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Original Article

Instructions the Modeling of Linear Ordinary Differential Equations using Matlab

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Abstract *- This study aim to introduce and expose the ties of matlab to the mathematician throughout the solutions of linear differential equations , with some of its physical applications. The study go throughout the linear differential equations and its types, also it explains what is modeling of linear ordinary differential equations also the study explained the solution of linear ordinary differential equations with matlab.*

Keywords *- The modeling, Linear ordinary differential equations, The matlab program, Physical application.*

1. Introduction

The linear ordinary differential equations have a vital role in solving problems of many scientific fields such as mathematics, physics, engineering [1,4, 8]. Matlab is programming platform designed specially for engineering, scientist and mathematician so it's a comprehensive software system aim to save time , power and energy, the program was developed by math work inc [2,3,6]. The aims of this paper is to develop and introduces matlab in the solution of the linear ordinary differential equations. The study aims to fined solutions for linear ordinary differential equations and some physical applications , throughout the mathematical application using the matlab program by explain the matlab and now it can be used [7,9,14] . The study reached an important result , which was the accuracy and sufficient of the results found out by matlab in some time how much it saves regarding manual efforts .Also the program has high accurate diagram and graphs for the functions curves. Computer software is one of the most widespread developments in mathematical solutions, which enables us to provide a more accurate and graphically representative solution very quickly[16,18,19]. Traditional solutions lead to inaccurate results, accompanied by many errors, and may be difficult to display and represent graphically . This study depends on collecting, describing, and analyzing information using several examples and types of the linear ordinary differential equations . Several stages of the proposed models are designed and implemented on the solutions programs used in MATLAB programming. The quality algorithm was used and developed to solve the differential equations of the covid-19 infection in River Nile State for 10 month . The proposed model contributed to providing an integrated computer solution for all stages of the solution starting from the stage of solving differential equations in the covid-19 infection and the stage of displaying and representing the results graphically in the Matlab program. This was done by solving two different types in type and rank to reach accurate readings that provide an ideal model applicable in many of the techniques that contribute to solving our daily problems.

2. The Linear Ordinary Differential Equations [5,10,11,12,21]:

The standard from of the linear ordinary differential equations is comes by re writing the linear ordinary differential equations .

$$u^n = q(x - p_{n-1}(x)u - p_{n-z} - \cdots {}^{p}_{o}(x)u^{(n-1)}$$

In the following from

$$u^n + p_{o(x1u^{n-1}} + \ldots + p_{n-2}(x)u' + p_{n-1}(x)u = g(x)$$

Is called homogenous not linear.

$$u' + \frac{1}{x}\ u' + (1 - \frac{n^2}{x^2})\ u = o$$

The equation is homogenous second order linear ordinary differential equations but equation $u'' + 4uu' = 0$ is a non-liner but also second order and non-homogenous.

3. The Modeling :[13,17]

Modeling is a description of physical or geometrical or any other problems which in linear ordinary differential equations to obtain the solutions of this problems throughout the mathematical solution of the equation In order to model a situation, we had to know about what involved in modeling process , the study look the at different situations as an example for modeling that was the mixing problem , population problem, and falling bodies problem .The three problems depict a chemical , social , and physics problem corresponding

4. Modeling the Linear Ordinary Differential Equations :[15,20,23]

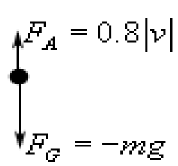
This part from this paper is to show how to model a physical problem using strategy that leads to find a linear ordinary differential equations that can give a solution to the problem. The strategy was as follows :

a) Visualize the problem and make sketch.
b) write down what you are asked to find.
c) identify the other variables in the problem .
d) write equations that relate the variables
e) use substitution and differentiation to get a single equation.

Example (4.1)

Set up the initial value problem that will give the velocity of a 60 kg sky diver that jumps out of a plane with no initial velocity and an air resistance of 0.8 v For this application assume that the positive direction is upward.

Solution:



Here are the forces that are acting on the sky diver

Because of the conventions the force due to gravity is negative and the force due to air resistance is positive. As set up, these forces have the correct sign and so the initial value problem is

$$mv' = -mg + 0.8|v| \quad v(0) = 0$$

The problem arises when you go to remove the absolute value bars.

