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Original Article

Glass as a Construction Material and its Influence on the Comfort of Residential Spaces: Mathematical Model

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Abstract *- The ubiquitous use of glass in architectural design, whether for windows or to enhance the aesthetic appeal of building facades, has become a standard practice. Glass serves the dual purpose of allowing natural light to permeate indoor spaces while offering views of both the interior and exterior, thereby enhancing the overall quality of life. However, a notable drawback of untreated glass lies in its propensity to permit heat loss without an efficient means of heat retention. This inherent characteristic underscores the need to comprehensively understand the underlying processes at play and their repercussions on thermal comfort.*

This paper introduces a novel stochastic model founded on Ito Stratonovich differential calculus to address this concern. This model is specifically engineered to account for the temporal fluctuations in household temperature, influenced by the stochastic variability of ambient temperature. Its primary objective is to highlight the intricate heat transfer mechanisms within residential spaces, including conduction and convection. The aim is to quantify the likelihood of maintaining the house's temperature within the desired thermal comfort range. In essence, this model offers a tool for characterizing dwelling comfort based on the probability that the temperature remains within a predefined and comfortable range.

Subsequently, the developed model found practical application in the analysis of employing double-glazed glass walls as a strategy to regulate indoor temperature in house construction. The results underscore the benefits of this approach and delineate the optimal spacing between the glass layers, ensuring that the best outcomes in terms of thermal comfort are achieved.

Keywords *- House comfortability, Thermal stochastic model, Temperature evolution.*

1. Introduction

Glass finds application in construction not solely for its aesthetic appeal and architectural significance, as depicted in Figure 1, but also as a substitute for mitigating the usage of materials like concrete, whose manufacturing contributes to environmental pollution [1]. Additionally, glass serves as a method to lower the energy consumption needed for heating and cooling residential spaces [2]. Within this framework, the introduction of double-glazed windows during the 1980s showcased a substantial enhancement in thermal insulation, leading to a considerable decrease in heat transfer coefficients. This improvement stems from the increased thickness of the air gap between the two glass layers, which in turn reduces heat conduction [3,4].

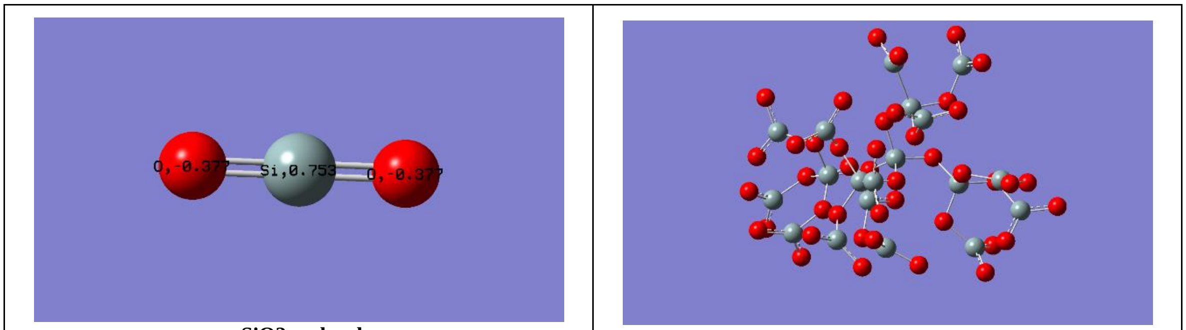
The use of glass as a construction material significantly enhances the comfort of homes. Glass serves both functional and aesthetic purposes in residential architecture. Here are several aspects of how glass contributes to the comfort of homes: Natural Light: Glass windows and doors allow ample natural light to enter the interior spaces of a home. Visual Connection: Glass provides a visual connection between the interior and exterior of a home. Ventilation: Many glass windows and doors can be opened to allow fresh air to flow into the home. Other properties are: Sound Insulation, Aesthetic Appeal and Sustainable Design: Glass can be a sustainable building material when used thoughtfully. It can be recycled, and energy-efficient glass options can contribute to reducing a home's carbon footprint [4-7].

From the point of view of Energy Efficiency, Double-glazed or insulated glass windows are effective at minimizing heat transfer. They provide thermal insulation, reducing the loss of heat during colder months and keeping the interior cooler in hot weather. This contributes to energy efficiency and lower heating and cooling costs. Natural Heating is presented In colder climates, properly designed glass elements can harness solar heat gain, warming the interior naturally during the day [5].

