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*Original Article*

# Gourava Indices of Certain Windmill Graphs

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***Abstract*** - *In this paper, we compute the first and second hyper Gourava indices, sum connectivity Gourava index, product connectivity Gourava index, general first and second Gourava indices of certain windmill graphs such as Kulli cycle windmill graph, Kulli path windmill graph, French windmill graph and Dutch windmill graph*.

***Keywords*** - *Gourava indices, Sum connectivity Gourava index, Product connectivity Gourava index, Windmill graphs.*

## 1. Introduction

Let $G$ be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. For a vertex $v$, the degree $d_G(v)$ is the number of vertices adjacent to $v$.

A topological index is a numerical parameters mathematically derived from the graph structure. It is a graph invariant. The topological indices have their applications in various disciplines of Science and Technology, see [1].

In [2], Kulli defined the first and second Gourava indices of a graph $G$ as

$$GO_1(G) = \sum_{uv\in E(G)} [d_G(u) + d_G(v) + d_G(u)d_G(v)],$$
$$GO_2(G) = \sum_{uv\in E(G)} (d_G(u) + d_G(v))\,(d_G(u)d_G(v))$$

The first and second hyper Gourava indices were introduced in [3] and they are defined as

$$HGO_1(G) = \sum_{uv\in E(G)} [d_G(u) + d_G(v) + d_G(u)d_G(v)]^2,$$
$$HGO_2(G) = \sum_{uv\in E(G)} [(d_G(u) + d_G(v))d_G(u)d_G(v)]^2$$

The sum connectivity Gourava index of a graph $G$ is defined as [4]

$$SGO(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v) + d_G(u)d_G(v)}},$$

In [5], Kulli proposed the product connectivity Gourava index of a graph $G$ and it is defined as

$$PGO(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{(d_G(u) + d_G(v))(d_G(u)d_G(v))}}$$

The general first and second Gourava indices [6] of a graph $G$ are defined as

$$GO_1^a(G) = \sum_{uv\in E(G)} [d_G(u) + d_G(v) + d_G(u)d_G(v)]^a, \tag{1}$$

$$GO_2^a(G) = \sum_{uv\in E(G)}[(d_G(u) + d_2(v))d_G(u)d_G(v)]^a, \tag{2}$$

where $a$ is a real number.
Recently, some indices were studied, for example, in [9,10,11,12,13,14,15,16,17,18].

In this paper, we compute Gourava indices, hyper Gourava indices, sum connectivity Gourava index, product connectivity Gourava index, general Gourava indices of certain windmill graphs.

## 2. Results for Kulli Cycle Windmill Graphs

The Kulli cycle windmill graph is the graph obtained by taking $m$ copies of the graph $K_1 + C_n$ for $n \geq 3$ with a vertex $K_1$ in common and it is denoted by $C_{n+1}^m$ This graph is presented in Fig. 1.

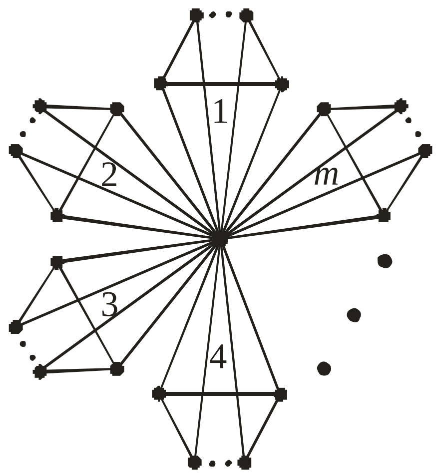


**Fig. 1 Kulli cycle windmill graph $C_{n+1}^m$**

Let $C = C_{n+1}^m$ be a wheel windmill graph with $mn$+1 vertices and 2$mn$ edges, $m \geq 2, n \geq 5$.

The graph $C$ has two types of edges as given Table 1.

**Table 1. Edge partition of $C_{n+1}^m$**

| $d_G(u), d_G(v) \setminus uv \in E(G)$ | (3, 3) | (3, $mn$ ) |
|---|---|---|
| **Number of edges** | $mn$ | $mn$ |

**Theorem 1.** The general first Gourava index of is $C_{n+1}^m$ given by

$$GO_1^a(C_{n+1}^m) = mn[15^a + (3 + mn)^a] \tag{3}$$

**Proof:** Let $C = C_{n+1}^m$ By using equation (1) and Table 1, we obtain

$$GO_1^a(C_{n+1}^m) = \sum_{uv\in E(C)}[d_G(u) + d_G(v) + d_G(u)d_G(v)]^a$$
$$= [(3 + 3) + (3 \times 3)]^a mn + [3 + mn + 3mn]^a mn$$
$$= [(3 + 3) + (3 \times 3)]^a mn + [3 + mn + 3mn]^a mn$$
$$= \mathrm{mn}[15^a + (3 + mn)^a]$$

We obtain the following results by using Theorem 1.

