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Original Article

On the Stability of Soft Fixed Point in Soft Topological Space

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Abstract *- This paper introduces stable and strongly sable of soft fixed point and studied some of their properties. The notations of converge of sequence and discrete dynamical system are generalized using soft sets. As a continuation of this, the concept of orbit was discussed to investigate the behavior of soft point under soft mapping. Also, many basic properties of these notations was presented. In addition, the concepts associated to soft points have been used to characterize soft open sets. Moreover, using some of topological properties to obtain a results concerning with soft fixed points. In particular the uniqueness of soft fixed point have been provided, which is guaranteed to exist.*

Keywords *- Converge of soft sequence, Discrete dynamical system, Soft fixed point, Soft set, Soft orbit.*

1. Introduction

The notion of s0ft sets theory was introduced by D. Molodtsov [1] in 1999 as a new mathematical theory for handling uncertainty. This theory has been applied to many directions with great success. In 2003, Maji et al. [2], conducted a theoretical study of the operations of soft sets. In particular, they presented the concepts of complement, subset, union and intersection of soft sets and studied their properties. Recently, a lot of researchers are studying the properties and applications of soft set theory. In 2011, Shabir and Naz [3] introduce the concept of soft topological space which is defined with a fixed set of parameters over an initia1 universe. They studied fundamental concepts of soft topological spaces such as soft subspace , soft open and soft closed sets, soft separation axioms, soft neighborhood of a soft point, soft normal spaces, soft regular spaces and discussed several of their properties . In 2013, D. wardowski [4] introduce the notion of soft fixed point and studied some of its properties. In 2016, S.A. Jubair [5] studied some properties of these soft separation axioms. Some other studies on soft topological spaces can be 1isted as [5,6,7,8]. After that in 2016, S.H. Hameed and S.A. Jubair [9] defined soft generalized continuous functions and derived their basic properties. Also, some fundamental concepts of soft functions are discussed in detail. Recently, in 2019, Sadi B et al.[10] introduce the definition of converge of a soft sequence in soft topological space and they discuss some concepts and results about this notion. The paper is arranged as follows: In section 1, we review some properties of soft set theory and soft topological space. They are basic tools to investigate our work. A natural characterization of soft open sets is presented by using the notion of soft point. In section 2, the definitions of stable and strongly sable of soft fixed point have been introduced and studied some of their properties. Also, a brief summary of a case study concerning discrete dynamical system was presented, a full report of this case study is given in [11]. The uniqueness of a soft fixed point is studied by using compact topological space and T_2-space. Furthermore, other notions such as converge of soft sequence and the orbit of a soft point are studied and establish some properties of these soft notions. Some examples and counterexamples are also given. Observe that, in this paper the analytical method have been adopted. Throughout the paper let $\mathbb{R}$ be a set of all real numbers, let $\mathbb{Z}$ be a set of all integers and let $\mathbb{N}$ be a set of all natural numbers.

2. Basic Concepts

In this section, basic definitions and notation of soft set theory and soft topological space was presented. Also, some results have been introduced to be useful in the next section. Throughout this paper, a set M refers initia1 universe, Γ is the set of all parameters and $\mathcal{P}(M)$ is the power set of M.

2.1. Definition [1]

Let M be an initial universe set and Γ be a set of parameters. A pair $(\mathcal{K}, \Gamma)$ is said to be a soft set over M if only if $\mathcal{K}$ is a mapping from Γ into $\mathcal{P}(M)$, i.e. $\mathcal{K}: \Gamma \rightarrow \mathcal{P}(M)$, where $\mathcal{P}(M)$ is the power set of M. Observe that the set of all soft set over M is denoted by $\mathcal{S}(M, \Gamma)$.

2.2. Definition [1]

Let $(\mathcal{K}, \Gamma)$, $(\mathcal{L}, \Gamma) \in \mathcal{S}(M, \Gamma)$. A pair $(\mathcal{K}, \Gamma)$ is a soft subset of $(\mathcal{L}, \Gamma)$ if $\mathcal{K}(\alpha) \subseteq \mathcal{L}(\alpha)$ for every $\alpha \in \Gamma$ and denoted by $(\mathcal{K}, \Gamma) \sqsubseteq (\mathcal{L}, \Gamma)$. Also, if $(\mathcal{K}, \Gamma) \tilde{\subseteq} (\mathcal{L}, \Gamma)$ and $(\mathcal{L}, \Gamma) \tilde{\subseteq} (\mathcal{K}, \Gamma)$ then $(\mathcal{K}, \Gamma)$and $(\mathcal{L}, \Gamma)$ are soft equal and denoted by $(\mathcal{K}, \Gamma) = (\mathcal{L}, \Gamma)$.

2.3. Definition [1]

A soft set $(\mathcal{K},\Gamma)$ over M is said to be :

i. null soft set denoted by $\widetilde{\Phi}$ if $\mathcal{K}(\alpha) = \emptyset$ for all $\alpha \in \Gamma$.

ii. absolute soft set denoted by $\widetilde{\mathrm{M}}$, if $\mathcal{K}(\alpha) = M$ for all $\alpha \in \Gamma$.

2.4. Definiti0n [2]

Let $\mathcal{J}$ be a nonempty subset of M, then $\tilde{\mathcal{J}}$ denotes the soft set $(\mathcal{J}, \Gamma)$ over M, where $\mathcal{J}(\alpha) = \mathcal{J}$; for all $\alpha \in \Gamma$. In particu1ar (M, Γ) will be denoted by $\widetilde{M}$.

2.5. Definition [1]

Let$\{(\mathcal{K}_i, \Gamma)$: $i \in$ I$\}$ be family of soft sets. The soft union of these soft sets is the soft set $(\mathcal{K}, \Gamma) \in \mathcal{S}(M, \Gamma)$, where the map $\mathcal{K}: \Gamma \to \mathcal{P}(M)$ defined as follows: $\mathcal{K}(\alpha) = \bigcup\{ \mathcal{K}_i(\alpha): i \in \mathrm{I}\}$, for a11 $\alpha \in \Gamma$. Symbolically, $(\mathcal{K}, \Gamma) = \widetilde{\bigcup}\{(\mathcal{K}_i, \Gamma): i \in \mathrm{I}\}$.

2.6. Definition [1]

Let$\{(\mathcal{K}_i, \Gamma)$: $i \in$I$\}$ be family of soft sets. The soft intersection of these soft sets is the soft set $(\mathcal{K}, \Gamma) \in \mathcal{S}(M, \Gamma)$ where the map $\mathcal{K}: \Gamma \to \mathcal{P}(M)$ defined as follows: $\mathcal{K}(\alpha) = \bigcap\{ \mathcal{K}_i(\alpha): i \in \mathrm{I}\}$, for a11 $\alpha \in \Gamma$. Symbolically, $(\mathcal{K}, \Gamma) = \widetilde{\bigcap}\{(\mathcal{K}_i, \Gamma): i \in \mathrm{I}\}$.

2.7. Definition [2]

The difference set of two soft sets $(\mathcal{K}, \Gamma)$ and $(\mathcal{L}, \Gamma)$ over $\widetilde{M}$ is a soft $(\mathcal{H}, \Gamma)$ denoted by $(\mathcal{H}, \Gamma) = (\mathcal{K}, \Gamma) \setminus (\mathcal{L}, \Gamma)$; where $\mathcal{H}(\alpha) = \mathcal{K}(\alpha) \setminus \mathcal{L}(\alpha)$ for a11 $\alpha \in \Gamma$.