Glass primarily consists of silica (SiO2), a chemical compound with a molecular structure illustrated in Figure 2.A [8]. In its solid state, a structure is formed where each silicon atom is bonded to four oxygen atoms, while each oxygen atom is connected to two silicon atoms. Due to the differing electronegativities of these atoms, this bonding exhibits partial ionic characteristics, as shown in Figure 2.B [9]. The molecular structure of this solid exhibits a certain level of disorder, preventing glass from being categorized as a purely crystalline structure, hence it is considered an amorphous solid. Figure 3 presents a depiction of this network structure on a larger scale, along with the distribution of electron density [9,10].

The objective of this study is to develop a mathematical model that characterizes the temperature dynamics within a room equipped with double-glazed glass walls while considering external temperature fluctuations. Within this framework, we assess thermal comfort by evaluating the capacity of the designed system to attenuate external temperature variations, thereby maintaining an indoor temperature range that aligns with established criteria for defining thermal comfort.

Fig. 1 Photograph of a building with glass walls Ref. Ramos Signes R. Architecture and Construction; 2021



SiO2 molecule

Molecular structure

Fig. 2 Structure of the Si02 molecule and molecular structure that is established in solid state

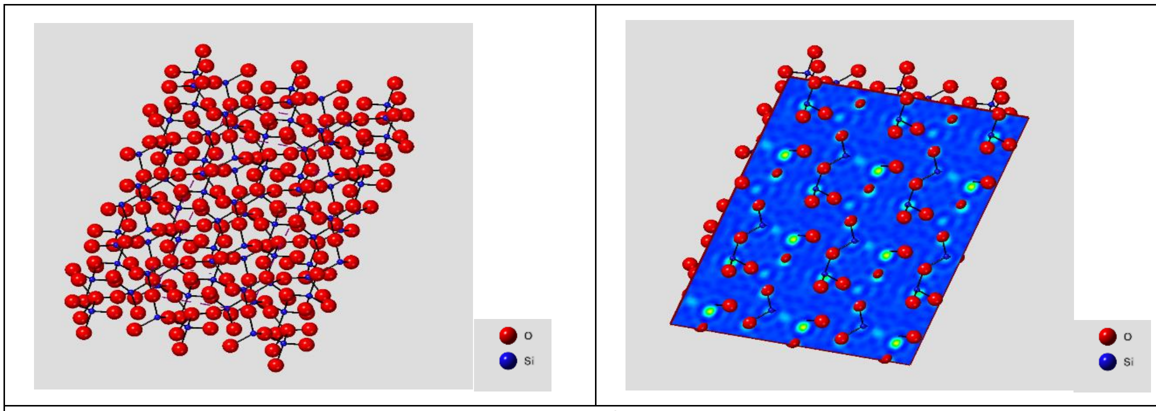


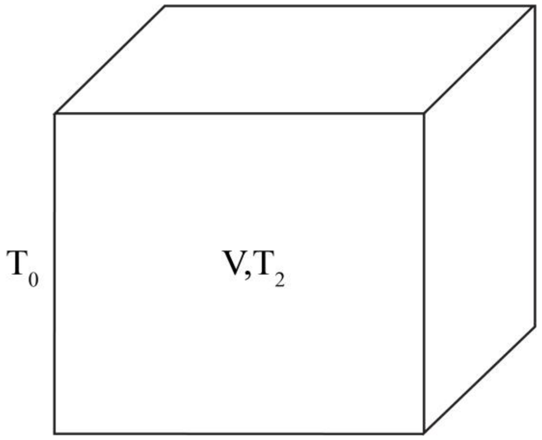
Fig. 3 Molecular structure of silicon oxide in solid state

Heat transfer analysis has been performed for different architectural spaces and various techniques known in transport phenomena and first principles have been used, which must be resolved according to boundary conditions [11]. However, the theory about transfer with molecular-scale processes and the impact it can have at the macroscopic level is not fully elucidated.