In order to do the problem they do need to be removed, this is where most of the students made their mistake, because they had forgotten about the convention and the direction of motion they just dropped the absolute value bars to get.

$$mv' = -mg + 0.8v$$

(incorrect initial value problem!!).

So, why is this incorrect? Well remember that the convention is that positive is up ward. however in this case the object is moving downward and so v is negative! Upon dropping the absolute value bars the air resistance became a negative force and hence was acting in the downward direction!

To get the correct initial value problem recall that because v is negative then

$$|v| = -v.$$

using this, the air resistance becomes

$$FA = -0.8v$$

and despite appearances this is a positive forces since the"-" cancels out against the velocity (which is negative) to get a positive force.

The correct initial value problem is then

$$mv' = -mg + 0.8v \quad v(0) = 0$$

Plugging in the mass gives

$$v' = -9.8 - \frac{v}{75} \qquad v(0) = 0$$

For the sake of completeness the velocity of the sky diver, at least until the parachute opens, which I didn't include in this problem is.

$$v(t) = -735 + 735e^{-\frac{t}{75}}$$

This mistake was made in part because the students were in a hurry and weren't paying attention, but also because they simply forgot about their convention and the direction of motion! don't fall into this mistake. always pay attention to your conventions and what is happening in the problems. Just to show you the difference here is the problem worked by assuming that down is positive.

5. The Matlab: [16,24,25]

Matlab is an integrated technical computing environment that combines numeric computation, advanced graphics and visualization, and a high level programming language. that statement encapsulates the view of the math works, Inc., the developer of matlab , you can use matlab, in conjunction with the word processing and desktop publishing features of Microsoft Word_, to combine mathematical computations with text and graphics to produce a polished, integrated, and interactive document. We will introduce the tools we need to begin using matlab effectively. these include some relevant information on computer platforms and software versions installation and location protocols; how to launch the program enter commands use online help and recover from hang-ups a roster of matlab's various windows and finally, how to quit the software.

5.1. Starting Matlab

you start matlab as you would do with any other software application. on a PC you access it via the Start menu, in Programs under a folder such as matlab R12 alternatively, you may have an icon set up that enables the matlab logo as well as some matlab product information and then a matlab desktop you to start matlab with a simple double-click window will launch. that window will contain a title bar a menu bar a tool bar and five embedded windows two of which are hidden. the largest and most important window is the command window on the right. there are also four windows the launch pad the workspace browser, the command history window and the current directory browser. the prompt will likely be a double caret (>> or _). if the command window is "active", its title bar will be dark, and the prompt will be followed by a cursor (a vertical line or box, usually blinking). That is the place where you will enter your matlab commands. if the command window is not active just click in it anywhere.

5.2. Typing in the Command Window

click in the command window to make it active, when a window becomes active its title bar darkens, outline form to solid, or from light to dark, or it may simply appear. now you can begin entering commands.

5.3. Matlab Windows

we have already described the matlab command window and the help browser and have mentioned in passing the command history window, current directory browser, workspace browser and launch Pad. these several other windows you will encounter as you work with matlab, will allow you to control files and folders that you and matlab will need to access write and edit the small matlab programs (that is, M-files) that you will utilize to run matlab most effectively keep track of the variables and functions that you define as you use matlab and design graphical models to solve problems and simulate processes. some of these windows launch separately, and some are embedded in the desktop. You can dock some of those that launch separately inside the desktop (through the View: Dock menu button) or you can separate windows inside your matlab desktop out to your computer desktop by clicking on the curved arrow in the upper right . Many of the commands you issue will generate graphics or pictures. These will appear in a separate window. Matlab documentation refers to these as figure. windows. Also call them graphics windows. graphics windows cannot be embedded into the Matlab desktop.

5.4. Matlab Basics

In this section , you will start learning how to use matlab to do mathematics. you should read this section at your computer, with matlab running. try the commands in a matlab command window as you go along. feel free to experiment with variants of the examples we present; the best way to find out how matlab responds to a command is to try it..

5.5. Arithmetic

As we have just seen, you can use matlab to do arithmetic as you would a calculator. you can use (+) to add (-) to subtract (*) to multiply (/) to divide and (ˆ) to exponentiate .