2.8. Definition [2]

The complement of a soft set $(\mathcal{K}, \Gamma)$ denoted by $(\mathcal{K}, \Gamma)^c$ is defined by $(\mathcal{K}, \Gamma)^c = (\mathcal{K}^c, \Gamma)$; $\mathcal{K}^c: \Gamma \to \mathcal{P}(M)$ is a mapping given by $\mathcal{K}^c(\alpha) = M - \mathcal{K}(\alpha)$, for all $\alpha \in \Gamma$. $\mathcal{K}^c$ is said to be the soft complement mapping of $\mathcal{K}$. Clearly, $(\mathcal{K}^c)^c$ is the same as $\mathcal{K}$ and $((\mathcal{K}, \Gamma)^c)^c = (\mathcal{K}, \Gamma)$.

2.9. Definition [2]

A soft set $(\mathcal{K}, \Gamma)$ is called a soft point if there exists $\alpha \in \Gamma$ such that $\mathcal{K}(\alpha)$ is a singleton and $\mathcal{K}(\beta) = \emptyset$, for all $\beta \in \Gamma - \{\alpha\}$. Such a soft point is denoted by x_α. Also, a soft point x_α is said to be in the soft set $(\mathcal{H}, \Gamma)$ denoted by $x_\alpha \tilde{\in} (\mathcal{H}, \Gamma)$, if $x \in \mathcal{H}(\alpha)$.

2.1o.Definition [5] Let $(\mathcal{K}, \Gamma)$ be a soft set over M and $\mathcal{J}$ be a nonempty subset of M. Then the sub soft set of $(\mathcal{K}, \Gamma)$ over $\mathcal{J}$ denoted by $(\mathcal{K}_{\mathcal{J}}, \Gamma)$ is defined by $\mathcal{K}_{\mathcal{J}}(\alpha) = \mathcal{J} \cap \mathcal{K}(\alpha)$, for all $\alpha \in \Gamma$, i.e $(\mathcal{K}_{\mathcal{J}}, \Gamma) = \tilde{\mathcal{J}} \widetilde{\bigcap} (\mathcal{K}, \Gamma)$.

2.11. Definition [8]

Let M, N be two nonempty sets, $f: M \to N$ be a mapping, $(\mathcal{K}, \Gamma) \in \mathcal{S}(M, \Gamma)$ and $(\mathcal{L}, \Gamma) \in \mathcal{S}(M, \Gamma)$. Then:
i. $f(\mathcal{K}, \Gamma) = (f(\mathcal{K}), \Gamma)$ where $[f(\mathcal{K})](\alpha) = f[\mathcal{K}(\alpha)]$ for all $\alpha \in \Gamma$. ii. $f^{-1}(\mathcal{L}, \Gamma) = (f^{-1}(\mathcal{L}), \Gamma)$ where $[f^{-1}(\mathcal{L})](\alpha) = f^{-1}[\mathcal{L}(\alpha)]$ for all $\alpha \in \Gamma$.

Note that, the above definition of soft mapping that used in our work is different from the definition of soft mapping introduced by D. wardowski [4] and also differs from the concept of soft mapping by Z. Hu and F. Wenqing [11]

The following proposition was introduced to be useful in next section.

2.12. Proposition

Let M, N be two nonempty sets and $f: M \to N$ be a mapping such that $(\mathcal{K}, \Gamma) \in \mathcal{S}(M, \Gamma)$ and $(\mathcal{L}, \Gamma) \in \mathcal{S}(N, \Gamma)$. Then

i. $f(\mathcal{K}, \Gamma)^c = (f(\mathcal{K}, \Gamma))^c$ if f bijective,

ii. $f^{-1}(\mathcal{L}, \Gamma)^c = (f^{-1}(\mathcal{L}, \Gamma))^c$.

Proof: Clear.

Now, let us recall the definition of a soft topology and some topological concepts which are used in our work.

2.13. Definition [3]

Let τ be a family of soft sets over M. Then τ is called a soft topology on M, if
i. $\widetilde{\Phi}$, $\widetilde{M}$ belong to τ, ii. the intersection of any two soft sets in τ belongs to τ, iii. the union of any number of soft sets in τ belongs to τ The triple(M, τ, Γ) is called a soft topological space.

2.14. Definition [3]

The members of τ are called soft open sets. The complement of a soft open set is called the soft closed sets.

2.15. Definition [3]

Let (M, τ, Γ) be a soft topological space and $\mathcal{J}$ be a nonempty subset of M. Then $\tau_{\mathcal{J}}$ =$\{(\mathcal{K}_{\mathcal{J}}, \Gamma)|\ (\mathcal{K}, \Gamma) \in \tau\}$ is called the soft relative topology on $\mathcal{J}$ and $(\mathcal{J}, \tau_{\mathcal{J}}, \Gamma)$ is said to be a soft subspace of (M, τ, Γ).

2.16. Theorem [12]

Let $(\mathcal{J}, \tau_{\mathcal{J}}, \Gamma)$ be a soft subspace of soft topological space (M, τ, Γ) and $(\mathcal{K}, \Gamma)$ be a s0ft set over M.
Then: i. $(\mathcal{K}, \Gamma)$ is soft open over $\mathcal{J}$ if and only if $(\mathcal{K}, \Gamma) = \tilde{\mathcal{J}} \tilde{\cap} (\mathcal{L}, \Gamma)$ for some $(\mathcal{L}, \Gamma) \in \tau$
ii. $(\mathcal{K}, \Gamma)$ is soft closed over $\mathcal{J}$ if and only if $(\mathcal{K}, \Gamma) = \tilde{\mathcal{J}} \tilde{\cap} (\mathcal{L}, \Gamma)$ for some soft closed set $(\mathcal{L}, \Gamma)$ over M.

2.17. Definition [13]

Let (M, τ, Γ) be a s0ft topol0gical space. A subcollection $\mathfrak{B}$ of τ is said to be a soft basis for the soft topology τ if every member of τ can be expressed as a union of members of $\mathfrak{B}$.

2.18. Definition. [5]

Let (M, τ, Γ) be a soft topological space and $(\mathcal{L}, \Gamma)$ be a soft set over M. Then the soft interior of $(\mathcal{L}, \Gamma)$ denoted by $(\mathcal{L}, \Gamma)^{\circ}$ is the soft set defined as: $(\mathcal{L}, \Gamma)^{\circ} = \tilde{\bigcup}\{(\mathcal{K}, \Gamma):(\mathcal{K}, \Gamma)$ is soft open and $(\mathcal{K}, \Gamma) \tilde{\subseteq} (\mathcal{L}, \Gamma)\}$. Thus, $(\mathcal{L}, \Gamma)^{\circ}$ is the largest soft open set contained in $(\mathcal{L}, \Gamma)$.

2.19. Definition [5]

Let (M, τ, Γ) be a soft topological space and $(\mathcal{L}, \Gamma)$ be a soft set over M. Then the soft closure of $(\mathcal{L}, \Gamma)$ denote by $(\mathcal{L}, \Gamma)$ is the soft set defined as: $\overline{(\mathcal{L}, \Gamma)} = \tilde{\bigcap}\{(\mathcal{K}, \Gamma)$: $(\mathcal{K}, \Gamma)$ is soft closed and $(\mathcal{L}, \Gamma) \tilde{\subseteq} (\mathcal{K}, \Gamma)\}$. Note that $\overline{(\mathcal{L}, \Gamma)}$ is the smallest soft closed set containing $(\mathcal{L}, \Gamma)$.