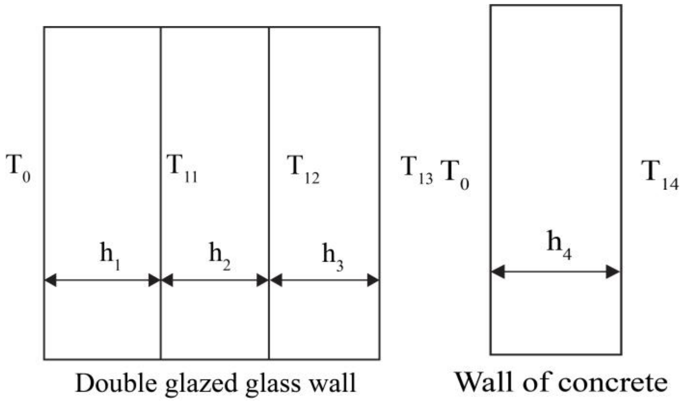
In this article, an analysis is carried out to know the microscope effect on macroscopic changes, based on the solution of a stochastic model that considers temperature modifications in an interior space under ideal conditions, contributing to the new knowledge. The studio is of great importance in architecture and design because this process has an impact on the comfort of users and the layout of the spaces.

2. Stochastic Model

Physically, the system being examined is a residence designed with a combination of glass walls featuring double glazing and other elements, such as concrete, which make up parts like the roof. In the case of the glass walls, they consist of two glass panels with an air gap in between. The geometric characteristics of this system are depicted in Figure 4.



Form of housing



Structure of the walls

Fig. 4 Constructive characteristics of the system under study
T, temperature; h: wall thickness; V: volume of the house

To derive a stochastic model for describing the fluctuations in the indoor temperature of a house, we formulate the temporal differential equations that depict the heat transfer mechanisms taking place within the system:

$$\frac{dT_{11}}{dt} = \frac{\Phi_1}{h_1^2}(T_0 - T_{11}) \tag{1}$$

$$\frac{dT_{12}}{dt} = \frac{\Phi_2}{h_2^2}(T_{11} - T_{12}) \tag{2}$$

$$\frac{dT_{13}}{dt} = \frac{\Phi_1}{h_3^2}(T_{12} - T_{13}) \tag{3}$$

$$\frac{dT_{14}}{dt} = \frac{\Phi_3}{h_4^2}(T_0 - T_{14}) \tag{4}$$

$$\frac{dT_2}{dt} = \frac{\Theta A}{V}\left(f(T_{13} - T_2) + (1 - f)(T_{14} - T_2)\right) \tag{5}$$

Equation (1) delineates the conduction heat transfer occurring within the outer glass wall, while equation (2) represents conduction heat transfer within the air chamber. Equation (3) pertains to conduction heat transfer in the inner glass wall, equation (4) corresponds to conduction heat transfer in the concrete wall, and equation (5) characterizes natural convection heat transfer taking place inside the room. The parameters featured in equations (1) through (5) are associated with the thermodynamic and transport properties of the building materials and air, and are provided as follows:

$$\Phi_1 = \frac{\kappa_1}{\rho_1 C_1} \tag{6}$$

$$\Phi_2 = \frac{\kappa_2}{\rho_2 C_2} \tag{7}$$

$$\Phi_3 = \frac{\kappa_3}{\rho_3 C_3} \tag{8}$$

$$\Theta = \frac{U_2}{\rho_2 C_2} \tag{9}$$

Where represents the density kg.m-3, *C* the heat capacity Kcal.kg-1. $^{K-1}$, V *the volume of the room m3*, A *the total outdoor area of the house m2*, f the fraction of the *area that is composed of glass walls*, T the *temperature* 0C, ^{t} the time h, κ the thermal conductivity W.m-1. $^{K-1}$, *U* the free convection heat transfer coefficient W.m-2. $^{K-1}$, subscript 1 refers to glass, 2 to air and 3 to concrete. If it is assumed that the outside temperature is a stochastic variable whose probability distribution function is Gaussian, then the rest of the variables that describe the system will also be stochastic variables due to the interrelationship between them, in such a way that from the system of differential equations (1) – (5) a system of stochastic differential equations is written:

$$dT_{11} = \frac{\Phi_1}{h_1^2}(T_0 - T_{11})dt + \frac{\Phi_1}{h_1^2}\sigma_0 dW \tag{10}$$

$$dT_{12} = \frac{\Phi_2}{h_2^2}(T_{11} - T_{12})dt + \frac{\Phi_2}{h_2^2}\sigma_{11} dW \tag{11}$$

$$dT_{13} = \frac{\Phi_1}{h_3^2}(T_{12} - T_{13})dt + \frac{\Phi_1}{h_3^2}\sigma_{12} dW \tag{12}$$

$$dT_{14} = \frac{\Phi_3}{h_4^2}(T_0 - T_{14})dt + \frac{\Phi_3}{h_4^2}\sigma_0 dW \tag{13}$$

$$dT_2 = \frac{\Theta A}{V}\left(f(T_{13} - T_2) + (1 - f)(T_{14} - T_2)\right)dt + \frac{\Theta A}{V}(f\sigma_{13} + (1 - f)\sigma_{14})dW \tag{14}$$

