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Research Article

Fuzzy Solution Model for Multi-Objective Multi-Resource Knapsack Problem

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Abstract *- Wholesaling vegetables stands as a major problem in the business market. The main role of the wholesaler is to supply the vegetables raw and fresh to retail supermarkets on a daily basis. The challenge here lies in getting done with this task efficiently with minimum time and maximum profit. The wholesaler has to collect different kinds of vegetables (objects) from different sources (resources), in order to sell the vegetables in different marketplaces (objects). Mathematically, this problem can be viewed as a multi-objective multi-resource (MOMR) problem, in which the ultimate goal is to reduce the time and increase the profit attained. For such a MOMR Knapsack problem, the Fuzzy solution model is applied in this work to present efficient wholesaling of vegetables. The defined objective function for the Fuzzy solution model considers possible profit, demand, and cost of travel. This work is proceeded with an assumption that all the vegetables are available for all the resources. The performance of the proposed model is then evaluated and compared with the existing works for validation.*

Keywords *- Knapsack problem, Fuzzy, Machine learning, Multi-objective problem, Vegetable wholesaling.*

Mathematics Subject Classification AMS 2020: *49Mxx, 65Kxx, 62A86, 90C70.*

1. Introduction

Based on the knowledge of communication about the operations of the retail market, on the domestic marketplace, as well as the importance is given to the markets of wholesale in the developed nations from the business aspect, it is known that even domestic trading can enhance the selling of agro-industrial products at wholesale marketplaces [1]. Considering the importance of the wholesale markets as a distribution channel, their main role is becoming more and more vital and crucial in the market development, but also their competitiveness relative to numerous kinds of market centers must have an evidently profiled competitive advantage strategy [2 - 4].

The multiple-choice multidimensional knapsack problem (MMKP) is a variant of the well-known 0–1 knapsack problem in which one is given different families of items and for each family, a set of mutually exclusive items is provided. The goal is to find a subset of items that maximize a given utility measure without violating a set of capacity constraints and ensuring that each family is exclusively represented by one item [5]. The multi-resource Knapsack problem (MRKP) is an extension of the 0–1 knapsack problem (KP). In the multi-resource problem, the objects or items are available for each resource and any object may be collected from any resources [6].

Therefore, the main goal of the wholesalers is to determine the demand and sell the vegetables with a minimum cost of travel and maximum profit. In this research, the vegetable wholesaler's problem is considered as the multi-objective multi-resource (MOMR) knapsack problem. This model is a formulation of the Fuzzy solution model which proposes an idea combining the wholesale marketplace's demand, cost and profit to provide a solution for minimized cost and maximized profit.
The rest of the sections of the research paper is arranged as: Section 2, where a review of the literature is presented; Section 3, where the proposed mathematical solution for the MOMR knapsack problem is provided; and Section 4 provides a conclusion for the proposed work.

2. Literature Review

Several researchers worked on bringing a solution to the vegetable wholesalers on minimizing the cost and maximizing the profit of the business.

Devkota, A. R., et al. (2014) [7] studied the losses of the fruits and vegetables wholesale markets in the regions of Nepal for identifying the wastage caused. The authors conducted a regression analysis to find that there was a lack of cold storage banks and the packaging procedure was found to be inappropriate. These two factors affected the loss produced in the wholesale market.

Nagabhushana, S., et al. (2016) [8], on the other hand, developed a model to predict the demand and the yield value of the future that will be closer to the actual demand and yield value. Using this model, it was proposed that food wastage can be controlled for the welfare of society. Bahebshi, R. O., & Almaktoom, A. T. (2017) [9], proposed a dynamic model for minimizing the impact of the delay by comparing 5 strategies of delivery while maintaining limited transport of resources. As a case study, different vegetables with differing rates of degradation, transportation capacity, and retailers were considered. Lee, B., et al. (2020) [10], examined the factors such as choice of marketing channel, direct customer outlets, wholesale markets, and sales inclusion to wholesalers, which affect the profitability of the farm. An extensive study on the mechanism behind these effects was conducted. The effects observed imply that the profit differentials are attributable to the use of fertilizers and pesticides.