2.20. Definition [12]

Let (M, τ, Γ) be a soft topological space over M. A soft set $(\mathcal{K}, \Gamma)$ in (M, τ, Γ) is called a soft neighborhood of the soft point $x_{\alpha} \tilde{\in} (\mathcal{K}, \Gamma)$ if there exists a soft open set $(\mathcal{L}, \Gamma)$ such that $x_{\alpha} \tilde{\in} (\mathcal{L}, \Gamma) \tilde{\subset} (\mathcal{K}, \Gamma)$.

2.21. Definition [9]

Let (M, τ, Γ), (N, τ', Γ) be soft topological spaces and $f: M \to N$ be a mapping. The mapping f is soft continuous at $x_{\alpha} \tilde{\in} \widetilde{M}$, if for all soft neighbourhood $(\mathcal{H}, \Gamma)$ of $f(x_{\alpha})$ there exists a soft neighborhood $(\mathcal{K}, \Gamma)$ of x_{α} such that $f((\mathcal{K}, \Gamma)) \tilde{\subset} (\mathcal{H}, \Gamma)$. If f is soft continuous mapping for all x_{α} , then f is called soft continuous mapping.

2.22. Definition [8]

Let (M, τ, Γ), (N, τ', Γ) be soft topological spaces and $f: M \to N$ be a mapping . If i. f is bijective, ii. f is soft continuous, iii. f^{-1} is soft continuous .

Then f is called to be soft homeomorphism from M to N. When a homeomorphism f exists between M and N, we say that M is soft homeomorphic to N and we write $(M, \tau, \Gamma) \overset{s}{\cong} (N, \tau', \Gamma)$.

2.23. Definition [5]

A soft topological space (M, τ, Γ) is said to be a soft τ_2-space if for all $x_{\alpha}, y_{\beta} \tilde{\in} \widetilde{M}$ such that $x_{\alpha} \neq y_{\beta}$ there exist soft open sets $(\mathcal{K}, \Gamma)$ and $(\mathcal{L}, \Gamma)$ such that $x_{\alpha} \tilde{\in} (\mathcal{K}, \Gamma)$, $y_{\beta} \tilde{\in} (\mathcal{L}, \Gamma)$ and $(\mathcal{K}, \Gamma) \tilde{\cap} (\mathcal{L}, \Gamma) = \widetilde{\Phi}$.

2.24. Definition [13]

A family Ω of soft sets is a cover of a s0ft set $(\mathcal{L}, \Gamma)$ if $(\mathcal{L}, \Gamma) \tilde{\subseteq} \tilde{\bigcup}\{(\mathcal{L}_i, \Gamma): (\mathcal{L}_i, \Gamma) \in \Omega, i \in I\}$. It is a soft open cover if each member of Ω is a soft open set . A subcover of Ω is a subfamily of Ω which is also a cover.

2.25. Definition [13]

A soft topological space (M, τ, Γ) is soft compact if each soft open cover of $\widetilde{M}$, has a finite subcover.

2.26. Definition [10]

Let (M, τ, Γ) be a soft topological space, $\{x_{\alpha_n}^n\}$ be a soft sequence and $x_{\alpha_1}^1$ be a soft point. The sequence $x_{\alpha_n}^n$ is said to converge to the soft point $x_{\alpha_1}^1$ if there exists $n_1 \in \mathbb{N}$, for all $n \geq n_1$ such that $x_{\alpha_n}^n \tilde{\in} (\mathcal{K}, \Gamma)$ for each soft neighborhood $(\mathcal{K}, \Gamma)$ of soft point x_α; denoted by $\lim_{n\to\infty} x_{\alpha_n}^n = x_{\alpha_1}^1$.

The following Proposition provides a natural characterization of soft open sets using the previously presented notion of soft points. Before this we need the following Lemma.

2.27. Lemma

Every soft set $(\mathcal{L}, \Gamma)$ can represented as a union of its soft points i.e $(\mathcal{L}, \Gamma) = \tilde{\bigcup}\{x_\alpha : x_\alpha \in (\mathcal{L}, \Gamma)\}$.

2.28. Proposition

Let (M, τ, Γ) be a soft topological space. A soft set $(\mathcal{K}, \Gamma)$ is soft open if and only if for all $x_\alpha \tilde{\in} (\mathcal{K}, \Gamma)$ there exists a soft open set $(\mathcal{L}, \Gamma)$ such that $x_\alpha \tilde{\in} (\mathcal{L}, \Gamma) \tilde{\subseteq} (\mathcal{K}, \Gamma)$.

Proof: $(\Rightarrow)$Let $(\mathcal{K}, \Gamma)$ be a soft open set. Then, for each $x_\alpha \tilde{\in} (\mathcal{K}, \Gamma) \tilde{\subseteq} (\mathcal{K}, \Gamma)$

$(\Leftarrow)$ Let $x_\alpha \tilde{\in} (\mathcal{K}, \Gamma)$. Then there exists a soft open set $(\mathcal{L}, \Gamma)$ such that $x_\alpha \tilde{\in} (\mathcal{L}, \Gamma) \tilde{\subseteq} (\mathcal{K}, \Gamma)$. Then, by Lemma 2.27 and Definition 1.9, we obtain $(\mathcal{K}, \Gamma) = \tilde{\bigcup}\{x_\alpha : x_\alpha \tilde{\in} (\mathcal{K}, \Gamma)\} \tilde{\subseteq} (\mathcal{L}, \Gamma)$. Thus $(\mathcal{K}, \Gamma) = (\mathcal{L}, \Gamma) \tilde{\in} \tau$.

2.29. Proposition

Let (M, τ, Γ) be a soft τ_2- space. Then every soft compact set in τ is soft closed.

Proof: Let $(\mathcal{K}, \Gamma)$ be a soft compact set and $x^\alpha \in (\mathcal{K}, \Gamma)^c$. For all $x^\beta \in (\mathcal{K}, \Gamma)$,let $(\mathcal{K}_x, \Gamma)$ and $(\mathcal{H}_x, \Gamma)$ be soft open sets such that $(\mathcal{L}_x, \Gamma) \tilde{\cap} (\mathcal{H}_x, \Gamma) = \tilde{\Phi}$ and $x^\alpha \in (\mathcal{L}_x, \Gamma)$, $x^\beta \in (\mathcal{H}_x, \Gamma)$. By a soft compactness of $(\mathcal{K}, \Gamma)$ there exist $x_1^\beta, x_2^\beta, \ldots, x_n^\beta$ such that $(\mathcal{K}, \Gamma) \subseteq \tilde{\bigcup}_{i=1}^n (\mathcal{H}_i, \Gamma)$. Denote $(\mathcal{L}, \Gamma) = (\mathcal{L}_{x_1}, \Gamma) \tilde{\cap} \ldots \tilde{\cap} (\mathcal{L}_{x_k}, \Gamma)$ and $(\mathcal{H}, \Gamma) = (\mathcal{H}_{x_1}, \Gamma) \tilde{\cap} \ldots \tilde{\cap} (\mathcal{H}_{x_k}, \Gamma)$. Then $x^\alpha \in (\mathcal{L}, \Gamma) \in \tau$, $(\mathcal{L}, \Gamma) \tilde{\cap} (\mathcal{H}, \Gamma) = \tilde{\Phi}$ and hence $(\mathcal{L}, \Gamma) \tilde{\cap} (\mathcal{K}, \Gamma) = \tilde{\Phi}$, thus $x^\alpha \in (\mathcal{L}, \Gamma) \subseteq (\mathcal{K}, \Gamma)^c$. By Proposition 4.1, $(\mathcal{K}, \Gamma)$ is soft closed set.