**Corollary 1.1.** The first Gourava index of $C_{n+1}^m$ is

$$GO_1(C_{n+1}^m) = 18mn + 4m^2n^2$$

**Corollary 1.2.** the first hyper Gourava index of $C_{n+1}^{m}$ is

$$HGO_1(C_{n+1}^{m}) = 16m^3n^3 + 24m^2n^2 + 234mn$$

**Corollary 1.3.** The sum connectivity Gourava index of $C_{n+1}^{m}$ is

$$SGO(C_{n+1}^{m}) = mn[\frac{1}{\sqrt{15}} + \frac{1}{\sqrt{3+4mn}}]$$

**Proof:** Put $a$ = 1, 2, –½ in equation (3), we obtain the desired results.

**Theorem 2.** The general second Gourava index of $C_{n+1}^{m}$ is given by

$$GO_2^a(C_{n+1}^{m}) = mn\ [54^a + (9mn + 3m^2n^2)^a] \quad (4)$$

**Proof:** Let $C = C_{n+1}^{m}$ By using equation (2) and Table 1, we deduce

$$GO_2^a(C_{n+1}^{m}) = \sum_{uv\in E(C)}[(d_2(u) + d_2(v))(d_2(u)d_2(v))]^a$$
$$= [(3+3)(3\times 3)]^a mn + [(3+mn)(3\times mn)]^a mn$$
$$= mn\ [54^a + (9mn + 3m^2n^2)^a]$$

The following results are obtained by using Theorem 2.

**Corollary 2.1.** The second Gourava index of $C_{n+1}^{m}$ is

$$GO_2(C_{n+1}^{m}) = 3m^3n^3 + 9m^2n^2 + 54mn$$

**Corollary 2.2.** The second hyper Gourava index of $C_{n+1}^{m}$ is

$$HGO_2(C_{n+1}^{m}) = 9m^5n^5 + 54m^4n^4 + 81m^3n^3 + 2916mn$$

**Corollary 2.3.** The product connectivity Gourava index of $C_{n+1}^{m}$ is

$$PGO(C_{n+1}^{m}) = mn[\frac{1}{\sqrt{54}} + \frac{1}{\sqrt{9mn+3m^2n^2}}]$$

Proof: Put $a$ = 1, 2, –½ in equation (4), we get the desired results.

## 3. Results For Kulli Path Windmill Graphs

The Kulli path windmill graph [20] is the graph obtained by taking $m$ copies of the graph $K_1+P_n$ with a vertex $K_1$ in common and it is denoted by $P_{n+1}^{m}$ This graph is shown in Fig. 2. The Kulli path windmill graph $P_3^m$ is a friendship graph.

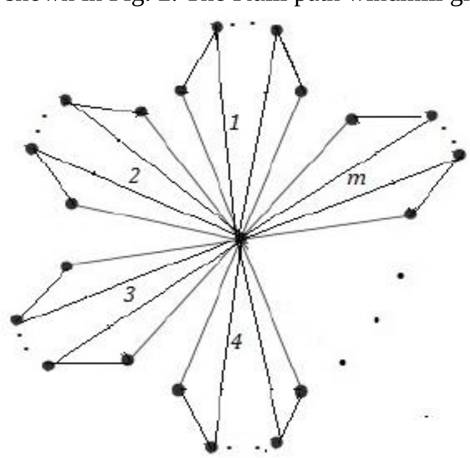


**Fig. 2 Kulli path windmill graph $P_{n+1}^{m}$**

Let $P = P_{n+1}^{m}$ $m\geq2$, $n\geq5$. Then $P$ has $mn$+1 vertices and 2$mn$ – $m$ edges. The graph $P$ has four types of 2-distance degree of edges as given Table 2.