The problem of vegetable wholesalers can be considered as the multiple knapsack problem. There are various methods proposed by the researchers to provide a solution to such knapsack problems. A few works related to solving multiple knapsack problems are discussed as follows.

Dell'Amico, M., et al. (2019) [11] presented a pseudo-polynomial mathematical formulation with a particularly tailored decomposition procedure for handling the practical difficulties of the problem considered. This is an NP-hard combinational problem of optimization that is relevant to managerial complications. Song, B., et al. (2018) [12], developed a repair-based approach by employing the recoverable robustness scheme that involves 3 main elements such as a static quadratic multiple knapsack problem algorithm, an instance generator, and a repair algorithm. This approach was built to solve the stochastic model of the quadratic multiple knapsack problem. Manicassamy, J., et al. (2018) [13], modeled and developed Gene Position based Suppression (GPS) for solving the convergence problem with the self-adaptability of a Genetic Algorithm (GA). This model works on the genes of the newly produced individuals, in order the maintain the solution inside the region of feasibility. On the other hand, Detti, P. (2021) [14], implemented a new upper bound for the multiple knapsack problem. This method applied multiple knapsack problems to sequential bounds, such that the item sizes of the problem are dividable. This approach of sequential relaxation is attained by replacing the items of the problem with the items of dividable sizes.

3. Proposed Methodology

3.1. Knapsack Problem

The knapsack problem is a problem based on combinatorial optimization in search for the solution to a problem under uncertainty [15]. The knapsack problem is a combinatorial optimization problem used widely in used in real-life applications: loading problems [16] and resource allocation [17]. This problem can be described as follow: Given t objects, each object has a profit value (p) and weight (w) [18]. The general form of the knapsack problem over the objective function can be given as

$$max/\min f(x) = \sum_{i=1}^{n} PR_i \tag{1}$$

Subject to the constraints $\sum_{i=1}^{n} w_i x_i \leq K\ ;\ x_i \in \{0,1\}, 1\ to\ t$

where, n is the object number, PR_i and w_i is profit and weight of object i and x_i demonstrates the selected or not selected object i.

3.2. Multi-Objective Multi-Resource Knapsack Problem

The MOMR knapsack problem is an extended version of the knapsack problem. For accommodating the MOMR knapsack problem with the business system of vegetable wholesalers, it is assumed that the objects are the vegetables and the resources are the villages. A time slice is considered for collecting vegetables from several villages. The main objective of the proposed MOMR knapsack problem is to determine a set of objects that minimize the total time for collecting the objects from numerous resources and maximize the total profit.

It is assumed that there is only n types of objects (vegetables) and m number of resources (villages). Let J = {1, 2,…, n} and I = {1, 2,…,m} with C > 0 for all j ∈ J and i ∈ I. The proposed MOMR knapsack problem can be modeled as:

$$\begin{cases} \text{Find a complete solution } (x_1, x_2, \dots, x_n) \text{ and respective resource } (r_1, r_2, \dots, r_n) \text{ such that} \\ \text{maximise } Z_1 = \sum_{j=1}^{n} p_{r_j j} x_j \ \text{ for } r_j \epsilon \{1,2,\dots,n\} \text{ and minimize } Z_2 = \sum_{j=1}^{n} t_{r_j j} x_j \ \text{ for } r_j \epsilon \{1,2,\dots,n\} \\ \text{subject to } \sum_{j=1}^{n} w_{r_j j} x_j \leq C, \text{where } r_j \epsilon \{1,2,\dots,n\} \text{ and } x_j \epsilon \{0,1\} \text{ for } j = 1,2,\dots,n \end{cases} \tag{2}$$

A set of items with profit $p_{ij} > 0$ is available in every resource i ∈ I and every object j ∈ J. In this context, t_{ij} and p_{ij} are the collection time and the profit of j^{th} item from i^{th} resource for available weight of w_{ij}. Every item j consumes an amount $w_{ij} \geq 0$ by laying expenses of time $t_{ij} \geq 0$ from i^{th} resource. The parameters x_j and r_j denote that x_j amount of j^{th} item is chosen from r_j-th resource. Therefore, the objective function is to determine subsets of resources and items that minimize total time and maximize total profit. But the chosen item should not go beyond the capacities and the total time consumed.

