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Original Article

Perpetuation of Eminent Forms of 2-Tuples into 3-Tuples Interlacing Some k-Polygonal Numbers with Appropriate Properties

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Abstract - *In this manuscript, the patterns of 3-tuples* $(u_{n_mk}, v_{n_mk}, w_{n_mk}), (v_{n_mk}, w_{n_mk}, x_{n_mk}), (w_{n_mk}, x_{n_mk}, y_{n_mk}),$ $(x_{n_mk}, y_{n_mk}, z_{n_mk})$ *etc,* $m = 1,2,3,4,5$ *where the elements are some* $k-$*polygonal numbers such that the product of two numbers enlarged by certain numbers remains a perfect square are appraised. Also, the MATLAB program for the verification of all the patterns of 3-tuples sustaining dissimilar properties is exemplified.*

Keywords - *Diophantine triples, Integer sequence, Polygonal numbers.*

1. Introduction

"A set of m positive integers $\{a_1, a_2 \dots . a_m\}$ is called a Diophantine $m-$tuples with the property $D(n), n \in Z - \{0\}$ if $a_i . a_j + n$ is a perfect square for all $1 \leq i < j \leq m$". In [3,4], the authors found few Diophantine triples with right properties. In [9,10], the authors extended Diophantine triples into quadruples involving Pell number and sequence of quadruples containing Gnomonic number by using the solutions of Pell equation. For finding various interesting numbers and for on extensive review one can refer [1,2,5-8,11-26]. In this communication, the patterns of 3-tuples $(u_{n_mk}, v_{n_mk}, w_{n_mk}), (v_{n_mk}, w_{n_mk}, x_{n_mk}),$ $(w_{n_mk}, x_{n_mk}, y_{n_mk}), (x_{n_mk}, y_{n_mk}, z_{n_mk})$ etc, $m = 1,2,3,4,5$ where the elements are some $k-$ polygonal numbers such that the multiplication of two of such numbers increased by some specific numbers stays a perfect square are audit. Also, the MATLAB program for the confirmation of all prescribed patterns of 3-tuples supporting disparate properties is demonstrated.

2. Diophantine 3-tuples involving polygonal numbers.

2.1. Creation of Diophantine 3-tuples concerning k-hexagonal numbers

The sequence of hexagonal number is defined by

$$h_s = 2s^2 - s, s \in N$$

The k-hexagonal sequence is activated from the hexagonal sequence by

$$6k + 1, 15k + 6, 28k + 15, \dots$$

The recurrence relation for k-hexagonal sequence is stated by

$$H_{s-2} = h_{s-2} + kh_{s-1}, s \geq 3 \text{ with } h_1 = 1, h_2 = 6.$$

Indicate $u_{n_1k} = H_{n-2} = 2n^2(k + 1) - n(5k + 9) + 3k + 10$ and

$v_{n_1k} = H_n = 2n^2(k+1) + n(3k-1) + k$ where $n \geq 3$ are two distinct positive integers such that $u_{n_1k}.v_{n_1k} + (k^2 - 14k + 1) = (2n^2(k+1) - n(k+5) - 2k - 1)^2$

Then, the couple of elements $(u_{n_1k}, v_{n_1k})$is named as a Diophantine 2-tuples with property $D_{n_1k}(k^2 - 14k + 1)$.

Deliberate w_{n_1k} is a new-fangled positive integer composed with the subsequent simultaneous equations

$$u_{n_1k}.w_{n_1k} + (k^2 - 14k + 1) = l^2 \quad (1)$$

$$v_{n_1k}.w_{n_1k} + (k^2 - 14k + 1) = m^2 \quad (2)$$

By executing a simple algebraic calculation, it is observed from (1) and (2) that

$$v_{n_1k}l^2 - u_{n_1k}m^2 = (v_{n_1k} - u_{n_1k})\,(k^2 - 14k + 1) \quad (3)$$

To estimate the conceivable value of w_{n_1k}, introduce the successive alternations in (3)

$$l = M + u_{n_1k}S\,, m = M + v_{n_1k}S \quad (4)$$

Then, the quadratic equation is conquered by

$$M^2 = u_{n_1k}.v_{n_1k}S^2 + (k^2 - 14k + 1) \quad (5)$$

Engaging the least positive root $(M, S) = (2n^2(k+1) - n(k+5) - 2k - 1,1)$ of (5) in (4), an apt option for w_{n_1k} is attained by $w_{n_1k} = 8n^2(k+1) - 4n(k+5) + 12$.

