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Original Article

Bianchi Type-VIII String Cosmological Model in the Presence of Magnetic Field in General Relativity

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Abstract - *Bianchi type-VIII string cosmological model in the presence of magnetic field in general relativity is investigated. We assume that the current is flowing along x-axis so magnetic field is in the yz- plane. Thus F_{23} is the at most surviving constituent of electromagnetic field tensor F_{ij}. For the complete deterministic of the model, we assume that $\sigma \propto \theta$ which leads to $A = B^n$ where A and B are the metric potentials and n is constant; σ is the scalar shear tensor and θ is the expansion in the model. The general solution of the Einstein's field equations for the cosmological model have been obtained under the assumption $\rho - \lambda = 0$ where ρ and λ are the rest energy density and string tension density of fluid, respectively. The physical and geometrical characteristics of the model in the presence of magnetic field are discussed.*

Keywords - *Bianchi type-VIII, Magnetic field, String, General relativity.*

1. Introduction

Bianchi type cosmological models are essential in the interpretation that these models contain isotropic special cases and they allow arbitrarily small anisotropic scales at some instant of cosmic times. Anisotropic universe has a greater generality than isotropic models. The Bianchi type cosmological models contribute significantly to the representation of the universe such as creation of galaxies during its starting point of natural process. The clarity of the field equations made Bianchi space time applicable in creating models of spatially homogeneous and anisotropic cosmologies. Consequently, these models are to be known as very much convenient models of our universe. As a consequence, perusal of these models creates considerable interest. Bianchi type VIII model is one of the important anisotropic cosmological models and hence it is broad studied in general relativity.

Cosmic strings are considered as topologically static objects which presumably existed during a stage transition in the early universe (Kibble [17]). The string theory is a useful abstraction before the creation of the particle in the universe. It is generally assumed that at a later time the Big Bang, the universe may have go through a series of phase transition as its temperature was lowered down below some particular temperature as forecasted by grand merged theories (Vilenkin [2], Zel'dovich et al., [19]). It is conceivable that the cosmic string gives rise to density perturbations which conduct to the creation of galaxies. The comprehensive relativistic usage of strings was initiated by Letelier [12, 13] and Stachel [9]. Adhav et. al. [10] have investigated Bianchi type VIII cosmological model with linear equation of state.

The magnetic field plays a consequential role at the cosmological scale and is present in astronomic and interstellar spaces. The present day magnitude of the magnetic energy is very small in comparison along the estimated matter density. It might not have been insignificant during early stage of the evolution of the universe. A cosmological model which contains a global magnetic field is inevitably anisotropic since the magnetic field vector identifies a chartered spatial direction. The significance of magnetic field in cosmology has been studied by several authors Tyagi et al. [1], Chhajed et al. [3], Lorentz [4], Trivedi and bhabor [5], Singh [7], Amirhashchi [8], Bali and Jain [14], Bali and Swati [15], Bali [16] and Rao et. al. [18] etc.

Impelled by the above mentioned studies, in this paper, we have investigated Bianchi type-VIII string cosmological model in the presence of magnetic field in general relativity under the condition $\rho - \lambda = 0$. The physical and geometrical characteristics of the model in the presence of magnetic field are discussed.

2. Field Equations

The Bianchi type VIII line element is given by

$$ds^2 = dt^2 - B^2dx^2 - A^2dy^2 - [A^2\sinh^2y + B^2\cosh^2y]dz^2 - 2B^2 \cosh y\, dxdz \qquad \dots (1)$$

where A and B are functions of time t only.

Einstein's field equation is given by

$$R_i^j - \frac{R}{2} g_i^j = -8\pi T_i^j \quad \dots (2)$$

where T_i^j is the energy momentum tensor for a cloud of strings given by Letelier (1979, 1983) as

$$T_i^j = (p + \rho) v_i v^j - p g_i^j - \lambda x_i x^j + E_i^j \quad \dots (3)$$

with

$$v_i v^i = -x_i x^i = 1 \text{ and } v^i x_i = 0 \quad \dots (4)$$

where λ is the string tension density, ρ is the matter density, p is the thermodynamical pressure, v^i is the four-velocity vector and x^i the direction of string. The particle density associated with the configuration is given by

$$\rho_p = \rho - \lambda \quad \dots (5)$$

E_i^j is the electromagnetic field given by

$$E_i^j = \frac{1}{4\pi}\left[g^{rs} F_{ir} F_s^j - \frac{1}{4} F_{rs} F^{rs} g_i^j\right] \quad \dots (6)$$