**Table 2. Edge partition of $P_{n+1}^{m}$**

| ***d2(u), d2(v) \ uv ∈ E(G)*** | (2, 3) | (3, 3) | (mn, 2) | (mn, 3) |
|---|---|---|---|---|
| ***Number of edges*** | 2m | mn – 3m | 2m | mn – 2m |

**Theorem 3.** The general first Gourava index of $P_{n+1}^{m}$ is

$$GO_1^a(P_{n+1}^m) = m[29(15)^a - 3(15)^a + 2(2+3mn)^a - 2(3+4mn)^a] + mn\,[(15)^a + (3+4mn)^a] \qquad (5)$$

Proof: Let $P = P_{n+1}^{m}$ By using equation (1) and Table 2, we deduce

$$GO_1^a(P_{n+1}^m) = \sum_{uv\in E(P)} [d_2(u) + d_2(v) + d_2(u)d_2(v)]^a$$

$$= [2+3+(2\times3)]^a 2m + [3+3+(3\times3)]^a(mn-3m) + [mn+2+(mn\times2)]^a 2m + [mn+3+(mn\times3)]^a(mn-2m)$$
$$= m[29(15)^a - 3(15)^a + 2(2+3mn)^a - 2(3+4mn)^a] + mn\,[(15)^a + (3+4mn)^a]$$

We obtain the following results by using Theorem 3.

**Corollary 3.1.** The first Gourava index of of $P_{n+1}^{m}$ is

$$GO_1(P_{n+1}^m) = 4m^2n^2 + 18mn - 25m - 2m^2n$$

**Corollary 3.2.** The first hyper Gourava index of $P_{n+1}^{m}$ is

$$HGO_1(P_{n+1}^m) = 16m^3n^3 - 14m^3n^2 + 24m^2n^2 - 24m^2n + 234mn - 443m$$

**Corollary 3.3.** The sum connectivity Gourava index of $P_{n+1}^{m}$

$$SGO(P_{n+1}^m) = m\left[\frac{2}{\sqrt{11}} - \frac{3}{\sqrt{15}} + \frac{2}{\sqrt{3mn+2}} - \frac{2}{\sqrt{4mn+3}}\right] + \text{mn}\left[\frac{1}{\sqrt{15}} + \frac{1}{\sqrt{3+4mn}}\right]$$

**Proof:** Put $a$ = 1, 2, –½ in equation (5), we get the desired results.

**Theorem 4.** The general second leap Gourava index of $P_{n+1}^{m}$ is

$$GO_2^a(P_{n+1}^m) = m[2(30)^a - 3(54)^a + 2(2+3mn)^a - 2(3+4mn)^a] + mn\,[(54)^a + (3+4mn)^a] \qquad (6)$$

**Proof:** Let $P = P_{n+1}^{m}$ . From equation (2) and by using Table 2, we obtain

$$GO_2^a(P_{n+1}^m) = \sum_{uv\in E(P)} \left[\big(d_2(u)+d_2(v)\big)\big(d_2(u)d_2(v)\big)\right]^a$$
$$= [(2+3)\times(2\times3)]^a 2m + [(3+3)\times(3\times3)]^a(mn-3m)$$
$$+[mn+2+(mn\times2)]^a 2m + [mn+3+(mn\times3)]^a(mn-2m)$$
$$= m[2(30)^a - 3(54)^a + 2(2+3mn)^a - 2(3+4mn)^a] + mn\,[(54)^a + (3+4mn)^a]$$

**Corollary 4.1.** The second Gourava index of $P_{n+1}^{m}$ is

$$GO_2(P_{n+1}^m) = 4m^2n^2 + 57\,mn - 2m^2n - 104m$$

**Corollary 4.2.** The second hyper Gourava index of $P_{n+1}^{m}$ is

$$HGO_2(P_{n+1}^m) = 16m^3n^3 + 24m^2n^2 - 14m^3n^2 - 24m^2n + 2925mn - 6958m$$

**Corollary 4.3.** The product connectivity Gourava index of $P_{n+1}^m$ is

$$PGO(P_{n+1}^m) = m\left[\frac{2}{\sqrt{30}} - \frac{3}{\sqrt{54}} + \frac{2}{\sqrt{3mn+2}} - \frac{2}{\sqrt{4mn+3}}\right] + \text{mn}\left[\frac{1}{\sqrt{54}} + \frac{1}{\sqrt{3+4mn}}\right]$$

**Proof:** Put $a$ = 1, 2, –½ in equation (6), we get the desired results.

## 4. Results for French Windmill Graphs

The French windmill graph $F_n^m$ is the graph obtained by taking $m \geq 2$ copies of $K_n$, $n \geq 2$with a vertex in common. The graph $F_n^m$ is presented in Fig. 3. The French windmill graph $F_3^m$ is called a friendship graph.