Note that, the triple

$$(u_{n_1k}, v_{n_1k}, w_{n_1k}) = (2n^2(k+1) - n(5k+9) + 3k + 10, 2n^2(k+1) + n(3k-1) + k, 8n^2(k+1) - 4n(k+5) + 12)$$

is a Diophantine triple sustaining the condition $D_{n_1k}(k^2 - 14k + 1)$.

Let (v_{n_1k}, w_{n_1k}) be a pair such that their product added with $k^2 - 14k + 1$ is a perfect square. Now, pick x_{n_1k} is another positive integer and applying the same procedure as described above by electing proper conversions, the choice of x_{n_1k} is exposed by

$$x_{n_1k} = 18n^2(k+1) + 3n(k-11) - k + 14$$

Similarly, if (w_{n_1k}, x_{n_1k}) is other 2-tuples such that multiplication of these two elements enlarged by $k^2 - 14k + 1$ is a square of an integer and y_{n_1k} is a third element for extending this 2-tuples into 3-tuples, then by repeating an analogous technique as declared above it is calculated by

$$y_{n_1k} = 50n^2(k+1) - n(5k+105) - 3k + 52$$

In this way if (y_{n_1k}, z_{n_1k}) is an other 2-tuples and z_{n_1k} is a next element for receiving

3-tuples, then it is noted that

$$z_{n_1k} = 128n^2(k+1) - 256n - 8k + 120$$

Consequently,$(u_{n_1k}, v_{n_1k}, w_{n_1k}), (v_{n_1k}, w_{n_1k}, x_{n_1k}), (w_{n_1k}, x_{n_1k}, y_{n_1k}), (x_{n_1k}, y_{n_1k}, z_{n_1k})$ etc is a pattern of triples in which the product of two numbers increased by $k^2 - 14k + 1$ is a square number.

Numerical calculations of this process for few values of n and k are listed in Table 2.1 below.

Table 2.1

n	k	D_{n_1k}	$(un1k, vn1k, wn1k)$	$(v_{n_1k}, w_{n_1k}, x_{n_1k})$	$(w_{n_1k}, x_{n_1k}, y_{n_1k})$	$(x_{n_1k}, y_{n_1k}, z_{n_1k})$
3	1	−12	(7,43,84)	(43,84,247)	(84,247,619)	(247,619,1648)
	2	−23	(13,71,144)	(71,144,417)	(144,147,1051)	(147,1051,2792)
	3	−32	(19,99,204)	(99,204,587)	(204,587,1483)	(587,1483,3936)
4	1	−12	(21,73,172)	(73,172,469)	(172,469,1209)	(469,1209,3184)
	2	−23	(36,118,284)	(118,284,768)	(284,768,1986)	(768,1986,5224)
	3	−32	(51,163,396)	(163,396,1067)	(396,1067,2763)	(1067,2763,7264)
5	1	−12	(43,111,292)	(111,292,763)	(292,763,1999)	(763,1999,5232)
	2	−23	(71,177,472)	(177,472,1227)	(472,1227,3221)	(1227,3221,8424)
	3	−32	(99,243,652)	(243,652,1691)	(652,1691,4443)	(1691,4443,11616)

2.2. Formation of Diophantine 3-tuples linking k-octagonal numbers

Describe the sequence of octagonal number by the symbol

$$c_s = 3s^2 - 2s, s \in N$$

The k-octagonal sequence is derived from octagonal sequence by

$$8k + 1 ,21k + 8,40k + 21, ...$$

The recurrence relation for k-octagonal sequence is prearranged by

$$C_{s-2} = c_{s-2} + kc_{s-1}, s \geq 3 \text{ with } c_1 = 1, c_2 = 8$$

Elect $u_{n_2k} = C_{n-1} = 2n^2(k + 1) - n(5k + 9) + 3k + 10$ and

$$v_{n_2k} = C_n \quad = 2n^2(k + 1) + n(3k - 1) + k \text{ where } n \geq 2$$

such that $u_{n_2k}.v_{n_2k} + k^2 - 7k + 1 = (3n^2(k + 1) + n(k - 5) - k + 1)^2$

