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Original Article

ve-Degree Sombor Indices of Certain Networks

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Abstract *- In this paper, we propose the modified ve-degree Sombor index, the modified ve-degree Sombor exponential and the ve-degree Sombor exponential of a graph. We determine the modified ve-degree Sombor index and its exponential for certain networks. Also we compute the ve-degree Sombor exponential for certain networks.*

Keywords *- Modified ve-degree Sombor index, Modified ve-degree Sombor exponential, Network.*

1. Introduction

Let G be a finite, simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The set of all vertices which adjacent to v is called the open neighborhood of v and denoted by $N(v)$. The closed neighborhood set of v is the set $N[v] = N(v) \cup \{v\}$. Let S_v denote the sum of the degrees of all vertices adjacent to a vertex v. For all further notation and terminology we refer the reader to [1].

In [2], Chellali et al. defined the *ve*-degree concept in graph theory as follows:

The *ve*-degree $d_{ve}(v)$ of a vertex v in a graph G is the number of different edges that incident to any vertex from the closed neighborhood of v.

A molecular graph or chemical graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edge to the bonds between atoms. Chemical Graph Theory is a branch of Graph Theory whose focus of interest is to finding topological indices of chemical graphs, which correlate well with chemical properties of the chemical molecules. Numerous topological indices have been considered in Theoretical chemistry, especially in QSAR and QSPR research, see [3].

The first *ve*-degree Zagreb beta index of a graph G is defined as

$$Ve_1(G) = \sum_{uv \in E(G)} \left[d_{ve}(u) + d_{ve}(v) \right].$$

The second *ve*-degree Zagreb index of a graph G is defined as

$$Ve_2(G) = \sum_{uv \in E(G)} d_{ve}(u) d_{ve}(v).$$

The above two *ve*-degree Zagreb indices were proposed by Ediz in [4]. Recently, some *ve*-degree topological indices were studied, for example, in [5, 6, 7, 8, 9, 10].

In [11], Ediz et al. introduced the *ve*-degree Sombor index of a graph and defined it as

$$SO_{ve}(G) = \sum_{uv \in E(G)} \sqrt{d_{ve}(u)^2 + d_{ve}(v)^2}.$$

We propose the *ve*-degree Sombor exponential of a graph G and it is defined as

$$SO_{ve}(G, x) = \sum_{uv \in E(G)} x^{\sqrt{d_{ve}(u)^2 + d_{ve}(v)^2}}.$$

Recently, some Sombor indices were studied in [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

We introduce the modified *ve*-degree Sombor index of a graph G and it is defined as

$$^{m}SO_{ve}(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_{ve}(u)^2+d_{ve}(v)^2}}.$$

We define the modified *ve*-degree Sombor exponential of a graph G as

$$^{m}SO_{ve}(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{d_{ve}(u)^2+d_{ve}(v)^2}}}.$$

In this paper, we compute the modified *ve*-degree Sombor index, the modified *ve*-degree Sombor exponential and the *ve*-degree Sombor exponential for certain networks.

2. Results for Dominating Oxide Networks

The family of dominating oxide networks is symbolized by $DOX(n)$. The molecular structure of a dominating oxide network is shown in Figure 1. Let G be the graph of a dominating oxide network.

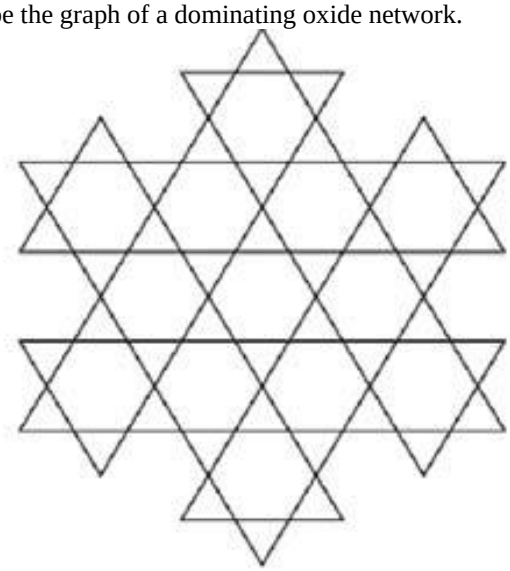

Fig. 1 The structure of a dominating oxide network

In [11], Ediz et al. obtained the *ve*-degree partition of the end vertices of edges for dominating oxide network in Table 1.