We assume that the current is flowing along x-axis so magnetic field is in the yz-plane. Thus F_{23} is the exclusive surviving component of $F_{ij.}$

The Maxwell's equation

$$\frac{\partial}{\partial x^j}\left(F^{ij}\sqrt{-g}\right) = 0 \quad \dots (7)$$

Equation (7) leads to

$$\frac{\partial}{\partial z}(F^{23} A^2 B \sinh y) = 0$$

which again leads to

$$\frac{B}{A^2}\frac{\partial}{\partial z}\left(\frac{F_{23}}{\sinh y}\right) = 0$$

Then we get,

$$\frac{F_{23}}{\sinh y} = K$$

Therefore

$$F_{23} = K(\sinh y) \quad \dots (8)$$

where K is constant.

The energy momentum tensor for the line element (1) using equation (3) is obtained as

$$T_1^1 = -p + \lambda - \frac{K^2}{8\pi A^4} \quad \dots (9)$$

$$T_2^2 = -p + \frac{K^2}{8\pi A^4} \quad \dots (10)$$

$$T_3^3 = -p + \frac{K^2}{8\pi A^4} \qquad \dots (11)$$

$$T_4^4 = \rho - \frac{K^2}{8\pi A^4} \qquad \dots (12)$$

In the above v^i is the flow vector satisfying

$$g_{ij} v^i v^j = 1 \qquad \dots (13)$$

and direction of string is along x-axis so that $x_1 \neq 0$, $x_2 = 0$, $x_3 = 0$, $x_4 = 0$.

In a comoving coordinate system, we have

$$v^i = (0,0,0,1)\,, \quad x^i = \left(\frac{1}{B}, 0,0,0\right) \qquad \dots (14)$$

The Einstein's field equation for the line element (1) lead to the under mentioned system of equations:

$$2\frac{A_{44}}{A} + \frac{{A_4}^2}{A^2} - \frac{1}{A^2} - \frac{3B^2}{4A^4} = -8\pi(p - \lambda) - \frac{K^2}{A^4} \qquad \dots (15)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{B^2}{4A^4} = -8\pi p + \frac{K^2}{A^4} \qquad \dots (16)$$

$$2\frac{A_4 B_4}{AB} + \frac{{A_4}^2}{A^2} - \frac{1}{A^2} - \frac{B^2}{4A^4} = 8\pi\rho - \frac{K^2}{A^4} \qquad \dots (17)$$

where suffix 4 stand for derivative with respect to t.

The scalar expansion (θ) is given by

$$\theta = 2\frac{A_4}{A} + \frac{B_4}{B} \qquad \dots (18)$$

The components of shear tensor (σ_i^j) and scalar shear (σ) are given by

$$\sigma_1^1 = \frac{2}{3}\left(\frac{B_4}{B} - \frac{A_4}{A}\right) \qquad \dots (19)$$

$$\sigma_2^2 = \sigma_3^3 = \frac{1}{3}\left(\frac{A_4}{A} - \frac{B_4}{B}\right) \qquad \dots (20)$$

$$\sigma_4^4 = 0 \qquad \dots (21)$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{A_4}{A} - \frac{B_4}{B}\right) \qquad \dots (22)$$

The directional Hubble parameters in the direction of x, y and z respectively are given by

$$H_x = H_y = \frac{A_4}{A} \;\text{ and }\; H_z = \frac{B_4}{B} \qquad \dots (23)$$

The generalized mean Hubble parameter H is given by

$$H = \frac{1}{3}\left(H_x + H_y + H_z\right) \qquad \dots (24)$$

A physical entity deceleration parameter (q) defined by

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \qquad \dots (25)$$

3. Solution of the Field Equations

The field equations (15) - (17) are three independent equations in five unknowns A, B, p, λ and ρ. Hence to get a determinate solution, two extra conditions are needed. For the complete estimation of the model, we assume that $\sigma \propto \theta$ which leads to

$$A = B^n \quad \dots (26)$$

From equations (15) and (16), we obtain

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{{A_4}^2}{A^2} - \frac{A_4 B_4}{AB} - \frac{B^2}{A^4} - \frac{1}{A^2} = 8\pi\lambda - 2\frac{K^2}{A^4} \quad \dots (27)$$

and we consider

$$\rho - \lambda = 0 \quad \dots (28)$$

Equation (28) corresponds to the state equation for a cloud of massless geometric (Nambu) strings given by ρ = λ. (Mohanty [6]).