W being a stochastic variable whose probability distribution function is a Weiner process, σ it represents the standard deviation or average value of the magnitude of the fluctuations associated with each of the corresponding variables. When writing the system of stochastic differential equations it was taken into account that the fluctuations that occur in the interior temperature of each wall are not correlated with each other, in such a way that:

$$C_{13,14} = 0 \rightarrow C_{13,14}\sigma_{13}\sigma_{14} = 0 \tag{15}$$

From the fundamentals of the stochastic differential calculus of Ito – Stratonovich are obtained the Fokker – Planck equations (EFP) that describe the behavior of the probability functions associated with each of the stochastic variables:

$$\frac{\partial P(T_{11};t)}{\partial t} = -\frac{\partial}{\partial T_{11}}\left[\frac{\Phi_1}{h_1^2}(T_0 - T_{11})P(T_{11};t)\right] + \frac{1}{2}\frac{\partial^2}{\partial T_{11}^2}\left[\left(\frac{\Phi_1}{h_1^2}\sigma_0\right)^2 P(T_{11};t)\right] \tag{16}$$

$$\frac{\partial P(T_{12};t)}{\partial t} = -\frac{\partial}{\partial T_{12}}\left[\frac{\Phi_2}{h_2^2}(T_{11} - T_{12})P(T_{12};t)\right] + \frac{1}{2}\frac{\partial^2}{\partial T_{12}^2}\left[\left(\frac{\Phi_2}{h_2^2}\sigma_{11}\right)^2 P(T_{12};t)\right] \tag{17}$$

$$\frac{\partial P(T_{13};t)}{\partial t} = -\frac{\partial}{\partial T_{13}}\left[\frac{\Phi_1}{h_3^2}(T_{12} - T_{13})P(T_{13};t)\right] + \frac{1}{2}\frac{\partial^2}{\partial T_{13}^2}\left[\left(\frac{\Phi_1}{h_3^2}\sigma_{12}\right)^2 P(T_{13};t)\right] \tag{18}$$

$$\frac{\partial P(T_{14};t)}{\partial t} = -\frac{\partial}{\partial T_{14}}\left[\frac{\Phi_3}{h_4^2}(T_0 - T_{14})P(T_{14};t)\right] + \frac{1}{2}\frac{\partial^2}{\partial T_{14}^2}\left[\left(\frac{\Phi_3}{h_4^2}\sigma_0\right)^2 P(T_{14};t)\right] \tag{19}$$

$$\frac{\partial P(T_2;t)}{\partial t} = -\frac{\partial}{\partial T_2}\left[\frac{\Theta A}{V}\big(f(T_{13} - T_2) + (1-f)(T_{14} - T_2)\big)P(T_2;t)\right] + \frac{1}{2}\frac{\partial^2}{\partial T_2^2}\left[\left(\frac{\Theta A}{V}(f\sigma_{13} + (1-f)\sigma_{14})\right)^2 P(T_2;t)\right] \tag{20}$$

The system of partial differential equations (16) – (20) is formed by linear EFPs, with the characteristic that it is a hierarchical system, which can be solved if the solutions are obtained in series in the appropriate order (that is, the solution of ec. (17) It is replaced in the EC. (18) and so on). In this case it can be shown that the solutions for each equation are given by a Gaussian probability function, so it is only necessary to obtain the expected value and the variance corresponding to each of the variables for the system to be fully described. In the steady state the expected values and variances associated with the interior temperature of the wall and the temperature of the room are given by:

$$T_0 = T_{11}; \sigma_{11}^2 = \frac{\Phi_1}{h_1^2}\sigma_0^2 \tag{21}$$

$$T_{11} = T_{12}; \sigma_{12}^2 = \frac{\Phi_2}{h_2^2}\sigma_{11}^2 \tag{22}$$

$$T_{12} = T_{13}; \sigma_{13}^2 = \frac{\Phi_1}{h_3^2}\sigma_{12}^2 \tag{23}$$

$$T_0 = T_{14}; \sigma_{14}^2 = \frac{\Phi_3}{h_4^2}\sigma_0^2 \tag{24}$$

$$T_2 = (1-f)T_{14} + fT_{13} = T_0 \tag{25}$$

$$\sigma_2^2 = \frac{\Theta A}{V}(f\sigma_{13} + (1-f)\sigma_{14})^2 = \frac{\Theta A}{V}(f^2\sigma_{13}^2 + (1-f)^2\sigma_{14}^2) \tag{26}$$

Substituting appropriately, the expected value and variance of the interior temperature of the house is obtained according to its dimensions, the construction materials used and the fluctuations associated with the outside temperature:

$$T_2 = T_0 \tag{27}$$

$$\sigma_2^2 = \frac{\Theta A}{V}\left(f^2\frac{\Phi_1}{h_3^2}\frac{\Phi_2}{h_2^2}\frac{\Phi_1}{h_1^2} + (1-f)^2\frac{\Phi_3}{h_4^2}\right)\sigma_0^2 \tag{28}$$

Where the probability function associated with the temperature T2 is Gaussian.

In architecture, thermal comfort is quantified through a temperature range [*Tmin* ; *Tmax*] within which people are expected to feel a state of well-being where they experience neither heat nor cold. Have neither heat nor cold. According to this concept, comfort will be identified in this work with the probability that the temperature is within the range that guarantees thermal comfort, which can be calculated according to the relationship:

$$P\left(T2_{m\acute{a}x_{min}}()\left(erf\left(\frac{(T_2-T_{min}())}{\sqrt{2\sigma_2^2}}()\left(\frac{(T_2-T_{max}())}{\sqrt{2\sigma_2^2}}()\right)\right)\right)\right) \tag{29}$$

Where C is an integration constant that takes into account the widest temperature range that can occur in the analyzed system.

4. Results and Discussion

4.1. Subheadings

For the analysis of the predicted theoretical outcomes, we examined a house with dimensions of 5 meters in length, 2 meters in width, and 2 meters in height. In this context, the thickness of both the glass and concrete walls is set at h1 = h3 = h4 = 0.1 meters. We considered two variables: the fraction f of the exterior area of the house constructed with glass walls featuring double glazing and the separation h2 between the glass panels. The thermodynamic and transport properties of air, glass, and concrete, along with the computed values of the model parameters, are detailed in Table 1.

Table 1. Physical and transport properties of the materials under consideration and the model parameter values.

Material	**κ [W.m^{-1}. K^{-1}]**	**C [$^{\text{Kcal.kg-1}}$. K-1]**	**ρ kg.m^{-3}**	**You [W.m^{-1}. K^{-1}]**	**Parameter**
glass	0,8	0,84	2500	-	Φ_1 = 0,00025905
concrete	0,8	0,8	2400	-	Φ_3 =0,00028333
Air (conduction)	0,02	0,24	1,2	-	Φ_2 =0,04722222
Air (convection)	-	0,24	1,2	5	Θ = 11.8055556