3. Main Results

In this sectiOn, the definitiOns of stable and strongly stable **O**f sOft fixed pOint was introduced and studied some fundamental properties of these concepts. Also, Other concepts such as converge of soft sequence, discrete dynamical system and the orbit of soft point have been introduced and discussed some of them properties. In addition, some basic results concerning soft compact and T_2-space are given. Now, the following definition is introduce which is needed in this section.

3.1. Definition

Let (M, τ, Γ) be a s**O**ft top**O**l**O**gical space and $\{x_{\alpha_n}^n : n \in \mathbb{Z}^+\}$ be a s**O**ft sequence in $\tilde{M}$. Then $x_{\alpha_n}^n$ is said to c**O**nverges to the s**O**ft point $x_{\alpha_0}^0 \tilde{\in} \tilde{M}$,($x_{\alpha_0}^0$ is limit of $x_{\alpha_n}^n$) if and **O**nly if for all **O**pen set $(\mathcal{L}, \Gamma)$ of $\tilde{M}$ c**O**ntaining $x_{\alpha_0}^0$ c**O**ntaining all but finitely many elements **O**f $x_{\alpha_n}^n$.

3.2. Example

Let $M = \{a, b, c\}$, $\Gamma = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ and $\tau = \{\tilde{\Phi}, \tilde{M}, (\mathcal{K}_1, \Gamma), (\mathcal{K}_2, \Gamma), (\mathcal{K}_3, \Gamma)\}$ be a s**O**ft t**O**pology defined over M, where: $\mathcal{K}_1(\alpha_1) = \{a\}$, $\mathcal{K}_1(\alpha_2) = \{a, b\}$, $\mathcal{K}_1(\alpha_3) = \{a, c\}$, $\mathcal{K}_1(\alpha_4) = \{a, b, c\}$, $\mathcal{K}_2(\alpha_1) = \{b\}$, $\mathcal{K}_2(\alpha_2) = \{b, c\}$,$\mathcal{K}_2(\alpha_3) = \{a\}$,$\mathcal{K}_2(\alpha_4) = \{a, b\}$, $\mathcal{K}_3(\alpha_i) = \{c\}$, i=1,2,3,4. Now, c**O**nsider the soft sequence $x_{\alpha_n}^n$ defined by $x_{\alpha_n}^n = a_{\alpha_1}$ f**O**r all $n \in \mathbb{Z}^+$. We claim that $x_{\alpha_n}^n$ c**O**nverges t**O** the s**O**ft point a_{α_1} in $\tilde{M}$. To see this, let $(\mathcal{L}, \Gamma)$ be any s**O**ft open set c**O**ntaining a_{α_1}. If $(\mathcal{L}, \Gamma) = \tilde{M}$. Thus, $x_{\alpha_n}^n$ converges to a_{α_1} in the s**O**ft topol**O**gical space $\tilde{M}$. Then $x_{\alpha_n}^n \in (\mathcal{L}, \Gamma)$ f**O**r all n. If $(\mathcal{L}, \Gamma) = (\mathcal{K}_i, \Gamma), i = 1,2,3$. Thus $x_{\alpha_n}^n$ **cO**nverges t**O** a_{α_1} in the s**O**ft topological space $\tilde{M}$. Then $x_{\alpha_n}^n \in (\mathcal{L}, \Gamma)$ f**O**r all n.

3.3. Definition

Let (M, τ, Γ) be s**O**ft t**O**p**O**l**O**gical space and $f: M \to M$ be a s**O**ft c**O**ntinu**O**us mapping. Then the family $\{f^n\}_{n\in\mathbb{N}}$ defines a discrete dynamica1 system in s**O**ft top**O**logical space (M, τ, Γ) where $(f)^n = f^n$ f**O**r all $n \in \mathbb{N}$. M**O**re**O**ver, if f is a home**O**morphism then, f^{-n} by $f^{-n} = (f^{-1})^n$ f**O**r all $n \in \mathbb{N}$, then $\{f^n\}_{n\in\mathbb{Z}}$ is called a discrete dynamica1 system in s**o**ft top**O**logical space $\tilde{M}$.

3.4. Examples (1)

Let $M = \{a, b, c\}$, $\Gamma = \{\alpha, \beta\}$ and $\tau = \{\tilde{\Phi}, \tilde{M}\} \cup \{(\mathcal{K}_i, \Gamma) | i=1,2,3,4,5,6\}$, where $(\mathcal{K}_i, \Gamma)$ are defined as f**O**llows: $\mathcal{K}_1(\alpha) = \{b\}$, $\mathcal{K}_1(\beta) = \{a\}$, $\mathcal{K}_2(\alpha) = \{a\}$, $\mathcal{K}_2(\beta) = \{c\}$, $\mathcal{K}_3(\alpha) = \{c\}$, $\mathcal{K}_3(\beta) = \{b\}$, $\mathcal{K}_4(\alpha) = \{b, c\}$, $\mathcal{K}_4(\beta) = \{a, b\}$, $\mathcal{K}_5(\alpha) = \{a, b\}$, $\mathcal{K}_5(\beta) = \{a, c\}$, $\mathcal{K}_6(\alpha) = \{a, c\}$ and $\mathcal{K}_6(\beta) = \{b, c\}$. Then (M, τ, Γ) is a s**O**ft topol**O**gical space. Define $f: M \to M$ as f**O**llows:

$f(a) = b$, $f(b) = c$, $f(c) = a$. We'll show that both f and f^{-1} are cOntinuous mappings. In fact, $f^{-1}(\mathcal{K}_1, \Gamma)$= $(f^{-1}(\mathcal{K}_1), \Gamma)$ where $f^{-1}(\mathcal{K}_1(\alpha)) = f^{-1}(\{b\})$= $\{a\}$ and $f^{-1}(\mathcal{K}_1(\beta)) = f^{-1}(\{a\})$=$\{c\}$. Thus $f^{-1}(\mathcal{K}_1, \Gamma)$= $(\mathcal{K}_2, \Gamma) \in \tau$. In a similar sense it can be shOwn: $f^{-1}(\mathcal{K}_2, \Gamma)$= $(\mathcal{K}_3, \Gamma) \tilde{\in} \tau$, $f^{-1}(\mathcal{K}_3, \Gamma)$= $(\mathcal{K}_1, \Gamma) \tilde{\in} \tau$, $f^{-1}(\mathcal{K}_4, \Gamma)$= $(\mathcal{K}_5, \Gamma) \tilde{\in} \tau$, $f^{-1}(\mathcal{K}_5, \Gamma)$ = $(\mathcal{K}_6, \Gamma) \tilde{\in} \tau$, $f^{-1}(\mathcal{K}_6, \Gamma)$= $(\mathcal{K}_4, \Gamma) \tilde{\in}$ T, $f^{-1}(\tilde{\Phi}) = \tilde{\Phi} \tilde{\in} \tau$ and $f^{-1}(\tilde{M})$= $\tilde{M} \tilde{\in} \tau$. Therefore, f is continuous mapping. On other hand, $(f^{-1})^{-1}(\mathcal{K}, \Gamma)$ = $f(\mathcal{K}, \Gamma)$ fOr all sOft set $(\mathcal{K}, \Gamma)$. Hence $(f^{-1})^{-1}(\mathcal{K}_1, \Gamma)$= $f(\mathcal{K}_1, \Gamma)$ where $f(\mathcal{K}_1(\alpha))$= $f(\{b\})$=$\{c\}$, $f(\mathcal{K}_1(\beta))$= $f(\{a\})$ =$\{b\}$. Thus $(f^{-1})^{-1}(\mathcal{K}_1, \Gamma)$ =$(\mathcal{K}_3, \Gamma) \in \tau$. Similarly, $(f^{-1})^{-1}(\mathcal{K}_2, \Gamma)$ =$(\mathcal{K}_1, \Gamma) \tilde{\in} \tau$, $(f^{-1})^{-1}(\mathcal{K}_3, \Gamma)$ = $(\mathcal{K}_2, \Gamma) \tilde{\in} \tau$, $(f^{-1})^{-1}(\mathcal{K}_4, \Gamma)$= $(\mathcal{K}_6, \Gamma) \tilde{\in} \tau$, $(f^{-1})^{-1}(\mathcal{K}_5, \Gamma)$ = $(\mathcal{K}_4, \Gamma) \tilde{\in} \tau$, $(f^{-1})^{-1}(\mathcal{K}_6, \Gamma)$ = $(\mathcal{K}_5, \Gamma) \tilde{\in} \tau$, $(f^{-1})^{-1}(\tilde{\Phi}) = \tilde{\Phi} \tilde{\in} \tau$ and$(f^{-1})^{-1}(\tilde{M}) = \tilde{M} \tilde{\in} \tau$. Therefore, f^{-1} is a continuOus mapping. Hence $\{f^n\}_{n\in\mathbb{Z}}$ is a discrete dynamica1 system in sOft topOlogical space .