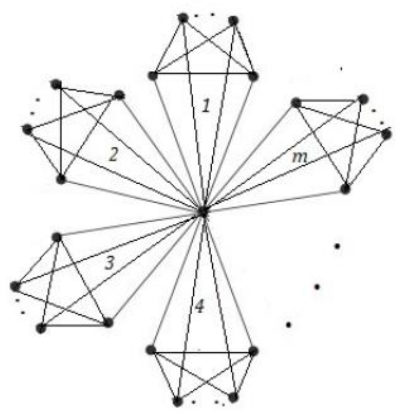


**Fig. 3 French windmill graph $F_n^m$**

Let $F$ be a French windmill graph $F_n^m$. Then $F$ has 1+ $m(n – 1)$ vertices and ½$mn(n – 1)$ edges, $m \geq 2$, $n \geq 2$. In $F$, there are two types of edges as given in Table 3.

**Table 3. Edge partition of *F*.**

| $d_G(u), d_G(v) \setminus uv \in E(G)$ | $((n-1), (n-1)m)$ | $((n-1), (n-1))$ |
|---|---|---|
| ***Number of edges*** | $m(n-1)$ | $\frac{1}{2}m(n-1)(n-2)$ |

**Theorem 5.** The general first Gourava index of $F_n^m$ is

$$GO_1^a(F_n^m) = [(n-1)(1+mn)]^a(n-1)m + [(n-1)(n+1)]^a\,\frac{1}{2}(n-1)(n-2) \qquad (7)$$

**Proof:** Let $F = F_n^m$ . From equation (1) and by using Table 3, we have

$$GO_1^a(F_n^m) = \sum_{uv\in E(F)} [d_2(u) + d_2(v) + d_2(u)d_2(v)]^a$$
$$= [(n-1) + (n-1)m + (n-1)(n-1)m]^a(n-1)m$$
$$+[(n-1) + (n-1) + (n-1)(n-1)]^a\frac{1}{2}m(n-1)(n-2)$$
$$= [(n-1)(1+mn)]^a(n-1)m + [(n-1)(n+1)]^a\,\frac{1}{2}(n-1)(n-2)$$

The following results are obtained by using Theorem 5.

**Corollary 5.1.** The first Gourava index of $F_n^m$ is

$$GO_1(F_n^m) = (n-1)^2\left[m + m^2n + \frac{1}{2}\,(n+1)(n-2)\right]$$

**Corollary 5.2.** The first hyper Gourava index of $F_n^m$ is

$$SGO(F_n^m) = (n-1)^3\left[(1+mn)^2 + \frac{1}{2}\ (n+1)^3(n-2)\right]$$

**Corollary 5.3.** The sum connectivity Gourava index of $F_n^m$ is

$$SGO(F_n^m) = \left[\frac{m\sqrt{n-1}}{\sqrt{1+mn}} + \frac{m(n-2)\sqrt{n-1}}{2\sqrt{n+1}}\right]$$

**Proof:** Put $a$ = 1, 2, –½ in equation (7), we get the desired results.

**Theorem 6.** The general second Gourava index of $F_n^m$ is

$$GO_2^a(F_n^m) = (n-1)^{3a+1}\ [\ m^{a+1}\ (1+m)^a + 2^{a-1}\ (n-2)] \tag{8}$$

**Proof:** Let $F = F_n^m$ By using equation (2) and Table 3, we obtain

$$GO_2^a(F_n^m) = \sum_{uv\in E(F)} \left[\big(d_2(u)+d_2(v)\big)\big(d_2(u)d_2(v)\big)\right]^a$$

$$= \left[\big((n-1)+(n-1)m\big)(n-1)(n-1)m\right]^a (n-1)m$$

$$+[((n-1)+(n-1))(n-1)(n-1)]^a \frac{1}{2} m(n-1)(n-2)$$

$$= (n-1)^{3a+1}\ [\ m^{a+1}\ (1+m)^a + 2^{a-1}\ (n-2)]$$

We obtain the following results by using theorem 6.

**Corollary 6.1.** The second Gourava index of $F_n^m$ is

$$GO_2(F_n^m) = (n-1)^4[n-2+m^2+m^3\ ]$$

**Corollary 6.2.** The second hyper Gourava index of $F_n^m$ is

$$HGO_2(F_n^m) = (n-1)^7[m^3(1+m)^2 + (2n-4)]$$

**Corollary 6.3.** The product connectivity Gourava index of $F_n^m$ is

$$PGO(F_n^m) = \frac{1}{\sqrt{n-1}}\left[\frac{\sqrt{m}}{\sqrt{1+m}} + 2^{-\frac{3}{2}}\ (n-2)\right]$$

**Proof:** Put $a$ = 1, 2, –½ in equation (8), we get the desired results

## 5. Results for Dutch Windmill Graphs

The Dutch windmill graph $D_n^m$ $m \in 2$, $n \in 5$ is the graph obtained by taking $m$ copies of the cycle $C_n$ with a vertex in common, see Fig. 4.

