (V, K) is an only soft nano open sent containing$f\big((A, E)\big)$, then $Ncl\Big(Nint\big(f(A, E)\big)\Big) \subset V$. Hence $f(A, E)$is SNwg- closed. Therefore $f\big((A, E)\big)$is SNwg- closed map.

Theorem 3.12

Every soft nano closed map is SNwg- closed map

Proof

Let $f: (\tau_R(X), U, E) \to (\sigma_{R'}(Y), V, K)$ be a soft nano closed map. Let (A, E) be a soft nano closed set in $(\tau_R(X), U, E)$. Then the image of (A, E) under the map f is soft nano closed $(\sigma_{R'}(Y), V, K)$. Since every soft nano closed set is SNwg- closed. $f\big((A, E)\big)$is SNwg- closed. Hence f is SNwg- closed.

Remark 3.13

The converse of the above theorem need not be true as seen from the following example.

Example 3.14

In example 3.4 g is SNwg- closed map. Consider the set $\{(h_1, \{a, b\}), (h_2, \{a, b\}), (h_3, \{a, b\})\}$,which is SNwg- closed set in $(\tau_{R'}(Y), V, K)$. But not in $(\tau_{R''}(Z), W, K)$

Theorem 3.15

Every SNg- closed map is SNwg- closed map not conversely.

Proof

Let $f: (\tau_R(X), U, E) \to (\sigma_{R'}(Y), V, K)$ be a soft nano closed map. Let (A, E) be a soft nano closed set in $(\tau_R(X), U, E)$. Then the image of (A, E) under the map f is SNg- closed in $(\sigma_{R'}(Y), V, K)$. Since every SNg- closed set is SNwg- closed set. $f\big((A, E)\big)$is SNwg- closed. Hence f is SNwg- closed.

Remark3.16

Let $f: (\tau_R(X), U, E) \to (\sigma_{R'}(Y), V, K)$ be any function. If $f\big((A, E)\big) \subset (B, E)$, (B, E) is soft nano open set in $(\sigma_{R'}(Y), V, K)$,$Nint\big(f(A, E)\big) = \emptyset, Ncl\big(f(A, E)\big) \not\subset (B, E)$. Then f is SNwg- closed but not SNg- closed. It follows from the following example.

Example 3.17

Let $U = V = \{a, b, c, d\}, E = H = \{h_1, h_2, h_3\}$ with $(\tau_R(X), U, E) = \{U, \emptyset, \{(h_1, \{a, b, d\}), (h_2, \{a, b, d\}), (h_3, \{a, b, d\})\}\}$, $(\sigma_{R'}(Y), V, H) = \{U, \emptyset, \{(h_1, \{a, b, c\}), (h_2, \{a, b, c\}), (h_3, \{a, b, c\})\}\}$. Let $f: (\tau_R(X), U, E) \to (\sigma_{R'}(Y), V, K)$ be an identity map. Then f is SNwg- closed but not SNg- closed. Since $cl(f(\{(h_1, \{a\}), (h_2, \{a\}), (h_3, \{a\})\})) = Ncl(\{(h_1, \{a\}), (h_2, \{a\}), (h_3, \{a\})\}) = (V, E) \not\subset \{(h_1, \{a, b, c\}), (h_2, \{a, b, c\}), (h_3, \{a, b, c\})\}$, $\{(h_1, \{a\}), (h_2, \{a\}), (h_3, \{a\})\}$ is not SNg- closed in $(\sigma_{R'}(Y), V, H)$.

4. Soft Nano Weakly Generalised Homeomorphisim and Soft Nano Weakly Generalised* Homeomorphisim

In this section is to define soft nano weakly generalised homeomorphism and soft nano weakly in generalised* homeomorphism in soft nano topological spaces

Definition 4.1

A function $f: (\tau_R(X), U, E) \to (\sigma_{R'}(Y), V, K)$ is said to be SNwg- homeomorphism if

(i) f is one-one and onto
(ii) f is SNwg- continuous function
(iii) f is SNwg- open(closed) mapping

Theorem 4.2

Every soft nano homeomorphism is SNwg- homeomorphism

Proof

Let $f: (\tau_R(X), U, E) \to (\sigma_{R'}(Y), V, K)$ be a soft nano homeomorphism. By the definition f is1-1 and onto, nano continuous and nano open map. Since every nano continuous is SNwg- continuous and every nano open map is SNwg- open map. $(\sigma_{R'}(Y), V, K)$ SNwg-homeomorphism.