From equations (17), (27) and (28), we obtain

$$\frac{B_{44}}{B} - \frac{A_{44}}{A} + 3\frac{A_4 B_4}{AB} + \frac{3B^2}{4A^4} - \frac{K^2}{A^4} = 0 \quad \dots (29)$$

Substituting equation (26) into equation (29), yields

$$2B_{44} + 2\frac{n(n-4)}{(n-1)}\frac{{B_4}^2}{B} = \frac{3}{2(n-1)B^{4n-3}} + \frac{2K^2}{(1-n)B^{4n-1}} \quad \dots (30)$$

Let $B_4 = f(B)$, $B_{44} = f\frac{df}{dB}$, in equation (30), we obtain

$$2ff^1 + 2\frac{n(n-4)}{(n-1)}\frac{f^2}{B} = \frac{3}{2(n-1)B^{4n-3}} + \frac{2K^2}{(1-n)B^{4n-1}} \quad \dots (31)$$

Equation (31) leads to

$$\frac{d}{dB}(f^2) + 2\frac{n(n-4)}{(n-1)}\frac{f^2}{B} = \frac{3}{2(n-1)B^{4n-3}} + \frac{2K^2}{(1-n)B^{4n-1}} \qquad \text{where } (n \neq 1) \quad \dots (32)$$

After integration, equation (32) reduces to

$$f^2 = {B_4}^2 = \frac{K^2}{(n^2+n+1)}B^{2(1-2n)} - \frac{3}{4(n^2+2)}B^{4(1-n)} + Q\,B^{\frac{2n(n-4)}{(1-n)}} \quad \dots (33)$$

where Q is the integrating constant.

Equation (33) leads to

$$\int \frac{dB}{\sqrt{\frac{K^2}{(n^2+n+1)}B^{2(1-2n)} - \frac{3}{4(n^2+2)}B^{4(1-n)} + Q\,B^{\frac{2n(n-4)}{(1-n)}}}} = \int dt + S = t + S \quad \dots (34)$$

where S is the integrating constant.

Suitable transformation of coordinates

B = T, x = X, y = Y, z = Z

The metric (1) reduces to,

$$ds^2 = \frac{dT^2}{\frac{K^2}{(n^2+n+1)T^{2(2n-1)}} - \frac{3}{4(n^2+2)T^{4(n-1)}} + \frac{Q}{T^{\frac{2n(n-4)}{(n-1)}}}} - T^2 dX^2 - T^{2n} dY^2 - [T^{2n}\sinh^2 Y + T^2\cosh^2 Y]dZ^2 - 2T^2\cosh Y\, dX\, dZ \quad \dots (35)$$

4. The Geometrical and Physical Characteristics of the Model

For the model (35), the energy density (ρ), string tension density (λ), isotropic pressure (p), particle density (ρ_p) attached to the string, the expansion (θ), shear (σ), Proper volume (V), Hubble directional parameters (H_x, H_y and H_z), Hubble parameter (H) and deceleration parameter (q) are given by

$$8\pi\rho = 8\pi\lambda = \frac{(n+1)(2n+1)K^2}{(n^2+n+1)\,T^{4n}} - \frac{(n+1)(2n+1)}{2(n^2+2)}\frac{1}{T^{2(2n-1)}} + n(n+2)\frac{Q}{T^{\frac{2(n^2-3n-1)}{n-1}}} - \frac{1}{T^{2n}} \quad \dots (36)$$

$$8\pi p = \frac{2n\,K^2}{(n^2+n+1)\,T^{4n}} + \frac{(n+1)(1-n)}{(n^2+2)}\frac{1}{T^{2(2n-1)}} + \frac{2n(n+2)}{(1-n)}\frac{Q}{T^{\frac{2(n^2-3n-1)}{n-1}}} \quad \dots (37)$$

$$\rho_p = 0 \quad \dots (38)$$

$$\theta = (2n+1)\sqrt{\frac{K^2}{(n^2+n+1)T^{4n}} - \frac{3}{4(n^2+2)T^{2(2n-1)}} + \frac{Q}{T^{\frac{2(n^2-3n-1)}{(n-1)}}}} \quad \dots (39)$$