Then, (u_{n_2k}, v_{n_2k}) is termed as Diophantine 2-tuples with the condition $D_{n_2k}(k^2 - 7k + 1)$

As in section 2.1, it is probable to scrutinize the pattern of Diophantine triples $(u_{n_2k}, v_{n_2k}, w_{n_2k}), (v_{n_2k}, w_{n_2k}, x_{n_2k}),$ $(w_{n_2k}, x_{n_2k}, y_{n_2k}), (x_{n_2k}, y_{n_2k}, z_{n_2k})$ etc with the property $D_{n_2k}(k^2 - 7k + 1)$, where

$$w_{n_2k} = 12n^2(k + 1) + 4n(k - 5) - k + 7$$

$$x_{n_2k} = 27n^2(k + 1) + n(18k - 36) + 9$$

$$y_{n_2k} = 75n^2(k + 1) + n(40k - 110) - 3k + 32$$

$$z_{n_2k} = 192n^2(k + 1) + n(112k - 272) - 5k + 75$$

The ensuing table 2.2 shows numerical samples of triples for some n and k.

Table 2.2

n	k	D_{n_2k}	$(u_{n_2k}, v_{n_2k}, w_{n_2k})$	$(v_{n_2k}, w_{n_2k}, x_{n_2k})$	$(w_{n_2k}, x_{n_2k}, y_{n_2k})$	$(x_{n_2k}, y_{n_2k}, z_{n_2k})$
2	1	-5	(9,29,70)	(29,70,189)	(70,189,489)	(189,489,1286)
	2	-9	(17,50,125)	(50.125,333)	(125,333,866)	(333,866,2273)
	3	-11	(25,71,180)	(71,180.477)	(180,477,1243)	(477,1243,3260)
3	1	-5	(29,61,174)	(61,174,441)	(174,441,1169)	(441,1169,3046)
	2	-9	(50,101,293)	(101,293,738)	(293,738,1961)	(738,1961,5105)
	3	-11	(71,141,412)	(141,412,1035)	(412,1035,2753)	(1035,2753,7164)
4	1	-5	(61,105,326)	(105,326,801)	(326,801,2149)	(801,2149,5574)
	2	-9	(101,170,533)	(170,533,1305)	(533,1305,3506)	(1305,3506,9089)
	3	-11	(141,235,740)	(235,740,1809)	(740,1809,4863)	(1809,4863,12604)

2.3. Construction of Diophantine triples in Decagonal numbers

The sequence of decagonal number is demarcated by the code

$$d_s = 4s^2 - 3s, s \in N$$

The k-decagonal sequence is estimated from decagonal sequence by

$$10k + 1, 27k + 10, 52k + 27, ...$$

The recurrence relation for k-decagonal sequence is provided by

$$D_{s-2} = d_{s-2} + kd_{s-1}, s \geq 3 \text{ with } d_1 = 1, d_2 = 10$$

Contemplate $u_{n_3k} = D_{n-1} = (4n^2(k+1) - n(3k+11) + 7$

$v_{n_3k} = D_n = 4n^2(k+1) + n(5k-3) + k, n \geq 2$ be k-decagonal numbers of rank $n-1$ and n respectively such that $u_{n_3k}.v_{n_3k} + (-k^2(4n^2+n-4) - k(-8n+15) - 4n^2 - 7n + 4) = (4n^2(k+1) + n(5k-3) + k)^2$