6. Physical Applications : [13,22,23]

In this part we are going to see that how linear ordinary differential equations are used to model many situations in physics and engineering . By solving these equations we can model many situations in a wide range of real life application and how we express its different behaviors like whether it is damped oscillations or over damped or critically damped. We look at how this works for systems of an object with mass attached to a vertical spring and an electric circuit containing a resistor, an inductor, and a capacitor connected in series, equations representing simple harmonic motion charge and current in an RLC series circuit. These models leads to more complicated situations like atomic and molecular bonding which are often modeled by approximation as springs that vibrate, as described by these same differential equations .

6.1. Simple Electric Circuits

(i) R, L *series circuit*.

(ii) Consider a circuit containing resistance R and inductance L in series with a voltage source (battery) E.

Let i be the current flowing in the circuit at any time *t*. Then by Kirchhoff's first law, we have

Sum of voltage drops across *R* and $L = E$

Equation $Ri + L\frac{di}{dt} = E$

Or $$\frac{di}{dt} + \frac{R}{L}\, i = \frac{E}{L} \qquad (1)$$

This is a Leibnitz's linear equation.

$$I.F = e^{\int \frac{R}{L} dt} = e^{Rt/L}$$

And, therefore, its solution is

$$i\,(I.F.) = \int \frac{E}{L}(I.F.)\, dt + c \qquad \text{Or} \qquad i.e^{Rt/L} = \int \frac{E}{L} e^{Rt/L}\, dt + \; = \frac{E}{L}.\frac{L}{R}.e^{Rt/L} + c$$

Whence

$$i = \frac{E}{R} + ce^{-Rt/L} \qquad (2)$$

If initially there is no current in the circuit, equation $i = 0,$

When $t = 0,$ we have $c = -E/R.$
We get (2) becomes

$$i = \frac{E}{R}(1 - e^{-Rt/L})$$

Which shows that *i* increases with *t* and attains the maximum value $E/R.$

(iii) R, L, C *series circuit*. Now consider a circuit containing resistance R, inductance *L* and capacitance *C* all in series with a constant.

If *i* be the current in the circuit at time *t*, then the charge *q* on the Condenser $= \int i\, dt, equation. i = \frac{dq}{dt}.$

Applying Kirchhoff's law, we have, sum of the voltage drops across R, L and $C = E. equation$ $Ri + L\frac{di}{dt} + \frac{q}{c} = E$ Or

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E$$

This is the desired differential equation of the circuit and will be solved .

Example: Simple Electric Circuits

Show that the differential equation for the current *t* in an electrical circuit containing an inductance *L* and resistance *R* in series and acted on by an electromotive force $E \sin \omega t$
Satisfies the equation

$$L\frac{di}{dt} + Ri = E \sin \omega t.$$

Find the value of the current at any time t, if initially there is no current in the circuit.
By Kirchhoff first law, we have sum of voltage drops across *R* and $sin\ \omega t$

$equation.$ $$Ri + L\frac{di}{dt} = E \sin \omega t.$$

This is the required differential equation which can be written as

$$\frac{di}{dt} + \frac{R}{L}\, i = \frac{E}{L} \sin \omega t$$

This is Leibnitz's equation.

Its $I.F. = e^{\int \frac{R}{L} dt} = e^{\frac{Rt}{L}}$

∴ The solution is

$i(I.F.) = \int \frac{E}{L} \sin \omega t. (I.F.)\, dt + c$ Or $ie^{Rt/L} = \frac{E}{L}\int e^{Rt/L} \sin \omega t\, dt + c$

$$= \frac{E}{L}\frac{e^{Rt/L}}{\sqrt{[(R/L)^2 + \omega^2]}} \sin\left(\omega t - tan^{-1}\frac{L\omega}{R}\right) + c$$

Or $$i = \frac{E}{\sqrt{(R^2 + \omega^2 L^2)}} \sin(\omega t - \emptyset) + ce^{-Rt/L} \quad (3)$$

Where $\tan \emptyset = L\omega/R$

Initially when $t = 0\,; i = 0$

$$\therefore \quad 0 = \frac{E\sin(-\emptyset)}{\sqrt{(R^2+\omega^2L^2)}} + c\ equation. \quad c = \frac{E\sin\emptyset}{\sqrt{(R^2+\omega^2L^2)}}$$

We get (3) takes the form

$$i = \frac{E\sin(\omega t-\emptyset)}{\sqrt{(R^2+\omega^2L^2)}} + \frac{E\sin\emptyset}{\sqrt{(R^2+\omega^2L^2)}}\,.e^{-Rt/L}$$

$$= \frac{E}{\sqrt{(R^2+\omega^2L^2)}}\,[\sin(\omega t-\emptyset) + \sin\emptyset . e^{-Rt/L}]$$

Which gives the current at any time *t*.