$$\sigma_1^1 = \frac{2(1-n)}{3}\sqrt{\frac{K^2}{(n^2+n+1)T^{4n}} - \frac{3}{4(n^2+2)T^{2(2n-1)}} + \frac{Q}{T^{\frac{2(n^2-3n-1)}{(n-1)}}}} \quad \dots (40)$$

$$\sigma_2^2 = \sigma_3^3 = \frac{(n-1)}{3}\sqrt{\frac{K^2}{(n^2+n+1)T^{4n}} - \frac{3}{4(n^2+2)T^{2(2n-1)}} + \frac{Q}{T^{\frac{2(n^2-3n-1)}{(n-1)}}}} \quad \dots (41)$$

$$\sigma_4^4 = 0 \quad \dots (42)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\sqrt{\frac{K^2}{(n^2+n+1)T^{4n}} - \frac{3}{4(n^2+2)T^{2(2n-1)}} + \frac{Q}{T^{\frac{2(n^2-3n-1)}{(n-1)}}}} \quad \dots (43)$$

From equations (39) and (43), we obtain

$$\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}(2n+1)} = \text{Constant}, \ \left(n \neq -\frac{1}{2}\right) \quad \dots (44)$$

$$V = A^2 B = T^{2n+1} \quad \dots (45)$$

$$H_x = H_y = n\left(\sqrt{\frac{K^2}{(n^2+n+1)T^{4n}} - \frac{3}{4(n^2+2)T^{2(2n-1)}} + \frac{Q}{T^{\frac{2(n^2-3n-1)}{(n-1)}}}}\right) \quad \dots (46)$$

$$H_z = \sqrt{\frac{K^2}{(n^2+n+1)T^{4n}} - \frac{3}{4(n^2+2)T^{2(2n-1)}} + \frac{Q}{T^{\frac{2(n^2-3n-1)}{(n-1)}}}} \quad \dots (47)$$

$$H = \frac{2n+1}{3}\left(\sqrt{\frac{K^2}{(n^2+n+1)T^{4n}} - \frac{3}{4(n^2+2)T^{2(2n-1)}} + \frac{Q}{T^{\frac{2(n^2-3n-1)}{(n-1)}}}}\right) \quad \dots (48)$$

$$q = -1 + \frac{3}{2n+1}\left\{\frac{\frac{2n\,K^2}{(n^2+n+1)T^{4n}} + \frac{3(1-2n)}{4(n^2+2)T^{2(2n-1)}} + \frac{(n^2-3n-1)}{(n-1)}\frac{Q}{T^{\frac{2(n^2-3n-1)}{(n-1)}}}}{\frac{K^2}{(n^2+n+1)T^{4n}} - \frac{3}{4(n^2+2)T^{2(2n-1)}} + \frac{Q}{T^{\frac{2(n^2-3n-1)}{(n-1)}}}}\right\} \qquad \ldots (49)$$

The magnitude of rotation

$$\omega = 0 \qquad \ldots (50)$$

5. Discussion

In the present paper, we have disquisited Bianchi type-VIII string cosmological model in the presence of magnetic field in general relativity. Einstein's field equations have been solved exactly with appropriate physical assumption for Nambu strings. When the universe is dominated by Nambu strings, the presence of electromagnetic field is essential as it accelerates the expansion of the universe. For the model (35), there is a Point Type singularity for $n > 0$ at $T = 0$ (MacCallum [11]). Since $T \to \infty$, $\frac{\sigma}{\theta} \neq 0$ so anisotropy is maintain throughout the progression of universe.

The model (35) starts with a Big Bang at $T = 0$. The expansion in the model decreases as time increases. The expansion tends to zero at $T = \infty$. As $T \to 0$, the spatial volume $(V) \to 0$ and V is increasing function of T for $n > -\frac{1}{2}$. The energy condition $\rho \geq 0$ is satisfied for all values of T. When $n = \frac{1}{2}$, T tends to infinity, deceleration parameter (q) tends to -1 therefore the model represent accelerating phase of universe.

We perused that the energy density, string tension density, pressure and Hubble parameter are decreasing function of time and tend to zero as $T \to \infty$. All physical parameters decrease more quickly in the presence of magnetic field. Generically, the present model describes shearing, expanding and non-rotating universe.

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