Thus, (u_{n_3k}, v_{n_3k}) is labelled as Diophantine 2-tuples with the property $D_{n_3k}(-k^2(4n^2+n-4) - k(-8n+15) - 4n^2 - 7n + 4)$. By the way of section 2.1 , it is feasible to dissect the pattern of Diophantine triples $(u_{n_3k}, v_{n_3k}, w_{n_3k}), (v_{n_3k}, w_{n_3k}, x_{n_3k}), (w_{n_3k}, x_{n_3k}, y_{n_3k}), (x_{n_3k}, y_{n_3k}, z_{n_3k})$ etc with the property $D_{n_3k}(-k^2(4n^2+n-4) - k(-8n+15) - 4n^2 - 7n + 4)$. Here

$$w_{n_3k} = 16n^2(k+1) + n(4k-28) - 3k + 11$$

$$x_{n_3k} = 36n^2(k+1) + n(21k-51) - 4k + 15$$

$$y_{n_3k} = 100n^2(k+1) + n(45k-115) - 15k + 52$$

$$z_{n_3k} = 256n^2(k+1) + n(128k-384) - 35k + 123$$

Table 2.3. Numerical specimens of 3-tuples for few values of n and k are records

n	k	D_{n_3k}	$(u_{n_3k}, v_{n_3k}, w_{n_3k})$	$(v_{n_3k}, w_{n_3k}, x_{n_3k})$	$(w_{n_3k}, x_{n_3k}, y_{n_3k})$	$(x_{n_3k}, y_{n_3k}, z_{n_3k})$
2	1	-7	(11,37,88)	(37,88,239)	(88,239,617)	(239,617,1624)
	2	-48	(21,64,157)	(64,157,421)	(157,421,1092)	(421,1092,2869)
	3	-117	(31,91,226)	(91,226,603)	(226,603,1567)	(603,1567,4114)
3	1	-7	(37,79,224)	(79,224,569)	(224,569,1507)	(569,1507,3928)
	2	-103	(64,131,377)	131,377,952)	(377,952,2527)	(952,2527,6581)
	3	-269	(91,183,530)	183,530,1335)	(530,1335,3547)	(1335,3547,9234)
4	1	-7	(79,137,424)	(137,424,1043)	(424,1043,2797)	(1043,2797,7256)
	2	-182	(131,222,693)	(222,693,1699)	(693,1699,4562)	(1699,4562,11829)
	3	-485	(183,307,962)	(307,962,2355)	(962,2355,6327)	(2355,6327,16402)

2.4. Conception of Diophantine triples in k-dodecagonal numbers

The general form of dodecagonal number is meant by

$$dd_s = 5s^2 - 4s, n \in N$$

The sequence of k-dodecagonal number framed from dodecagonal sequence is

$$12k + 1, 33k + 12, 64k + 33, \ldots$$

The recurrent relation for k-dodecagonal sequence is agreed by

$$DD_{s-2} = dd_{s-2} + k\, dd_{s-1}, s \geq 3 \text{ where } dd_1 = 1, dd_2 = 12.$$

Undertake $u_{n_4k} = DD_{n-1} = 5n^2(k+1) - n(4k+14) + 9$

$$v_{n_4k} = DD_n = 5n^2(k+1) + n(6k-4) + k, n \geq 2 \text{ together with}$$

$$u_{n_4k}.v_{n_4k} + 4k^2 - 17k + 4 = (5n^2(k+1) + n(k-9) - 2k + 2)^2$$

Thus, (u_{n_4k}, v_{n_4k}) is characterized as Diophantine 2-tuples with the property $D_{n_4k}(4k^2 - 17k + 4)$. By the mode of section 2.1, it is viable to treasure the form of Diophantine triples $(u_{n_4k}, v_{n_4k}, w_{n_4k}), (v_{n_4k}, w_{n_4k}, x_{n_4k}), (w_{n_4k}, x_{n_4k}, y_{n_4k})$,etc with the property $D_{n_4k}(4k^2 - 17k + 4)$. Here

$$w_{n_3k} = 20n^2(k+1) + n(4k-36) - 3k + 13$$

$$x_{n_3k} = 45n^2(k+1) + n(24k-66) - 4k + 17$$

$$y_{n_3k} = 125n^2(k+1) + n(50k-200) - 15k + 60$$

$$z_{n_3k} = 320n^2(k+1) + n(144k-496) - 35k + 141$$

Table 2.4. Numerical computations of 3-tuples for limited number of n and k are documented.