6.2. Moving Bodies

(1) Let a body of mass *m* start moving from *O* along the straight line *OX* under the action of a force *F*. After any time *t*, let

(i) moving at P ,where $OP = x$, then

(ii) Its velocity $(v) = \frac{dx}{dt}$

(iii) Its acceleration (a) $= \frac{dv}{dt}$ or $\frac{vdv}{ds}$ or $\frac{d^2s}{dt^2}$

If, however, the body be moving along a curve, then

(i) its velocity $(v) = ds/dt$ and

(ii) its acceleration (a) $= \frac{dv}{dt},\ v\frac{dv}{ds}$ or $\frac{d^2s}{dt^2}$

The quantity mv is called the $momentum$.

(2) Newton's second law states that

$$F = \frac{d}{dt}(mv)$$

If m is constant, then

$$F = m\frac{dv}{dt} = ma$$

Equation net force = mass x acceleration.

(3) Hooke's law states that tension of an elastic string (or a spring) is proportional to extension of the string (or the spring) beyond its natural length.

We get $T = \lambda e/l,$

Where e is the extension beyond the natural length l and λ is the modulus of elasticity. Sometimes for a spring, we write $T = ke$ where e is the extension beyond the natural length and k is the stiff $-$ ness of the spring.

7. Systems of units

(a) F.P.S. (foot ($ft.$), pound ($lb.$), second ($sec.$)]system. If mass m is in pounds and acceleration a sin ft/sec^2, then the force $F(=ma)$ is in $poundals$.

(b) C.G.S. [centimeter (cm.) , gram (g), second (sec)] system. If mass m is in grams and acceleration a in cm/sec^2, then the force $F(=ma)$ is in dynes.

(c) M.K.S. [meter (m), kilogram (kg.), second (sec.)] system. If mass m is in kilograms and acceleration a in m/sec^2, then the force $F(=ma)$ is in $newtons\ (nt)$.

These are called absolute units. If g is the acceleration due to gravity and w is the weight of the body, then w/g is the mass of the body in gravitational units.

$$g = 32\,ft/sec^2 = 980\,m/sec^2 \text{approx.}$$

8. Proposed Solutions by Matlab

We give some comprise solution presented a general application of ordinary differential equations for covid-19 infection in River Nile state and apply the main result of algorithm.

Example (7.1):

Solve the differential equation $\frac{dy}{dx} + 3x^2\text{y} = 6x^2$

Solution :

```
clear all
clc
syms x y dy dx f
f =6*(x^2)-3*(x^2)*y
t = (1:10)
x=1;
y0= [68 42 24 9 11 103 117 67 103 75]
[x,y] = ode45(@(x,y) 6*(x^2)-3*(x^2)*y, t, y0)
figure
% title ('Application of Ordinary Differential Equations for covid-19
infection in River Nile State')
subplot(3,1,1);
plot(x,y0)
xlabel('Time with month')
title ('A')
grid
subplot(3,1,2);
bar(y0)
xlabel('Time with month')
ylabel('The rate of infection with COVID-19')
title ('B')
grid
subplot(3,1,3);
bar(y)
title ('C')
xlabel('Repetition according to the number of months and the differential
equation')
grid
```