Table 1. The *ve*-degree of the end vertices of edges for *DOX* networks

$d_{ve}(u)$, $d_{ve}(v)\backslash uv\square E(G)$	Number of edges
(7, 10)	$12n$
(7, 12)	$12n-12$
(10, 10)	6
(10, 12)	$12n-12$
(12, 14)	$24n-24$
(14, 14)	$54n^2-114n+60$

Theorem 1. The modified *ve*-degree Sombor index of a dominating oxide network is

$$^{m}SO_{ve}(G)=\frac{27}{7\sqrt{2}}n^2+\left(\frac{12}{\sqrt{149}}+\frac{12}{\sqrt{193}}+\frac{6}{\sqrt{61}}+\frac{12}{\sqrt{85}}-\frac{57}{7\sqrt{2}}\right)n$$
$$-\left(\frac{12}{\sqrt{193}}-\frac{6}{10\sqrt{2}}+\frac{6}{\sqrt{61}}+\frac{12}{\sqrt{85}}-\frac{30}{7\sqrt{2}}\right).$$

Proof: From the definition and Table 1, we have

$$
\begin{aligned}
{}^{m}SO_{ve}(G) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{d_{ve}(u)^2 + d_{ve}(v)^2}} \\
&= \frac{12n}{\sqrt{7^2+10^2}} + \frac{12n-12}{\sqrt{7^2+12^2}} + \frac{6}{\sqrt{10^2+10^2}} + \frac{12n-12}{\sqrt{10^2+12^2}} \\
&\quad + \frac{24n-24}{\sqrt{12^2+14^2}} + \frac{54n^2-114n+60}{\sqrt{14^2+14^2}}
\end{aligned}
$$

gives the desired result after simplification.

Theorem 2. The *ve*-degree Sombor exponential of a dominating oxide network is

$$
\begin{aligned}
SO_{ve}(G,x) &= 12nx^{\sqrt{149}} + (12n-12)x^{\sqrt{193}} + 6x^{10\sqrt{2}} + (12n-12)x^{2\sqrt{61}} \\
&\quad + (24n-24)x^{2\sqrt{85}} + \left(54n^2-114n+60\right)x^{14\sqrt{2}}.
\end{aligned}
$$

Proof: Using definition and Table 1, we obtain

$$
\begin{aligned}
SO_{ve}(G,x) &= \sum_{uv\in E(G)} x^{\sqrt{d_{ve}(u)^2+d_{ve}(v)^2}} \\
&= 12nx^{\sqrt{7^2+10^2}} + (12n-12)x^{\sqrt{7^2+12^2}} + 6x^{\sqrt{10^2+10^2}} + (12n-12)x^{\sqrt{10^2+12^2}} \\
&\quad + (24n-24)x^{\sqrt{12^2+14^2}} + \left(54n^2-114n+60\right)x^{\sqrt{14^2+14^2}}.
\end{aligned}
$$

After simplification, we obtain the desired result.

Theorem 3. The modified *ve*-degree Sombor exponential of a dominating oxide network is

$$
\begin{aligned}
{}^{m}SO_{ve}(G,x) &= 12nx^{\frac{1}{\sqrt{149}}} + (12n-12)x^{\frac{1}{\sqrt{193}}} + 6x^{\frac{1}{10\sqrt{2}}} + (12n-12)x^{\frac{1}{2\sqrt{61}}} \\
&\quad + (24n-24)x^{\frac{1}{2\sqrt{85}}} + \left(54n^2-114n+60\right)x^{\frac{1}{14\sqrt{2}}}.
\end{aligned}
$$

Proof: From the definition and by using Table 1, we get

$$
\begin{aligned}
{}^{m}SO_{ve}(G,x) &= \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{d_{ve}(u)^2+d_{ve}(v)^2}}} \\
&= 12nx^{\frac{1}{\sqrt{7^2+10^2}}} + (12n-12)x^{\frac{1}{\sqrt{3^2+6^2}}} + 6x^{\frac{1}{\sqrt{10^2+10^2}}} + (12n-12)x^{\frac{1}{\sqrt{10^2+12^2}}} \\
&\quad + (24n-24)x^{\frac{1}{\sqrt{12^2+14^2}}} + \left(54n^2-114n+60\right)x^{\frac{1}{\sqrt{14^2+14^2}}}.
\end{aligned}
$$

After simplification, we get the desired result.

3. Results for Regular Triangulate Oxide Networks *RTOX*(*n*)

The family of regular triangulate oxide networks is denoted by *RTOX*(*n*), $n \geq 3$. The molecular structure of a regular triangulate oxide network is shown in Figure 2. Let *G* be the graph of a regular triangulate oxide network.

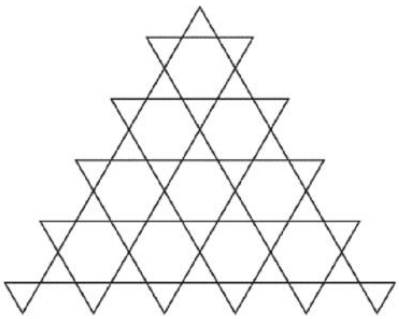

Fig. 2 The structure of a regular triangulate oxide network

In [11], Ediz et al. obtained the ve-degree partition of the end vertices of edges for regular triangulate oxide network in Table 2.