n	k	D_{n_3k}	$(u_{n_3k}, v_{n_3k}, w_{n_3k})$	$(v_{n_3k}, w_{n_3k}, x_{n_3k})$	$(w_{n_3k}, x_{n_3k}, y_{n_3k})$	$(x_{n_3k}, y_{n_3k}, z_{n_3k})$
2	1	−9	(13,45,106)	(45,106,289)	(106,289,745)	(289,745,1962)
	2	−14	(25,78,191)	(78,191,513)	(191,513,1330)	(513,1330,3495)
	3	−11	(37,111,276)	(111,276,737)	(276,737,1915)	(737,1915,5028)
3	1	−9	(45,97,274)	(97,274,697)	(274,697,1845)	(697,1845,4810)
	2	−14	(78,161,463)	(161,463,1170)	(463,1170,3105)	(1170,3105,8087)
	3	−11	(111,225,652)	(225,652,1643)	(652.1643,4365)	(1643,4365,11364)
4	1	−9	(97,169,522)	(169,522,1285)	(522,1285,3445)	(1285,3445,8938)
	2	−14	(161,274,855)	(274,855,2097)	(855,2097,5630)	(2097,5630,14599)
	3	−11	(225,379,1188)	(379,1188,2909)	(1188,2909,7815)	(2909,7815,20260)

2.5. Determination of 3-tuples in k-tetradecagonal numbers

The formula for tetradecagonal number is specified by

$$t_s = 6s^2 - 5s, s \in N$$

The sequence of k-tetradecagonal number obtained from tetradecagonal sequence is

$$14k + 1, 39k + 14, 76k + 3, \ldots$$

The recurrence relation for k-tetradecagonal is provided by

$$T_{s-2} = t_{s-2} + kt_{s-1}, s \geq 3 \text{where } t_1 = 1, t_2 = 14.$$

Let us take $u_{n_5k} = T_{n-1} = 6n^2(k+1) - n(5k+17) + 11$

$v_{n_5k} = T_n = 6n^2(k+1) + n(7k-5) + k$, $n \geq 2$ with the succeeding equation

$$u_{n_5k}.v_{n_5k} + k^2(-6n^2 - n + 9) + k(12n - 29) + 6n^2 - 11n + 9 = (6n^2(k+1) + n(k-11) - 3k + 3)^2$$

Accordingly,$(u_{n_5k}, v_{n_5k})$is categorized as Diophantine 2-tuples with the right property $D_{n_5k}(k^2(-6n^2 - n + 9) + k(12n - 29) + 6n^2 - 11n + 9)$. As the explanation given in section 2.1, it is workable to get the arrangement of Diophantine triples $(u_{n_5k}, v_{n_5k}, w_{n_5k}), (v_{n_5k}, w_{n_5k}, x_{n_5k}), (w_{n_5k}, x_{n_5k}, y_{n_5k}), (x_{n_5k}, y_{n_5k}, z_{n_5k})$ etc, with the property $D_{n_5k}(k^2(-6n^2 - n + 9) + k(12n - 29) + 6n^2 - 11n + 9)$. where

$$w_{n_3k} = 24n^2(k+1) + n(4k - 44) - 5k + 17$$

$$x_{n_3k} = 54n^2(k+1) + n(27k - 81) - 8k + 23$$

$$y_{n_3k} = 150n^2(k+1) + n(55k - 245) - 27k + 80$$

$$z_{n_3k} = 384n^2(k+1) + n(160k - 608) - 65k + 189$$

Table 2.5. Numerical calculations of triples for restricted values of n and k are recognized in the table 2.5 given below.

n	k	D_{n_3k}	$(u_{n_3k}, v_{n_3k}, w_{n_3k})$	$(v_{n_3k}, w_{n_3k}, x_{n_3k})$	$(w_{n_3k}, x_{n_3k}, y_{n_3k})$	$(x_{n_3k}, y_{n_3k}, z_{n_3k})$
2	1	−11	(15,53,124)	(53,124,339)	(124,339,873)	(339,873,2300)
	2	−67	(29,92,223)	(92,223,601)	(223,601,1556)	(601,1556,4091)
	3	−157	(43,131,322)	(131,322,863)	(322,863,2239)	(863,2239,5882)
3	1	−11	(53,115,324)	(115,324,825)	(324,825,2183)	(825,2183,5692)
	2	−148	(92,191,547)	(191,547,1384)	(547,1384,3671)	(1384,3671,9563)
	3	−381	(131,267,770)	(267,770,1943)	(770,1943,5159)	(1943,5159,13434)
4	1	−11	(115,201,620)	(201,620,1527)	(620,1527,4093)	(1527,4093,10620)
	2	−265	(191.326,1015)	(326,1015,2491)	(1015,2491,6686)	(2491,6686,17339)
	3	−701	(267,451,1410)	(451,1410,3455)	(1410,3455,9279)	(3455,9279,24058)