3.3. Fuzzy Logic

Fuzzy logic is a well-known concept and it makes sense to process the vagueness value and deals with Boolean logic. Each possible value of a feature has a range between 0 to 1 through a membership function. The triangular membership function assists to fuzzify the data and acquire more knowledge on given data. The triangular curve is a function of a vector, a and it depends on three scalar parameters [19] [20], x, y, and z, as given by

$$\mu_T(a: x, y, z) = \begin{Bmatrix} \frac{a-x}{y-x} & x < a \leq y \\ \frac{a-b}{z-x} & b < a \leq z \\ 0 & otherwise \end{Bmatrix} \quad (3)$$

3.4. Fuzzy Solution Model for Multi-Objective Multi-Resource Knapsack Problem

"Main steps of the modeling are:

(1') To define the attributes and their weights for order selection problem by using the fuzzy group decision making philosophy.

(2') To compute the overall ranking score of each candidate orders with Fuzzy solution method,

(3') The final Fuzzy solution ranking score of each order is transferred to knapsack problem model's objective function as the coefficient to build the knapsack problem model and maximize the total profit and

(4') To determine the optimal order quantities by maximizing the total profit.

(a) Assumptions

$$p_i = (pr_i - c_i) \times b_{si} \quad (4)$$

$$f_i = d_i \times b_{si} \quad (5)$$

$$prls_i = pr_i \times 2 \quad (6)$$

Where,

p_i is the profit of the item i, pr_i is the price of the item i, c_i is the cost of the item i, b_{si} is the lot sizes of the item i, f_i is the travel cost, d_i is the distance travelled, and $prls_i$ is the lost sales of the item i.

(b) Objective function

The objective functions attempt to

(1) maximize the profit

$$\sum_{t=1}^{T}\sum_{i=1}^{n} R_i p_i x_{it}$$

Where R_i is the final rating of the item i, x_{it} number of items to be sold in the time period t.

(2) minimize the travel costs

$$\sum_{t=1}^{T}\sum_{i=1}^{n} (1 - R_i) f_i y_{it}$$

Where y_{it} is the probability of selling the item i within the time period t.

(3) minimize the cost of the left overs

$$\sum_{t=1}^{T} O_t ov$$

Where O_t is the number of vegetables left over without selling within the period t, and ov is the cost of the items left over.

Then, the combined Fuzzy solution knapsack problem 0-1 integer programming model can be formulated as follows:

Final model

$$\max \sum_{t=1}^{T}\sum_{i=1}^{n} R_i p_i x_{it} - \sum_{t=1}^{T}\sum_{i=1}^{n}(1-R_i) f_i y_{it} - \sum_{t=1}^{T} O_t ov \tag{7}$$

Subject to

$$\sum_{i=1}^{n}(w_i x_{it} + s_i y_{it}) \le W + O_t\,, \qquad t = 1,2,3,\dots,T$$

$$O_t \le 6, \qquad t = 1,2,3,\dots,T$$

$$\left(\frac{D_i - mbs_i}{bs_i}\right) y_{it} \le x_{it} \le \left(\frac{D_i}{bs_i}\right) y_{it} \times 1.2, \qquad i = 1,2,3,\dots,n; t = 1,2,3,\dots,T$$

$$x_{it}, O_t \text{ integer}$$

$$y_{it} \epsilon \{0,1\}$$

Where w_i is the weight of the item i, s_i is the capacity of the left over item i, W is the total capacity, D_i is the total demand of the item i, and mbs_i is the minimum lot size of the item i.

4. Conclusion

Thus the proposed research work presented a mathematical model on vegetable wholesalers trading by considering it as the multi-objective multi-resource knapsack problem. A Fuzzy solution model was proposed in this work for achieving maximum profit and minimum cost. The simple and flexible nature of Fuzzy logic have paved a solution for the competitive market of wholesaler's by considering demand, travel cost and possible profit.

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