Table 2. The ve-degree of the end vertices of edges for *RTOX* networks

$d_{ve}(u)$, $d_{ve}(v)\backslash uv \in E(G)$	Number of edges
(5, 5)	2
(5, 10)	4
(7, 10)	4
(7, 12)	$6n$–8
(10, 10)	1
(10, 12)	6
(12, 12) (12, 14) (14, 14)	$6n$–9 $6n$–12 $3n^2$–$12n$+12

Theorem 4. The modified *ve*-degree Sombor index of a regular triangulate oxide network is

$$^{m}SO_{ve}(G)=\frac{3}{14\sqrt{2}}n^2+\left(\frac{6}{\sqrt{193}}+\frac{3}{\sqrt{85}}-\frac{5}{14\sqrt{2}}\right)n$$
$$+\left(\frac{17}{28\sqrt{2}}+\frac{4}{5\sqrt{5}}+\frac{4}{\sqrt{149}}-\frac{8}{\sqrt{193}}+\frac{3}{\sqrt{61}}-\frac{6}{\sqrt{85}}\right).$$

Proof: From the definition and Table 2, we have

$$^{m}SO_{ve}(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_{ve}(u)^2+d_{ve}(v)^2}}$$
$$=\frac{2}{\sqrt{5^2+5^2}}+\frac{4}{\sqrt{5^2+10^2}}+\frac{4}{\sqrt{7^2+10^2}}+\frac{6n-8}{\sqrt{7^2+12^2}}+\frac{1}{\sqrt{10^2+10^2}}$$
$$+\frac{6}{\sqrt{10^2+12^2}}+\frac{6n-9}{\sqrt{12^2+12^2}}+\frac{6n-12}{\sqrt{12^2+14^2}}+\frac{3n^2-12n+12}{\sqrt{14^2+14^2}}$$

gives the desired result after simplification.

Theorem 5. The *ve*-degree Sombor exponential of a dominating oxide network is

$$SO_{ve}(G,x)=2x^{5\sqrt{2}}+4x^{5\sqrt{5}}+4x^{\sqrt{149}}+(6n-8)x^{\sqrt{193}}+1x^{10\sqrt{2}}+6x^{2\sqrt{61}}$$
$$+(6n-9)x^{12\sqrt{2}}+(6n-12)x^{2\sqrt{85}}+\left(3n^2-12n+12\right)x^{14\sqrt{2}}.$$

Proof: Using definition and Table 2, we obtain

$$SO_{ve}(G,x)=\sum_{uv\in E(G)}x^{\sqrt{d_{ve}(u)^2+d_{ve}(v)^2}}$$
$$=2x^{\sqrt{5^2+5^2}}+4x^{\sqrt{5^2+10^2}}+4x^{\sqrt{7^2+10^2}}+(6n-8)x^{\sqrt{7^2+12^2}}+1x^{\sqrt{10^2+10^2}}$$
$$+6x^{\sqrt{10^2+12^2}}+(6n-9)x^{\sqrt{12^2+12^2}}+(6n-12)x^{\sqrt{12^2+14^2}}+\left(3n^2-12n+12\right)x^{\sqrt{14^2+14^2}}.$$

After simplification, we obtain the desired result.

Theorem 6. The modified *ve*-degree Sombor exponential of a dominating oxide network is

$${}^{m}SO_{ve}(G,x)=2x^{\frac{1}{5\sqrt{2}}}+4x^{\frac{1}{5\sqrt{5}}}+4x^{\frac{1}{\sqrt{149}}}+(6n-8)x^{\frac{1}{\sqrt{193}}}+1x^{\frac{1}{10\sqrt{2}}}+6x^{\frac{1}{2\sqrt{61}}}$$
$$+(6n-9)x^{\frac{1}{12\sqrt{2}}}+(6n-12)x^{\frac{1}{2\sqrt{85}}}+\left(3n^2-12n+12\right)x^{\frac{1}{14\sqrt{2}}}.$$

Proof: From the definition and by using Table 1, we get

$${}^{m}SO_{ve}(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{d_{ve}(u)^2+d_{ve}(v)^2}}}$$
$$=2x^{\frac{1}{\sqrt{5^2+5^2}}}+4x^{\frac{1}{\sqrt{5^2+10^2}}}+4x^{\frac{1}{\sqrt{7^2+10^2}}}+(6n-8)x^{\frac{1}{\sqrt{7^2+12^2}}}+1x^{\frac{1}{\sqrt{10^2+10^2}}}$$
$$+6x^{\frac{1}{\sqrt{10^2+12^2}}}+(6n-9)x^{\frac{1}{\sqrt{12^2+12^2}}}+(6n-12)x^{\frac{1}{\sqrt{12^2+14^2}}}+\left(3n^2-12n+12\right)x^{\frac{1}{\sqrt{14^2+14^2}}}.$$

After simplification, we get the desired result.

4. Conclusion

In this paper, we have computed the modified *ve*-degree Sombor index, the modified *ve*-degree Sombor exponential and the *ve*-degree Sombor exponential for dominating oxide networks and regular triangulate oxide networks.

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