All the above sequences of 3-tuples can be substantiated for other options of k by the succeeding MATLAB program.

```
clc; clear all; close all;
n = input('enter the value of n:');
k = input('enter the value of k:');
disp('hexagonal number');
un1k = (2 * (n^2) * (k + 1)) - (5 * n * k) - (n * 9) + (3 * k) + 10;
vn1k = (2 * (n^2) * (k + 1)) + (n * 3 * k) - n + k;
wn1k = (8 * (n²) * (k + 1)) - (4 * n * k) - (20 * n) + 12;
xn1k = (18 * (n^2) * (k + 1)) + (3 * n * k) - (33 * n) - k + 14;
yn1k = (50 * (n^2) * (k + 1)) - (5 * n * k) - (105 * n) - (3 * k) + 52;
zn1k = (128 * (n^2) * (k + 1)) - (256 * n) - (8 * k) + 120;
Dn1k = (1 * k^2) - (14 * k) + 1; fprintf('un1k = %d\n', un1k);
fprintf('vn1k = %d\n', vn1k); fprintf('wn1k = %d\n', wn1k);
fprintf('xn1k = %d\n', yn1k); fprintf('yn1k = %d\n', yn1k);
fprintf('zn1k = %d\n', zn1k); fprintf('Dn1k = %d\n', Dn1k);
disp('octagonal number');
un2k = (3 * (n^2) * (k + 1)) - (2 * n * k) - (8 * n) + 5;
vn2k = (3 * (n^2) * (k + 1)) + (4 * n * k) - (2 * n) + k;
```

$wn2k\ = (12 * (n^2) * (k+1)) + (4 * n * k) - (20 * n) - k + 7;$

$xn2k\ = (27 * (n^2) * (k+1)) + (18 * n * k) - (36 * n) + 9;$

$yn2k\ = (75 * (n^2) * (k+1)) + (40 * n * k) - (110 * n) - (3 * k) + 32;$

$zn2k\ = (192 * (n^2) * (k+1)) + (112 * n * k) - (272 * n) - (5 * k) + 75;$

$Dn2k\ = (1 * k^2) - (7 * k) + 1; fprintf('un2k\ =\ \%d\backslash n', un2k);$

$fprintf('vn2k\ =\ \%d\backslash n', vn2k); fprintf('wn2k\ =\ \%d\backslash n', wn2k);$

$fprintf('xn2k\ =\ \%d\backslash n', xn2k); fprintf('yn2k\ =\ \%d\backslash n', yn2k);$

$fprintf('zn2k\ =\ \%d\backslash n', zn2k); fprintf('Dn2k\ =\ \%d\backslash n', Dn2k);$

$disp('Decagonal\ number');$

$un3k\ = (4 * (n^2) * (k+1)) - (3 * n * k) - (11 * n) + 7;$

$vn3k\ = (4 * (n^2) * (k+1)) + (5 * n * k) - (3 * n) + k;$

$wn3k\ = (16 * (n^2) * (k+1)) + (4 * n * k) - (28 * n) - (3 * k) + 11;$

$xn3k\ = (36 * (n^2) * (k+1)) + (21 * n * k) - (51 * n) - (4 * k) + 15;$

$yn3k\ = (100 * (n^2) * (k+1)) + (45 * n * k) - (155 * n) - (15 * k) + 52;$

$zn3k\ = (256 * (n^2) * (k+1)) + (128 * n * k) - (384 * n) - (35 * k) + 123;$

$Dn3k\ = (-1 * k^2) * ((4 * (n^2)) + n - 4) + (-1 * k) * ((-8 * n) + 15) + ((-4 * (n^2)) - (7 * n) + 4);$