Remark 4.3

The converse of the above theorem need not be true

Example 4.4

In example 3.2 f is SNwg- homeomorphism. But not soft nano homeomorphism.

Theorem 4.5

Every SNg-homeomorphism is SNwg- homeomorphism

Proof

The proof of the theorem follows from the fact that every SNg- continuous function is SNwg-continuous function and every SNg-open map is SNwg- open map.

Remark 4.6

The converse of the above theorem need not be true as shown in the following example.

Example4.7

In example 3.17 f is not SNg- homeomorphism but SNwg- homeomorphism

Theorem 4.8

Let $f: (\tau_R(X), U, E) \to (\sigma_R(Y), V, K)$ be a bijective SNwg-continuous function. Then the following are equivalent

(i) f is SNwg- open map
(ii) f is SNwg- homeomorphism
(iii) f is SNwg- closed map

Proof

(i)⇒(ii) By the assumption and (i) f is bijective SNwg-continuous function and SNwg- open map. By the definition of SNwg- homeomorphism f is SNwg- homeomorphism.

(ii)⇒(iii) Since f isSNwg- homeomorphism, it is 1-1, onto, SNwg-continuous function and SNwg- open map. Let (A, E) be a soft nano closed set in $(\tau_R(X), U, E)$. Then $f((U, E) - (A, E))$ is SNwg- open in $(\sigma_R(Y), V, K)$. $f((U, E) - (A, E)) = f((U, E)) - f((A, E)) = (V, E) - f((A, E))$ is SNwg-open. Hence $f((A, E))$ is SNwg- closed map in $(\sigma_R(Y), V, K)$.

(iii)⇒(i) Let (A, E) be a soft nano open set in $(\tau_R(X), U, E)$. Then $f((U, E) - (A, E))$ is SNwg- closed in $(\sigma_R(Y), V, K)$. That is $f((A, E))$ is SNwg- open map in $(\sigma_R(Y), V, K)$. Therefore f is SNwg- open map.

Definition 4.9

A function $f:(\tau_R(X),U,E)\rightarrow(\sigma_R(Y),V,K)$ is said to be SNwg*- homeomorphism if both f and f^{-1} are SNwg- irresolute.

Remark 4.10

If a function $f:(\tau_R(X),U,E)\rightarrow(\sigma_R(Y),V,K)$ is said to be SNwg*- homeomorphism if

(i) f is one-one and onto

(ii) f is SNwg- irresolute function

(iii) f is SNwg- open(closed) map

Theorem 4.11

Every SNwg*-homeomorphism is SNwg- homeomorphism

Proof

The proof of the theorem follows from the definition and by the theorem 2.7

Remark 4.12

The converse of the above theorem need not be true, Since SNwg- continuous function need not be a SNwg- irresolute function.

Example 4.13

In example 3.2 image of open sets are $\emptyset, V, (J_1,K), (J_9,K), (J_6,K)$. Where $\emptyset, V, (J_1,K), (J_9,K)and(J_6,K)$ are SNwg- open sets in$(\tau_{R'}(Y),V,K)$. Hence $f$is SNwg-open function. Hence $f$is SNwg-homeomorphism but $f$is not SNwg- irresolute. Since the set $f^{-1}\big((J_1,K)\big)=(I_1,K)$ is not SNwg- closed in $(\tau_R(X),U,E)$. Hence $f$isnot SNwg*-homeomorphism.

5. Conclusion

The concept of a soft nano weakly generalised closed map is introduced here, and some of its properties are discussed. Also, soft nano weakly generalised homeomorphisim and soft nano weakly generalised* homeomorphisim is defined and its properties are studied.

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