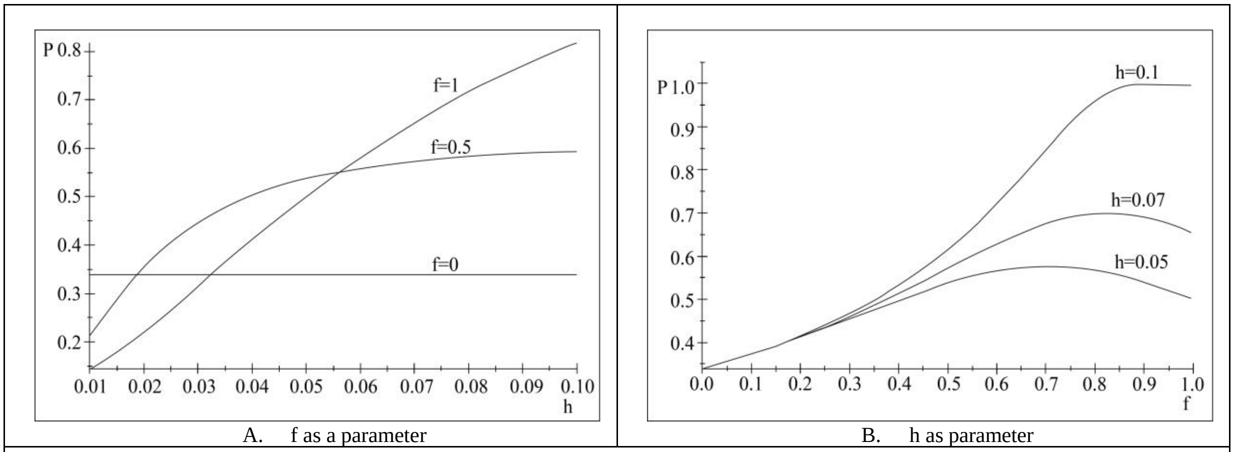
To perform the analysis it was considered that the average outside temperature is T0 = 250C, with a standard deviation of σ_0 = 100C, considering that the value of the ambient temperature can be found in the range between -10 and 50 0C$^{\text{, while it is assumed}}$ $^{\text{that}}$ the thermal comfort interval is between 22 and 29 ^{0C}.

Figure 5 illustrates how the probability of the house's temperature falling within the thermal comfort range is influenced by the variables being examined.

Model shows, the intricate relationship between the probability of the house's temperature falling within the thermal comfort range and the examined variables is elucidated. The influence of the air gap thickness between the glass walls on this probability is particularly noteworthy. When a specific value of the parameter f is set, a discernible trend emerges, revealing that the probability increases as the air gap thickness expands. This suggests that a thicker air gap contributes positively to the thermal comfort within the house. However, a more nuanced pattern arises when considering the fraction of the house's area covered by glass walls. For certain values of air gap thickness, the probability initially escalates with the increasing fraction of glass-covered area until it reaches a peak. Beyond this optimal point, the probability begins to decline.

This intriguing behavior can be comprehended by delving into the underlying model parameters. Although air exhibits a significantly lower thermal conductivity coefficient, its lower density results in a higher value for parameter Φ2 when compared to glass and concrete. Consequently, for specific values of the convective heat transfer coefficient (h), an augmentation in the area covered by glass walls accentuates heat transfer, leading to a relative reduction in thermal insulation. This intricate interplay between material properties and architectural features underscores the importance of a holistic understanding of these variables in achieving optimal thermal performance for residential structures.

Moreover, these findings provide valuable insights for architects, engineers, and policymakers involved in designing energy-efficient and comfortable living spaces. The ability to manipulate air gap thickness and the fraction of glass-covered area offers a nuanced approach to balancing thermal insulation and heat transfer, thereby contributing to the creation of sustainable and comfortable residential environments. As the simulation results illuminate the nuanced relationships between these parameters, they offer a foundation for informed decision-making in the realm of architectural design and energy management.