Result:

```
6*x^2 - 3*x^2*y

t =
     1     2     3     4     5     6     7     8     9    10
y0 =
    68    42    24     9    11   103   117    67   103    75

x =
     1
     2
     3
     4
     5
     6
     7
     8
     9
    10
```

```
y =
68.0000   42.0000   24.0000    9.0000   11.0000  103.0000  117.0000   67.0000
103.0000   75.0000
2.0606    2.0367    2.0202    2.0064    2.0083    2.0928    2.1057    2.0597
2.0928    2.0671
1.9998    1.9999    1.9999    2.0000    2.0000    1.9997    1.9997    1.9998
1.9997    1.9998
1.9996    1.9997    1.9999    2.0000    1.9999    1.9993    1.9992    1.9996
1.9993    1.9995
1.9999    2.0000    2.0000    2.0000    2.0000    1.9999    1.9999    1.9999
1.9999    1.9999
2.0000    2.0000    2.0000    2.0000    2.0000    1.9999    1.9999    2.0000
1.9999    2.0000
1.9994    1.9996    1.9998    1.9999    1.9999    1.9991    1.9990    1.9994
1.9991    1.9993
1.9988    1.9993    1.9996    1.9999    1.9998    1.9982    1.9979    1.9988
1.9982    1.9987
2.0001    2.0001    2.0000    2.0000    2.0000    2.0002    2.0002    2.0001
2.0002    2.0001
2.0003    2.0002    2.0001    2.0000    2.0000    2.0004    2.0005    2.0003
2.0004    2.0003
```