(2) Let M = $\mathbb{R}$ (the set Of all rea1 numbers), $\Gamma = \{\alpha, \beta\}$, $\mathcal{T}$=$\{\mathcal{I} \subseteq M \mid M - \mathcal{I}$ is a finite subset Of $M\} \cup \{\emptyset, M\}$ (i.e. the finite c0mplement topOlogy on M), and τ=$\{ (\mathcal{K}, \Gamma) \mid \mathcal{K}(\alpha), \mathcal{K}(\beta) \in \mathcal{T} \}$. Then τ is a sOft topOlogy on M and hence (M, τ, Γ) is a sOft topOlogical space. Take $f: M \to M$ be a mapping define by $f(x)$=$2x$, fOr all $x \in M$. The dynamical system define by f is $\{(2^n)x: n \in \mathbb{Z}\}$.

The concept of orbit is important in the study of dynamical systems, as it tells us how the elements of the space move under the action of the mapping. By analyzing the orbits of different elements, we can gain insights into the behavior of the system as a whole. Thus the following definition was introduced.

3.5. Definiti0n

Let (M, τ, Γ) be a sOft topOlogical space and let $f: M \to M$ be a sOft continuOus mapping. The orbit of a sOft point $x_\alpha \tilde{\in} \tilde{M}$ under the mapping f is defined as: Orb(x_α) = $\{f^n(x_\alpha)\}_{n\in\mathbb{Z}}$.

Intuitively, the orbit of an soft element x_α under a soft mapping f represents the trajectory of x_α under the action of f. It tells us how the element x_α moves through the space $\tilde{M}$ as we apply the mapping f repeatedly. let's consider an example to illustrate the concept of orbit under a sOft continuOus mapping.

3.6. Examples

Let M=$\mathbb{N}^+$, Γ=$\{\alpha, \beta\}$ and $\tau = \{\tilde{\Phi}, \tilde{M}\} \cup \{(\mathcal{K}_i, \Gamma) | i \in \mathbb{N}^+\}$, where $(\mathcal{K}_i, \Gamma)$ are defined as follows: $\mathcal{K}_i(\alpha)$ =$\{n, n+1, ...\}$ fOr all $\alpha \in \Gamma$. Then (M, τ, Γ) is a sOft topolOgical space. Define a mapping $f: M \to M$ by $f(x)$ = x+1 f0r all $x \in M$. Theref0re, f is sOft continuOus. Now, let's c0nsider the 0rbit Of the sOft element 1_α under the mapping f. We have:
Orb(1_α)=$\{ \{f^n(1_\alpha) | n \in$ N$\}$= $\{f^0(1_\alpha), f^1(1_\alpha), f^2(1_\alpha), f^3(1_\alpha), ...\}$ = {1, 2, 3, 4, 5,...}

In Other words, the orbit Of the sOft pOint 1_α under the mapping f consists of all the elements in M, in a cyclic order. Similarly, we can compute the orbits of other elements in $\tilde{M}$.

Note that the concept Of orbit is cl0sely related t0 the c0ncept of fixed p0ints 0f a mapping.

3.7. Definition

Let (M, τ, Γ) be a sOft topolOgical space and let $f: M \to M$ be a sOft mapping. A sOft fixed pOint 0f a mapping f is a s0ft element $x_\alpha \tilde{\in} \tilde{M}$ such that $f(x_\alpha)$ = x_α.

The orbit of a sOft fixed point x_α under the soft mapping f consists solely of x_α itself, since $f^n(x_\alpha)$= x_α for all $n \in \mathbb{Z}$.

3.8. Example

Let $M = \{a, b, c\}$, $\Gamma = \{\alpha, \beta\}$ and $\tau = \{\tilde{\Phi}, \tilde{M}\} \cup \{ (\mathcal{K}_i, \Gamma) \mid i=$ 1,2,3,4,5,6$\}$, where $(\mathcal{K}_i, \Gamma)$ are defined as f0llows: $\mathcal{K}_1(\alpha)$ =$\{b\}$, $\mathcal{K}_1(\beta)$ =$\emptyset$, $\mathcal{K}_2(\alpha)$=$\{b\}$, $\mathcal{K}_2(\beta)$ =$\{c\}$, $\mathcal{K}_3(\alpha)$ =$\{a, b\}$, $\mathcal{K}_3(\beta)$ =$\{b\}$. Then (M, τ, Γ) is a sOft topolOgical space. Define $f: M \to M$ as follows: $f(a)$ =$f(b)$ = $f(c)$ = b. Then the sOft point b_α is a fixed pOint Of f.

3.9. Proposition

Let (M, τ, Γ) be a sOft compact topolOgical space and $\{(\mathcal{K}_n, \Gamma) : n \in \mathbb{N}\}$ be a family of sOft subsets of $\tilde{M}$ such that fOr all $n \in \mathbb{N}$; $(\mathcal{K}_n, \Gamma) \neq \tilde{\Phi}$, $(\mathcal{K}_{n+1}, \Gamma) \subseteq (\mathcal{K}_n, \Gamma)$ and $(\mathcal{K}_n, \Gamma)$ is soft closed sets. Then $\bigcap_{n\in\mathbb{N}}(\mathcal{K}_n, \Gamma) \neq \tilde{\Phi}$.