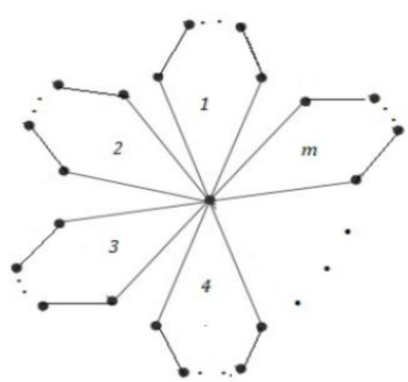


**Fig. 4 Dutch windmill graph $D_n^m$**

Let $G = D_n^m$ be a Dutch windmill graph with 1 + mn – m vertices and mn edges, $m \geq 2, n \geq 5$. Then G has two types of edges as given in Table 4.

**Table 4. Edge partition of $D_n^m$**

| $d_G(u), d_G(v) \setminus uv \in E(G)$ | (2, 2) | (2, $m$ ) |
|---|---|---|
| ***Number of edges*** | $m(n-2)$ | $2m$ |

**Theorem 7.** The general first Gourava index of $D_n^m$ is

$$GO_1^a(D_n^m) = 8^a m\,(n-2) + (2+3m)^a\, 2m \qquad (9)$$

**Proof:** Let $G = D_n^m$ . From equation (1) and by using Table 4, we have

$$GO_1^a(D_n^m) = \sum_{uv \in E(G)} [d_G(u) + d_G(v) + d_G(u)d_G(v)]^a$$

$$= [2 + 2 + (2 \times 2)]^a (n-2)m + [2 + m + 2m]^a 2m$$

$$= 8^a m\,(n-2) + (2+3m)^a\, 2m$$

We obtain the following results by using Theorem 7.

**Corollary 7.1.** The first Gourava index of $D_n^m$ is

$$GO_1(D_n^m) = 6m^2 + 8mn + 4m - 16$$

**Corollary 7.2.** the first hyper Gourava index of $D_n^m$ is

$$HGO_1(D_n^m) = 18m^3 + 24m^2 + 64mn + 8m - 128$$

**Corollary 7.3.** The sum connectivity Gourava index of $D_n^m$ is

$$SGO(D_n^m) = \left[\frac{m(n-2)}{\sqrt{8}} + \frac{2m}{\sqrt{2+3m}}\right]$$

**Proof:** Put $a$ = 1, 2, –½ in equation (9), we obtain the desired results.

**Theorem 8.** The general second Gourava index of $D_n^m$ is

$$GO_2^a(D_n^m) = 8^a m\,(n-2) + (2m)^{m+1}\,(2+m)^a \qquad (10)$$

**Proof:** Let $G = D_n^m$ . From equation (1) and by using Table 4, we have

$$GO_2^a(P_{n+1}^m) = \sum_{uv \in E(P)} \left[\big(d_2(u) + d_2(v)\big)\big(d_2(u)d_2(v)\big)\right]^a$$

$$= [(2+2) \times (2 \times 2)]^a (n-2)m + [(2+m) \times (2 \times m)]^a 2m$$

$$= 8^a m\,(n-2) + (2m)^{m+1}\,(2+m)^a$$

The following results are obtained by using Theorem 8.

**Corollary 8.1.** The second Gourava index of $D_n^m$ is

$$GO_2(D_n^m) = 4m^3 + 8m^2 + 8mn - 16$$

**Corollary 8.2.** The second hyper Gourava index of $D_n^m$ is

$$HGO_2(D_n^m) = 8m^5 + 32m^4 + 32m^3 + 64mn - 128$$

**Corollary 8.3.** The product connectivity Gourava index of $D_n^m$ is

$$PGO(D_n^m) = \left[\frac{m(n-2)}{\sqrt{8}} + \frac{\sqrt{2m}}{\sqrt{2+m}}\right]$$

Proof: Put *a* = 1, 2, –½ in equation (10), we get the desired results.

## 6. Conclusion

The first and second hyper Gourava indice computed , sum connectivity Gourava index, product connectivity Gourava index, general first and second Gourava indices of certain windmill graphs such as Kulli cycle windmill graph, Kulli path windmill graph, French windmill graph and Dutch windmill graph.

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