8.1. Represent the Solution Graphically

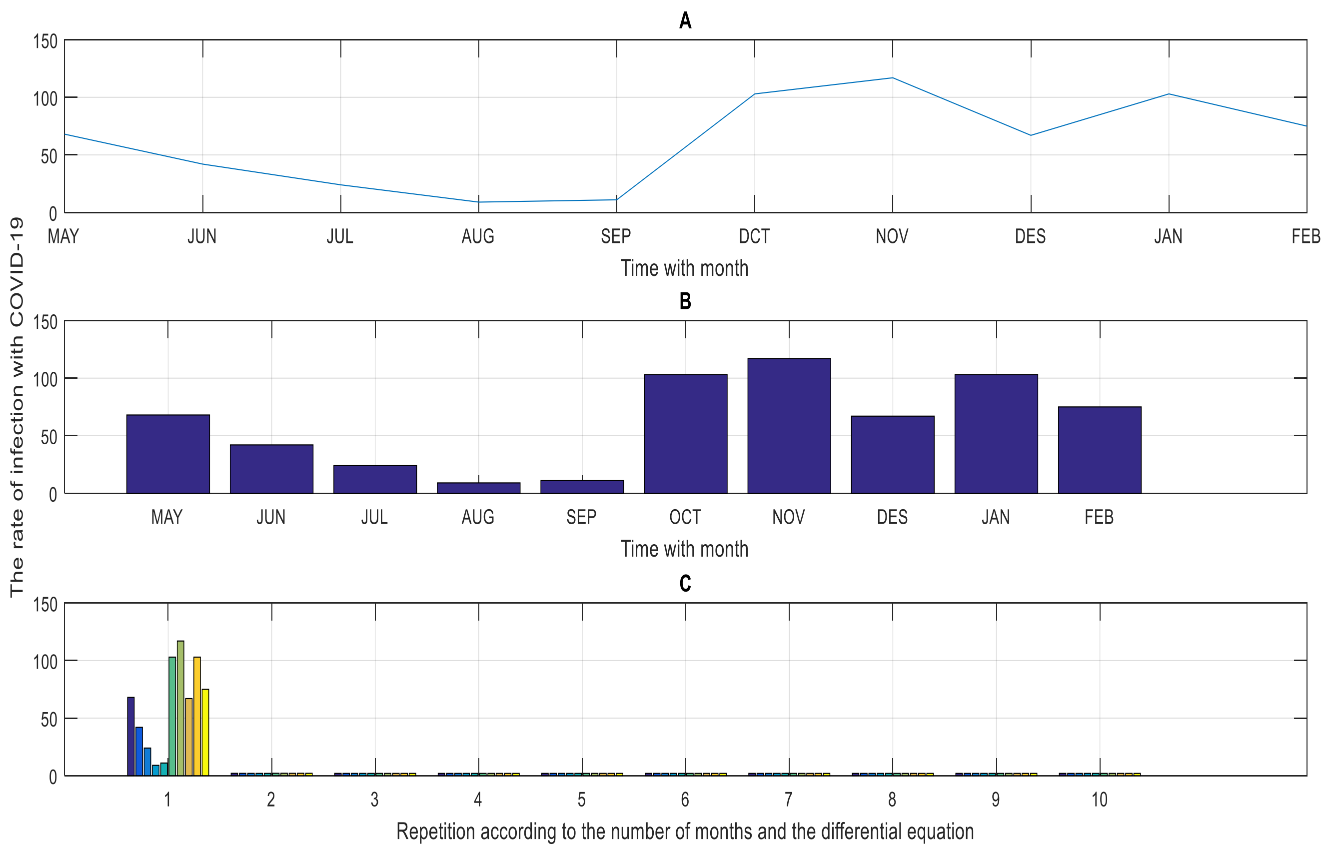


Fig. 1 Apply FODE of covid-19 infection in River Nile State for 10 month

Example (7.2): Solve the differential equation $\frac{dy}{dx} + x^2 = y$

Solution :

```
clear all
clc
syms x y dy dx
f =(-x)^2-y
x = (1:10)
y0 = [68 42 24 9 11 103 117 67 103 75]
[x,y] = ode45(@(x,y) (-x)^2-y, x, y0)
figure
subplot(3,1,1);
plot(x,y0)
xlabel('Time with month')
title ('Application of Ordinary Differential Equations for covid-19 infection in
River Nile State')
title ('A')
grid
subplot(3,1,2);
bar(y0)
xlabel('Time with month')
ylabel('The rate of infection with COVID-19')
title ('B')
grid
subplot(3,1,3);
bar(y)
title ('C')
xlabel('Repetition according to the number of months and the differential
equation')
grid
```

Result

```
f =
x^2 - y

x =
     1     2     3     4     5     6     7     8     9    10

y0 =
    68    42    24     9    11   103   117    67   103    75

x =
     1
     2
     3
     4
     5
     6
     7
     8
     9
    10

y =
   68.0000   42.0000   24.0000    9.0000   11.0000  103.0000  117.0000   67.0000
103.0000   75.0000
   26.6546   17.0872   10.4637    4.9440    5.6800   39.5338   44.6854   26.2866
```

```
39.5338   29.2304
   14.0689   10.5496    8.1131    6.0827    6.3534   18.8065   20.7016   13.9336
18.8065   15.0165
   13.3361   12.0413   11.1450   10.3980   10.4976   15.0790   15.7761   13.2863
15.0790   13.6846
   18.2278   17.7513   17.4214   17.1465   17.1831   18.8692   19.1258   18.2095
18.8692   18.3561
   26.4525   26.2770   26.1556   26.0543   26.0678   26.6887   26.7831   26.4457
26.6887   26.4997
   37.1672   37.1026   37.0579   37.0206   37.0256   37.2542   37.2890   37.1647
37.2542   37.1846
   50.0619   50.0381   50.0216   50.0079   50.0097   50.0939   50.1067   50.0609
50.0939   50.0683
   65.0227   65.0139   65.0079   65.0028   65.0035   65.0345   65.0392   65.0224
65.0345   65.0250
   82.0089   82.0057   82.0034   82.0016   82.0018   82.0132   82.0150   82.0088
82.0132   82.0098
```