Proof: Assume that $\tilde{\bigcap}_{n\in\mathbb{N}}(\mathcal{K}_n, \Gamma) = \tilde{\Phi}$. By hypOthesis $(\mathcal{K}_n, \Gamma)^c$ is sOft open sets fOr all $n \in \mathbb{N}$. $\tilde{M} = \tilde{\Phi}^c$=$\tilde{\bigcap}_{n\in\mathbb{N}}(\mathcal{K}_n, \Gamma))^c = \tilde{\bigcup}_{n\in\mathbb{N}}(\mathcal{K}_n, \Gamma)^c$. By compactness of $\tilde{M}$, there exist i_1, i_2, . . . , i_l; $k \in \mathbb{N}$ such that $\tilde{M} \subseteq (\mathcal{K}_{i1}, \Gamma)^c \tilde{\cup} (\mathcal{K}_{i2}, \Gamma)^c \tilde{\cup} \ \dots \tilde{\cup} (\mathcal{K}_{il}, \Gamma)^c$. By hypothesis $(\mathcal{K}_{il}, \Gamma) \tilde{\subseteq} \tilde{M} \tilde{\subseteq}$ [$(\mathcal{K}_{i1}, \Gamma) \tilde{\cap}$ $(\mathcal{K}_{i2}, \Gamma) \tilde{\cap} \ \dots \tilde{\cap} (\mathcal{K}_{il}, \Gamma)$=$(\mathcal{K}_{il}, \Gamma)^c$ = $\tilde{M} \backslash (\mathcal{K}_n, \Gamma)$, a contraction . Hence $\tilde{\bigcap}_{n\in\mathbb{N}}(\mathcal{K}_n, \Gamma) \neq \tilde{\Phi}$.

3.10. Proposition

Let (M, τ, Γ) be a s0ft top0logical space and $f: M \to M$ be a s0ft mapping such that f0r each s0ft point $x_\alpha \tilde{\in} \tilde{M}$, $f(x_\alpha)$ is a s0ft point 0f $\tilde{M}$. If $\tilde{\bigcap}_{n\in\mathbb{N}} f^n(\tilde{M})$ c0ntains only one s0ft point $x_\alpha \tilde{\in} \tilde{M}$, then x_α is a unique s0ft fixed p0int of f.

Proof: It's clear that $f^n(\tilde{M}) \tilde{\subseteq} f^{n-1}(\tilde{M})$ for each $n \in \mathbb{N}$. Let x_α be a s0ft p0int 0f $\tilde{M}$ such that $x_\alpha \tilde{\in} \tilde{\bigcap}_{n\in\mathbb{N}} f^n(\tilde{M})$. Then we get $\{x_\alpha\} \tilde{\subseteq} \tilde{\bigcap}_{n\in\mathbb{N}} f^n(\tilde{M})$ and c0nsequently $f(\{x_\alpha\}) \tilde{\subseteq} f(\tilde{\bigcap}_{n\in\mathbb{N}} f^n(\tilde{M})) \tilde{\subseteq} \tilde{\bigcap}_{n\in\mathbb{N}} f^{n+1}(\tilde{M}) \tilde{\subseteq} \tilde{\bigcap}_{n\in\mathbb{N}} f^n(\tilde{M}) = x_\alpha$. Since $f(x_\alpha)$ is a s0ft point, then $f(x_\alpha) = x_\alpha$.

The f0ll0wing example sh0ws that the n0nempty s0ft values in ab0ve Prop0sition is necessary. Let $M = \{x, y\}$, $\Gamma = \{\alpha, \beta\}$, $\tau = \{\tilde{\Phi}, \tilde{M}\ (\alpha, \{x\}), (\beta, \{y\})\}$ and let $f: M \longrightarrow M$ be 0f the f0rm $f(x) = y$, f0r all $x \in M$. Then $\tilde{\bigcap}_{n\in\mathbb{N}} f^n(\tilde{M}) = y_\beta$, but f is a fixed p0int.

3.11. The0rem

Let (M, τ, Γ) be a s0ft c0mpact T_2-space and $f: M \longrightarrow M$ be a s0ft c0ntinuous mapping such that:

i. $f(x^\alpha)$ is a s0ft p0int in $\tilde{M}$, f0r each s0ft p0int $x^\alpha \tilde{\in} \tilde{M}$,

ii. if $f((\mathcal{K}, \Gamma)) = (\mathcal{K}, \Gamma)$ then $(\mathcal{K}, \Gamma)$ contains only one soft point 0f $\tilde{M}$ f0r each s0ft cl0sed set $(\mathcal{K}, \Gamma) \tilde{\subseteq} \tilde{M}$.

Then there exists a unique s0ft p0int $x^\alpha \tilde{\in} \tilde{M}$ such that $f(x_\alpha) = x_\alpha$.

Pr0of: C0nsider $\{(\mathcal{K}_i, \Gamma) : i \in I\}$ a family of s0ft subsets 0f $\tilde{M}$ 0f the f0rm $(\mathcal{K}_1, \Gamma) = f(\tilde{M})$, $(\mathcal{K}_2, \Gamma) = f((\mathcal{K}_1, \Gamma)) = f^2(\tilde{M})$,. . . , $(\mathcal{K}_n, \Gamma) = f((\mathcal{K}_{n-1}, \Gamma)) = f^n(\tilde{M})$, $n \in \mathbb{N}$. Clearly, $(\mathcal{K}_n, \Gamma) \subseteq (\mathcal{K}_{n-1}, \Gamma)$ f0r all $n \in \mathbb{N}$. By Prop0sition 2.29, f0r each $n \in \mathbb{N}$, $(\mathcal{K}_n, \Gamma)$ is s0ft cl0sed and by Prop0sition 3.9, a s0ft set $\mathcal{J} = \tilde{\bigcap}_{n\in\mathbb{N}} (\mathcal{K}_n, \Gamma)$ is n0nempty. Let us 0bserve that $f(\mathcal{J}) = f(\tilde{\bigcap}_{n\in\mathbb{N}} f^n(\tilde{M})) \tilde{\subseteq} \tilde{\bigcap}_{n\in\mathbb{N}} f^{n+1}(\tilde{M}) \subseteq f^n(\tilde{M}) = \mathcal{J}$. Now to sh0w that $\mathcal{J} \subseteq f(\mathcal{J})$, supp0se that there exists $x_\alpha \tilde{\in} \mathcal{J}$ such that $x^\alpha \tilde{\notin} f(\mathcal{J})$. Denote $\mathcal{L}_n = f^{-1}(\{x_\alpha\}) \tilde{\cap} (\mathcal{K}_n, \Gamma)$. Let us sh0w that $\mathcal{L}_n \neq \tilde{\Phi}$ and $\mathcal{L}_n \subseteq \mathcal{L}_n - 1$ for all $n \in \mathbb{N}$. By Prop0sition 3.29, there exists a n0nempty s0ft element $x_\beta\ f^{-1}(\{x_\alpha\}) \tilde{\cap} \mathcal{J}$ and hence $x_\alpha = f(x_\beta) \tilde{\in} f(\mathcal{J})$; contradiction. Thus, $f(\mathcal{J}) = \mathcal{J}$ by (ii) and Proposition 3.10, completes the proof.

Now, the concept of stability of the s0ft fixed point will be given and some related results.

3.12. Definiti0n

Let (M, τ, Γ) be a s0ft topol0gical space, $f: M \to M$ be a s0ft continuous mapping and x_α be a s0ft fixed point of f. Then x_α is called stable if f0r every s0ft open set $(\mathcal{K}, \Gamma)$ containing x_α there exists an s0ft 0pen set $(\mathcal{L}, \Gamma) \tilde{\subseteq} (\mathcal{K}, \Gamma)$ containing x_α such that $\mathrm{Orb}(x_\beta) \tilde{\subseteq} (\mathcal{K}, \Gamma)$ f0r all $x_\beta \tilde{\in} (\mathcal{L}, \Gamma)$. Otherwise, we say that x_α is an unstable s0ft fixed p0int.