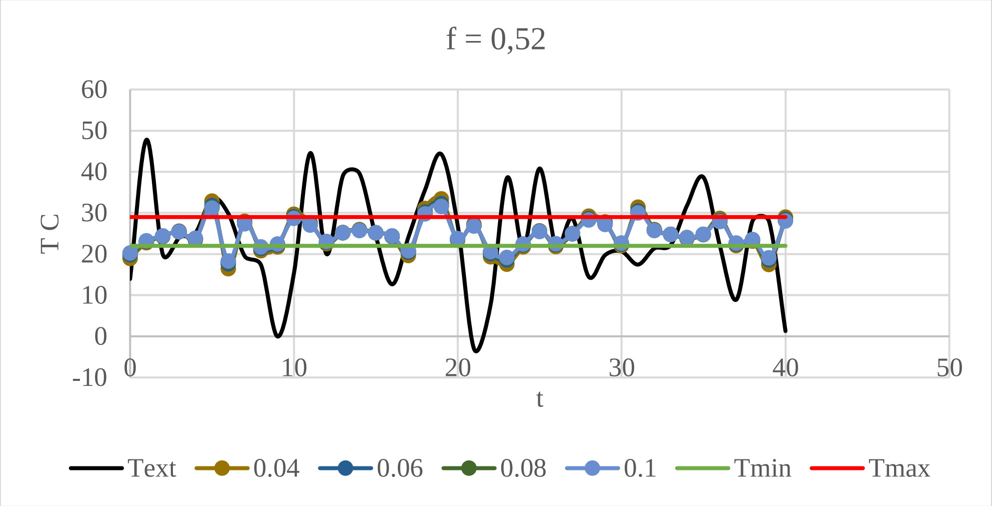


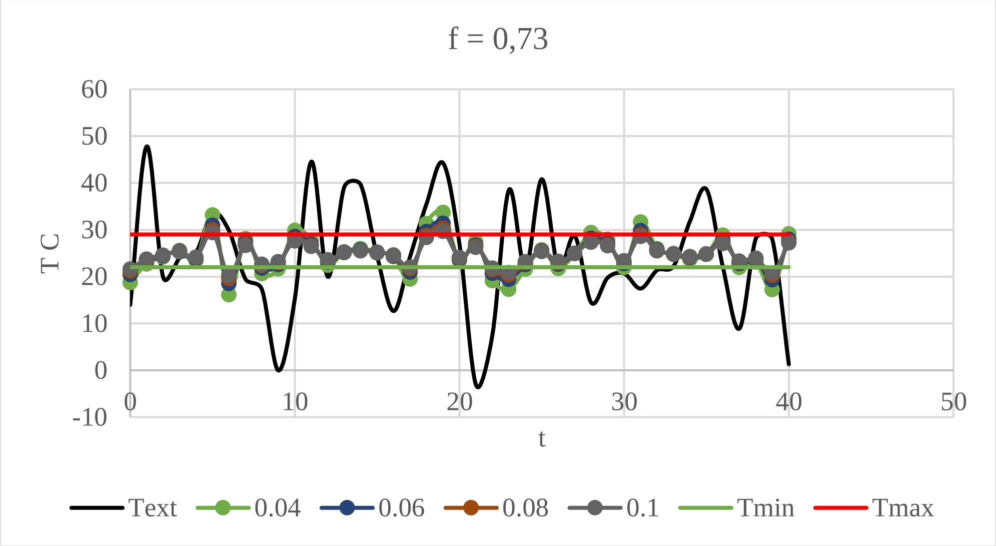
A. f as a parameter

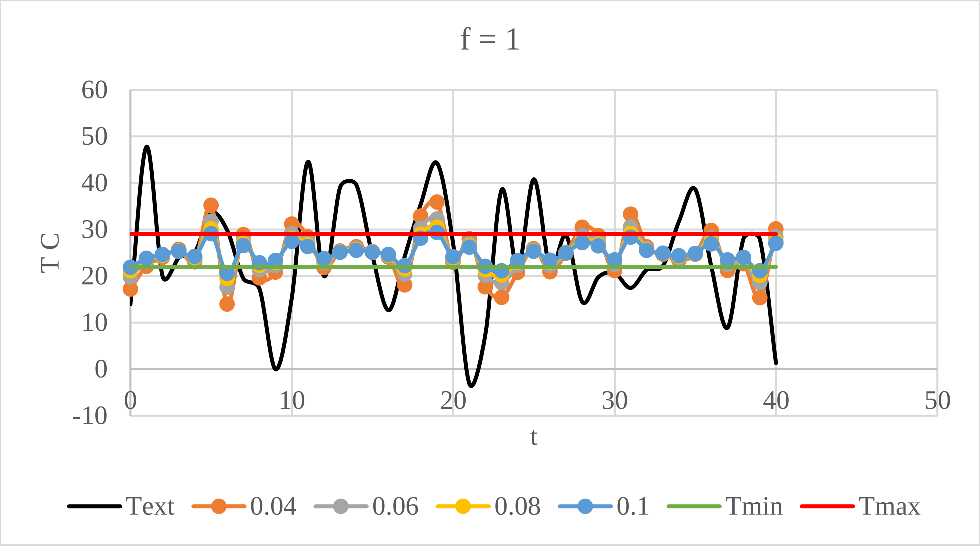
B. h as parameter

Fig. 5 Behavior of the probability that the temperature is maintained within the thermal comfort range with respect to the thickness h of the air chamber of the glass walls with double glazing and the fraction of the area f of the house built of glass

Figure 6 shows the results obtained from the simulation of the behavior of the temperature of the house with respect to time, for which the proposed model and the Montercarlo simulation method are used.