8.2. Represent the solution graphically

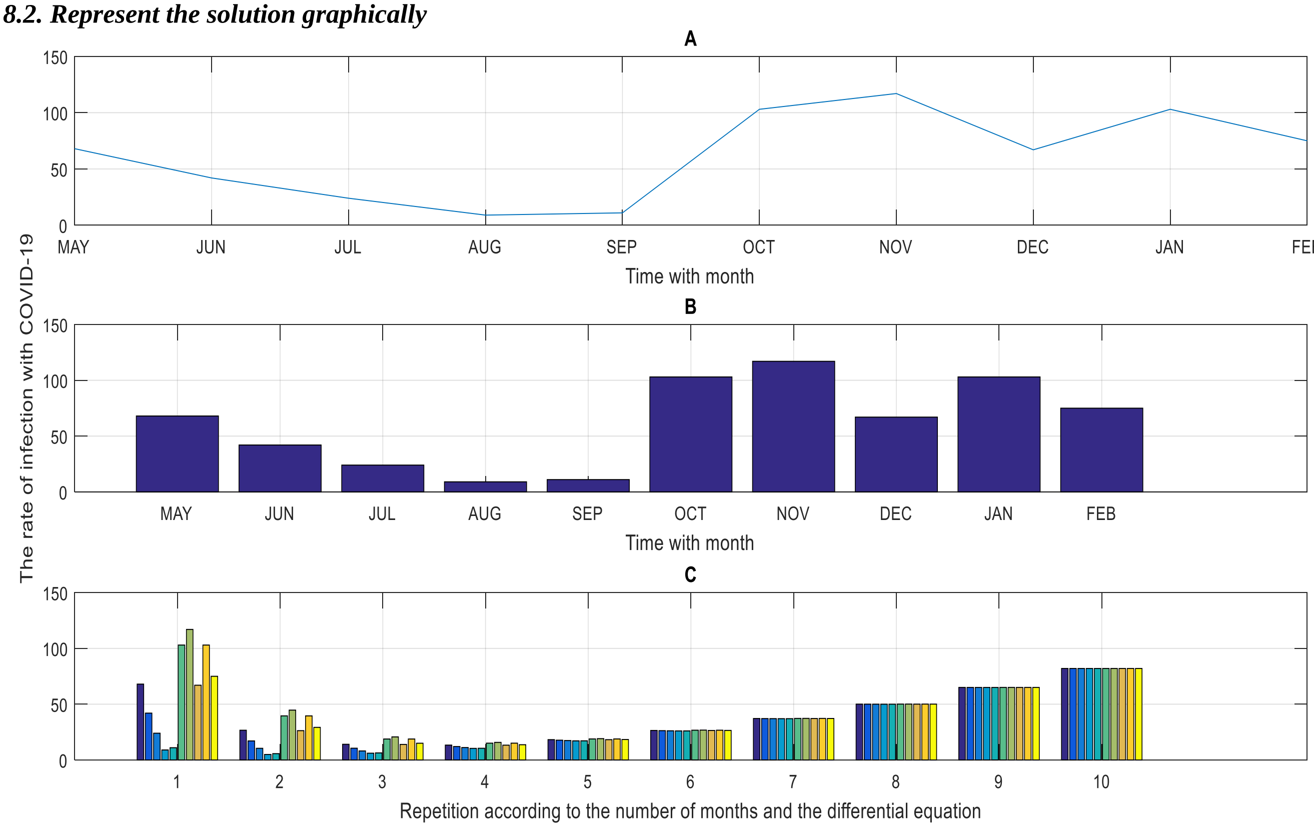


Fig. 2 Apply ODE of covid-19 infection in River Nile State for 10 month

9. Results

After performing all the steps for the two examples described in the previous sections, the model is evaluated and the final results discussed. The results obtained from the proposed model automatically during implementation and the testing process to obtain accurate results using MATLAB by applying the experiment twice for the two examples using the values collected from the Ministry of Health in River Nile State for those infected with Covid-19 for the period from May 2021 to February 2022 by passing the data on the algorithms Ordinary differential equations that were modeled, where each realization resulted in a curve to the graphs. The experiment was repeated with two different examples for accuracy to get excellent accuracy due to repetition, training and verification in each example. Indeed, it became clear that the infection rate in the first example Fig 1 -A and Fig 1-B that infection rates are high in the months when the temperature is low, which is from October to February next year, as well as in the second example Fig2 -A and Fig2-B. However, the difference after applying the solution of ordinary differential equations by repetition according to the number of ten months, the value diminished by a constant value in the first example Fig1-C. However, in the second example, which was used, an exponential differential equation increased the rates to hit with this solution as shown in Fig2-C.

10. Discussion

In this study, the performance of the computational solution model of ordinary differential equations was evaluated through the graphs presented in the previous sections for validation. This study used a new method to solve ODE by making a powerful algorithm for the results and automatically representing them in graphics unlike other studies that focused on a specific application, such as ODE: MATLAB / Simulink Solutions. etc.This study achieved great success in solving many types of ordinary differential equations and directing these solutions for application in the Corona pandemic, but it failed to provide a mathematical solution to the seriousness of the spread, as it was satisfied with the solution only and represented it graphically. This problem was solved by increasing the solution of two different examples so that each application finds the solution that best fits the problem in our daily life (eg industrial, agricultural, animal and economic production problems) and thus the model achieved high accuracy.

11. Conclusion

The paper dealt with the possible to model most of the applications in the form of linear ordinary differential equations, and through the matlab we can deal with most problems that occurred form applications , and the matlab program is a very significant in for reaches especially mathematician and physical , ,and the layout and exposure of the matlab result throughout graph, and such alike are more easier and a tractive for the perception of studiers and researchers which saves time and power.

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