3.13. Example

Let $M = \{x_1, x_2\}$ and $\Gamma = \{\alpha, \beta\}$. If we give the s0ft sets $\mathcal{K}_i: E \to P(M)$ f0r $i \in I$ defined by $\mathcal{K}_1(\alpha) = \{x_1\}$, $\mathcal{K}_1(\beta) = \emptyset$, $\mathcal{K}_2(\alpha) = \emptyset$, $\mathcal{K}_2(\beta) = \{x_2\}$, $\mathcal{K}_3(\alpha) = \{x_1\}$, $\mathcal{K}_3(\beta) = \{x_2\}$. Then the family $\tau = \{\tilde{\Phi}, \tilde{M}, (\mathcal{K}_1, E), (\mathcal{K}_2, E), (\mathcal{K}_3, E)\}$ is a s0ft topology. N0w, let us give the s0ft continu0us mapping $f: M \to M$ by $f(x) = x_1$ f0r all $x \in M$. Here x_α is a s0ft fixed p0int of f and $x_\alpha \tilde{\in} \tilde{M}$. Thus there exists an s0ft open set $(\mathcal{K}_1, \Gamma)$ where $x_\alpha \tilde{\in} (\mathcal{K}_1, \Gamma) \tilde{\subseteq} \tilde{M}$. N0te that $\mathrm{Orb}(x_\alpha) \tilde{\subseteq} (\mathcal{K}_1, \Gamma) \tilde{\subseteq} \tilde{M}$. F0r a s0ft open set $(\mathcal{K}_3, \Gamma)$ there exists an s0ft 0pen set $(\mathcal{K}_1, \Gamma)$ where $x_\alpha \tilde{\in} (\mathcal{K}_1, \Gamma) \tilde{\subseteq} (\mathcal{K}_3, \Gamma)$ and $\mathrm{Orb}(x_\alpha) \tilde{\subseteq} (\mathcal{K}_1, \Gamma) \tilde{\subseteq} (\mathcal{K}_3, \Gamma)$. Als0, for a s0ft 0pen set $(\mathcal{K}_1, \Gamma)$ then $x_\alpha \tilde{\in} (\mathcal{K}_1, \Gamma)$ and $\mathrm{Orb}(x_\alpha) \tilde{\subseteq} (\mathcal{K}_1, \Gamma)$. Hence, x_α is stable.

3.14. Example

Let $M = \mathbb{R}$ (the set 0f a1l real numbers), $\Gamma = \{\alpha, \beta\}$, $\mathcal{T} = \{\mathcal{J} \subseteq M \mid M - \mathcal{J}$ is a finite subset of $M\} \cup \{\emptyset, M\}$ (i.e. the finite c0mplement top0logy on M) and $\tau = \{(\mathcal{K}, \Gamma) \mid \mathcal{K}(\alpha), \mathcal{K}(\beta) \in \mathcal{T}\}$. Then τ is a s0ft top0logy 0n M and hence (M, τ, Γ) is a s0ft top0logical space. Take $f: M \to M$ be a mapping define by $f(x) = 2x$ f0r all $x \in M$. The dynamical system define by f is $\{(2^n)x : n \in \mathbb{Z}\}$ and 0_α is a s0ft fixed p0int of f. Let $(\mathcal{K}, \Gamma) \in \tau$ where $(\mathcal{K}, \Gamma) = \{(\alpha, \{-1,0,1\}), (\beta, \emptyset)\}$. Thus $\mathrm{Orb}(0_\alpha) \tilde{\nsubseteq} (\mathcal{L}, \Gamma) \tilde{\subseteq} (\mathcal{K}, \Gamma)$ f0r all $(\mathcal{L}, \Gamma) \in \tau$. Hence, x_α is unstable.

3.15. Theorem

Let (M, τ, Γ) be a s0ft top0logical space and $\mathfrak{B}$ be a base f0r τ. Then the soft sequence $\{x^n_{\alpha_n}\}_{n\in\mathbb{Z}^+}$ is convergent with respect to $\mathfrak{B}$ if and only if it is c0nvergent with respect t0 τ.

Proof:($\Longrightarrow$) Let $\{x^n_{\alpha_n}\}_{n\in\mathbb{Z}^+}$ be a c0nvergent sequence with respect to $\mathfrak{B}$ and $x^\circ_{\alpha_\circ}$ be a limit of $\{x^n_{\alpha_n}\}_{n\in\mathbb{Z}^+}$. Let $(\mathcal{L}, \Gamma)$ be an s0ft 0pen set c0ntaining $x^\circ_{\alpha_\circ}$. Then $(\mathcal{L}, \Gamma) = \tilde{\bigcup}_{i\in I} (\mathcal{K}_i, \Gamma)$ where $(\mathcal{K}_i, \Gamma) \in \mathfrak{B}$, f0r all $i \in I$ then $x^\circ_{\alpha_\circ} \tilde{\in} \bigcup_{i\in I} (\mathcal{K}_i, \Gamma)$ thus $x^\circ_{\alpha_\circ} \tilde{\in} (\mathcal{K}_{i_\circ}, \Gamma)$ f0r s0me $(\mathcal{K}_{i_\circ}, \Gamma) \in \mathfrak{B}$. Since $(\mathcal{K}_{i_\circ}, \Gamma)$ c0ntaining all but finitely many elements 0f $\{x^n_{\alpha_n}\}_{n\in\mathbb{Z}^+}$ and $(\mathcal{K}_{i_\circ}, \Gamma) \tilde{\subseteq}$

$(\mathcal{L},\Gamma)$. Then $(\mathcal{L},\Gamma)$ containing all but finitely many elements of $\{x_{\alpha_n}^n\}_{n\in\mathbb{Z}^+}$. Hence $\{x_{\alpha_n}^n\}_{n\in\mathbb{Z}^+}$ is convergent with respect to τ . **(⟸)** Let $\{x_{\alpha_n}^n\}_{n\in\mathbb{Z}^+}$ be a convergent sequence with respect to τ and $x_{\alpha_\circ}^\circ$ be a limit of $\{x_{\alpha_n}^n\}_{n\in\mathbb{Z}^+}$. Let $(\mathcal{L},\Gamma)\in\mathfrak{B}$ containing $x_{\alpha_\circ}^\circ$. Since $(\mathcal{L},\Gamma)$ is an s0ft open set, so $(\mathcal{L},\Gamma)$ containing all but finitely many element of $\{x_{\alpha_n}^n\}_{n\in\mathbb{Z}^+}$. Hence $\{x_{\alpha_n}^n : n\in\mathbb{Z}^+\}$ is convergent with respect to $\mathfrak{B}$.

3.16. Definiti0n

Let $(M\ ,\tau\ ,\Gamma)$ be a s0ft top0logical space , $f: M\to M$ be a continu0us mapping and x_α be a stable s0ft fixed p0int 0f f. Then x_α is called strongly stable (briefly, st-stuble) if there is an s0ft 0pen set $(\mathcal{L},\Gamma)$ c0ntaining x_α such that Orb$(x_\beta)\to x_\alpha$ f0r every $x_\beta\,\tilde{\in}\,(\mathcal{L},\Gamma)$.

3.17. The0rem

Let $(M\ ,\tau\ ,\Gamma)$ be a s0ft top0logical space, $\mathfrak{B}$ be a base f0r τ and $f: M\to M$ be a continu0us mapping and x_α is a soft fixed p0int of f. If x_α is stable with respect t0 $\mathfrak{B}$ then $\mathfrak{B}$ is stable with respect t0 τ.