$fprintf('un3k\ =\ \%d\backslash n', un3k); fprintf('vn3k\ =\ \%d\backslash n', vn3k);$

$fprintf('wn3k\ =\ \%d\backslash n', wn3k); fprintf('xn3k\ =\ \%d\backslash n', xn3k);$

$fprintf('yn3k\ =\ \%d\backslash n', yn3k); fprintf('zn3k\ =\ \%d\backslash n', zn3k);$

$fprintf('Dn3k\ =\ \%d\backslash n', Dn3k);$

$disp('Dodacagonal\ number');$

$un4k\ = (5 * (n^2) * (k+1)) - (4 * n * k) - (14 * n) + 9;$

$vn4k\ = (5 * (n^2) * (k+1)) + (6 * n * k) - (4 * n) + k;$

$wn4k\ = (20 * (n^2) * (k+1)) + (4 * n * k) - (36 * n) - (3 * k) + 13;$

$xn4k\ = (45 * (n^2) * (k+1)) + (24 * n * k) - (60 * n) - (4 * k) + 17;$

$yn4k\ = (125 * (n^2) * (k+1)) + (50 * n * k) - (200 * n) - (15 * k) + 60;$

$zn4k\ = (320 * (n^2) * (k+1)) + (144 * n * k) - (496 * n) - (35 * k) + 141;$

$Dn4k\ = (4 * k^2) - (17 * k) + 4; fprintf('un4k\ =\ \%d\backslash n', un4k);$

$fprintf('vn4k\ =\ \%d\backslash n', vn4k); fprintf('wn4k\ =\ \%d\backslash n', wn4k);$

$fprintf('xn4k\ =\ \%d\backslash n', xn4k); fprintf('yn4k\ =\ \%d\backslash n', yn4k);$

$fprintf('zn4k\ =\ \%d\backslash n', zn4k); fprintf('Dn4k\ =\ \%d\backslash n', Dn4k);$

$disp('Tetragonal\ number');$

$un5k\ = (6 * (n^\wedge 2) * (k + 1)) - (5 * n * k) - (17 * n) + 11;$

$vn5k\ = (6 * (n^\wedge 2) * (k + 1)) + (7 * n * k) - (5 * n) + k;$

$wn5k\ = (24 * (n^\wedge 2) * (k + 1)) + (4 * n * k) - (44 * n) - (5 * k) + 17;$

$xn5k\ = (54 * (n^\wedge 2) * (k + 1)) + (27 * n * k) - (81 * n) - (8 * k) + 23;$

$yn5k\ = (150 * (n^\wedge 2) * (k + 1)) + (55 * n * k) - (245 * n) - (27 * k) + 80;$

$zn5k\ = (384 * (n^\wedge 2) * (k + 1)) + (160 * n * k) - (608 * n) - (65 * k) + 189;$

$Dn5k\ = (-1 * k^\wedge 2) * ((6 * (n^\wedge 2)) + n - 9) + (1 * k) * ((12 * n) - 29) + ((6 * (n^\wedge 2)) - (11 * n) + 9);$

$fprintf('un5k\ =\ \%d\backslash n', un5k); fprintf('vn5k\ =\ \%d\backslash n', vn5k);$

$fprintf('wn5k\ =\ \%d\backslash n', wn5k); fprintf('xn5k\ =\ \%d\backslash n', xn5k);$

$fprintf('yn5k\ =\ \%d\backslash n', yn5k); fprintf('zn5k\ =\ \%d\backslash n', zn5k);$

$fprintf('Dn5k\ =\ \%d\backslash n', Dn5k);$

3. Conclusion

In this paper, the patterns of 3-tuples $(u_{n_m k}, v_{n_m k}, w_{n_m k}), (v_{n_m k}, w_{n_m k}, x_{n_m k}), (w_{n_m k}, x_{n_m k}, y_{n_m k}), (x_{n_m k}, y_{n_m k}, z_{n_m k})$ etc, $m = 1,2,3,4,5$ comprising some $k -$ polygonal numbers in which the product of two numbers added by precise numbers become a perfect square are reviewed. All the patterns of 3-tuples satisfying various properties are also verified by the code of MATLAB program. In this way, one can search other triples, quadruples etc with some other properties concerning various numbers.

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