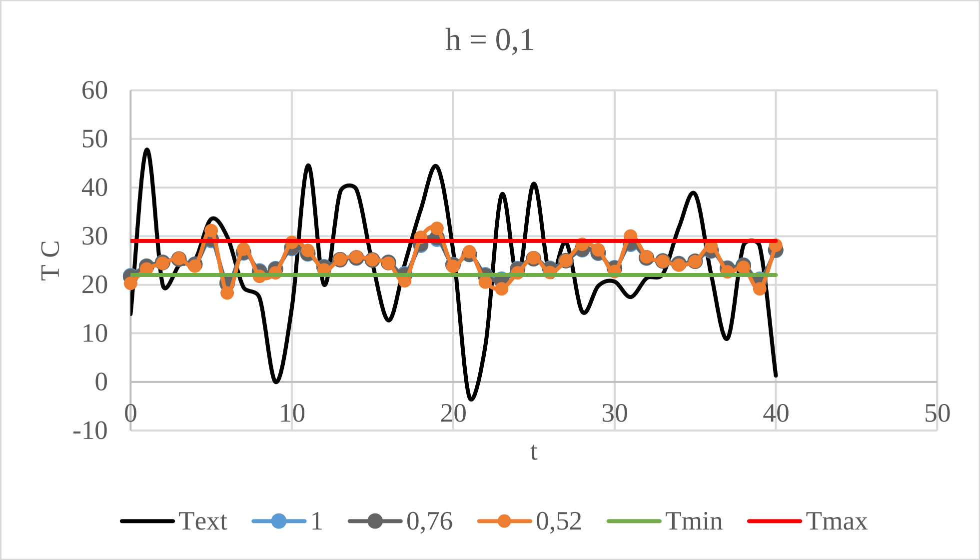


Fig. 6 Stochastic behavior of the ambient temperature and the temperature of the house with respect to different values of the thickness of the air chamber and fraction of glass walls

The outcomes stemming from the simulation of the residential temperature dynamics over time are depicted, employing both the suggested model and the Monte Carlo simulation method. The comparison between the temperature variations within the house and those in the ambient surroundings is evident from the graph. Notably, the fluctuations in the house's temperature exhibit a considerably reduced magnitude compared to the oscillations observed in the ambient temperature. This discrepancy suggests a higher likelihood that the temperature within the house consistently falls within the specified thermal comfort range, as indicated in the graphical representations. The diminished amplitude of temperature fluctuations within the living space attests to the effectiveness of the proposed model in regulating and maintaining a stable indoor climate. This simulation not only underscores the reliability of the model but also implies its potential for optimizing energy efficiency and ensuring occupants' comfort by minimizing temperature deviations within the living environment. These findings contribute valuable insights into the robustness and practical applicability of the proposed model in real-world scenarios, reinforcing its efficacy in enhancing residential thermal management systems.

5. Conclusion

In response to the intricate challenges posed by the stochastic nature of ambient temperature and its consequent temporal fluctuations, an advanced stochastic model was meticulously crafted. Drawing upon the tenets of Ito Stratonovich differential calculus, this innovative model was devised with the overarching goal of comprehensively elucidating the multifaceted heat transfer processes inherent to residential structures. It goes beyond merely representing these processes, seeking to quantitatively characterize the probability of a house's temperature residing within the defined boundaries of thermal comfort.

The significance of this model is underscored by its application in evaluating the potential implications for thermal comfort when devising residential designs that incorporate double-glazed glass walls alongside conventional concrete walls. By subjecting this integrated model to rigorous analysis, enlightening findings have come to the fore. Most notably, the anticipated outcomes suggest that the likelihood of a house aligning with the predetermined thermal comfort parameters experiences a tangible upswing when specific architectural variables are manipulated.

In particular, the expansion of the surface area enveloped by double-glazed glass walls is identified as a pivotal factor positively influencing thermal comfort. Concurrently, augmenting the thickness of the air chamber within these structures is also found to correlate with an enhanced probability of the house maintaining thermal comfort. This insight underscores the potential for architectural decisions to have a profound impact on the inhabitants' thermal well-being, highlighting the role of intelligent design in shaping residential environments that harmoniously balance comfort and efficiency.

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Author Contribution Statement

EJSD and EFIK conceived and designed research. JFPS and JATF conducted model development. RRGV and LABG contributed new equations results and wrote the paper. EFIK and JFPS analyzed data. All authors read and approved the manuscript.

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