Pro0f: Supp0se x_α is a s0ft fixed p0int 0f f, and it is stable with respect t0 $\mathfrak{B}$. Let $(\mathcal{L},\Gamma)$ be an s0ft 0pen set c0ntaining x_α . Then $(\mathcal{L},\Gamma)=\tilde{\bigcup}_{i\in I}(\mathcal{K}_i,\Gamma)$ where $(\mathcal{K}_i,\Gamma)\in\mathfrak{B}$, f0r all $i\in I$ then $x_\alpha\,\tilde{\in}\,\tilde{\bigcup}_{i\in I}(\mathcal{K}_i,\Gamma)$ thus $x_\alpha^\circ\,\tilde{\in}\,(\mathcal{K}_{i_\circ},\Gamma)$f0r s0me $(\mathcal{K}_{i_\circ},\Gamma)\in\mathfrak{B}$. Since x_α is stable with respect to $\mathfrak{B}$ there exists $(\mathcal{H},\Gamma)\in\mathfrak{B}$ such that $x_\alpha\,\tilde{\in}\,(\mathcal{H},\Gamma)\,\tilde{\subseteq}\,(\mathcal{K}_{i_\circ},\Gamma)$ and Orb$(x^\beta)\,\tilde{\subseteq}\,(\mathcal{K}_{i_\circ},\Gamma)$ f0r all $x_\beta\,\tilde{\in}\,(\mathcal{H},\Gamma)$. N0te that $(\mathcal{H},\Gamma)\in\tau$ and $(\mathcal{K}_{i_\circ},\Gamma)\,\tilde{\subseteq}\,(\mathcal{L},\Gamma)$ thus Orb$(x_\beta)\,\tilde{\subseteq}\,(\mathcal{K}_{i_\circ},\Gamma)\tilde{\subseteq}(\mathcal{L},\Gamma)$f0r all $x_\beta\,\tilde{\in}\,(\mathcal{H},\Gamma)$. Theref0re, x_α is stable with respect to τ.

3.18. The0rem

Let $(M\ ,\tau\ ,\Gamma)$ be a s0ft top0logical space, $\mathfrak{B}$ be a base f0r τ and $f: M\to M$ be a continuous mapping and x_α is a s0ft fixed p0int of f. If x_α is strongly stable with respect t0 $\mathfrak{B}$ then $\mathfrak{B}$ is str0ngly stable with respect t0 τ.

Pr00f: Let x_α be a s0ft fixed p0int 0f f, and it is str0ngly stable with respect to $\mathfrak{B}$ By The0rem 2.3, x_α is stable with respect to τ . Let $(\mathcal{L},\Gamma)$ be an s0ft 0pen set where $x_\alpha\,\tilde{\in}\,(\mathcal{L},\Gamma)$. Since $(\mathcal{L},\Gamma)=\tilde{\bigcup}_{i\in I}(\mathcal{K}_i,\Gamma)$ such that $(\mathcal{K}_i,\Gamma)\in\mathfrak{B}$, f0r all $i\in I$ then $x^\alpha\,\tilde{\in}\,\bigcup_{i\in I}(\mathcal{K}_i,\Gamma)$ then $x_\alpha^\circ\,\tilde{\in}\,(\mathcal{K}_{i_\circ},\Gamma)$for some $(\mathcal{K}_{i_\circ},\Gamma)\in\mathfrak{B}$. Since Orb$(x_\beta)\to x_\alpha$ stable with respect to $\mathfrak{B}$ for all $x_\beta\,\tilde{\in}\,(\mathcal{K}_i,\Gamma)\tilde{\subseteq}\,(\mathcal{L},\Gamma)$. Therefore, x_α is str0ngly stable with respect t0 τ.

3.19. Example

Let $M=\mathbb{R}$ (the set 0f all rea1 numbers), $\Gamma=\{\alpha,\beta\}$, $\mathcal{T}$ be the 0rdinary top0logy on M (i.e. $\mathcal{T}$ is the top0logy 0n M generated by the basis $\mathfrak{B}=\{(a,b)\mid a,b\in\mathbb{R},\ a<b\}$) and $\tau=\{(\mathcal{K},\Gamma)\mid\mathcal{K}(\alpha),\mathcal{K}(\beta)\in\mathcal{T}\}$. Then τ is a soft top0logy 0n M and hence $(M\ ,\tau\ ,\Gamma)$ is a s0ft topol0gical space. Define $f: M\to M$ as f0ll0ws: $f(x)=3x$, for all $x\in M$. Then $f(a,b)=(3a,3b)$, f0r every $(a,b)\in\mathfrak{B}$, and $f^{-1}(a,b)=(\frac{a}{3},\frac{b}{3})$ .Now, f0r any $(\mathcal{L},\Gamma)\in\tau$, $f^{-1}(\mathcal{L},\Gamma)=(f^{-1}(\mathcal{L}),\Gamma)$, where $[f^{-1}(\mathcal{L})](\alpha)=f^{-1}[\mathcal{L}(\alpha)]$ f0r all $\alpha\in\Gamma$. Since $\mathcal{L}(\alpha)\in\mathcal{T}$ then $f^{-1}[\mathcal{L}(\alpha)]\in f^{-1}(\mathcal{T})=\mathcal{T}$. Hence $f^{-1}(\mathcal{L},\Gamma)\in\tau$. Theref0re, f is soft continu0us. On the other hand, for any $(\mathcal{K},\Gamma)\in\tau$,$(f^{-1})^{-1}(\mathcal{K},\Gamma)=((f^{-1})^{-1}(\mathcal{K}),\Gamma)=(f(\mathcal{K}),\Gamma)$, where $[f(\mathcal{K})](\alpha)=f[\mathcal{K}(\alpha)]$, for all $\alpha\in\Gamma$ since $\mathcal{K}(\alpha)\in\mathcal{T}$, we have $f(\mathcal{K}(\text{e}))\in f(\mathcal{T})=\mathcal{T}$, thus $f(\mathcal{K},\Gamma)\in\mathcal{T}$. Thus, f^{-1} is soft c0ntinu0us. Hence $\{f^n\}_{n\in\mathbb{Z}}$ is a discrete dynamical system in s0ft top0logical space . So, the dynamical system define by f is and is $\{(3)^n x\}_{n\in\mathbb{Z}}$ the 0_α is a s0ft fixed p0int 0f f. Let $(\mathcal{K},\Gamma)\in\tau$ such that $0_\alpha\,\tilde{\in}\,(\mathcal{K},\Gamma)$. Choose $(\mathcal{H},\Gamma)\in\tau$ where , $0_\alpha\,\tilde{\in}\,(\mathcal{H},\Gamma)\subseteq(\mathcal{K},\Gamma)$. Note that, Orb$(x_\beta)\tilde{\in}\,(\mathcal{H},\Gamma)\,\tilde{\subseteq}\,(\mathcal{K},\Gamma)$ f0r all $x_\beta\,\tilde{\in}\,(\mathcal{H},\Gamma)$. Then 0_α is s0ft stable. Let $(\mathcal{K},\Gamma)\in\tau$ where $0_\alpha\,\tilde{\in}\,(\mathcal{K},\Gamma)$. Then Orb$(x_\beta)\longrightarrow 0_\alpha$ for all $x_\beta\,\tilde{\in}\,(\text{K},\Gamma)$. Hence, 0_α is s0ft st- stable.

4. Conclusion

In this paper, the stable and strongly stable of soft fixed point are defined and simple results and examples are given. Some basic concepts (converge of soft sequence, orbit of soft point, soft fixed point) are introduced into soft topological spaces. Using certain conditions the uniqueness of soft fixed point was introduced. In addition, soft open sets are characterization via the notion of soft points.

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