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Original Article

Fair Doubly Connected Domination in the Corona and the Cartesian Product of Two Graphs

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Abstract - *Let G be a nontrivial connected graph. A dominating set $S \subseteq V(G)$ is called a doubly connected dominating set of G if both $\langle S\rangle$ and $\langle V(G)\backslash S\rangle$ are connected. If every distinct vertices u and v from $V(G)\backslash S$, $|N_G(u) \cap S| = |N_G(v) \cap S|$, then S is called a fair doubly connected dominating set of G. Furthermore, the fair doubly connected domination number, denoted by $\gamma_{fcc}(G)$, is the minimum cardinality of a fair doubly connected dominating set of G. A fair doubly connected dominating set of cardinality $\gamma_{fcc}(G)$ is called γ_{fcc}-set. In this paper, we characterized the fair doubly connected domination in the corona and Cartesian product of two graphs and give some important results.*

Keywords - *Dominating set, Doubly connected dominating set, Fair dominating set, Fair doubly connected dominating set.*

1. Introduction

Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [1]. Following an article [2] by Ernie Cockayne and Stephen Hedetniemi in 1977, the domination in graphs became an area of study by many researchers. A subset S of $V(G)$ is a *dominating set* of G if for every $v \in V(G)\backslash S$, there exists $x \in S$ such that $xv \in E(G)$, that is, $N[S] = V(G)$. The *domination number* $\gamma(G)$ of G is the smallest cardinality of a dominating set of G. Some studies on domination in graphs were found in the papers [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

One variant of domination is the doubly connected domination in graphs. A dominating set $S \subseteq V(G)$ is called a *doubly connected dominating set* of G if both $\langle S\rangle$ and $\langle V(G)\backslash S\rangle$ are connected. The minimum cardinality of a doubly connected dominating set of G, denoted by $\gamma_{cc}(G)$, is called the *doubly connected domination number* of G. A doubly connected dominating set of cardinality $\gamma_{cc}(G)$ is called a γ_{cc} −set of G. Doubly connected domination in graphs is found in the papers [25, 26, 27].

In 2011, Caro, Hansberg and Henning [28] introduced fair domination and k-fair domination in graphs. A dominating subset S of $V(G)$ is a *fair dominating set* in G if all the vertices not in S are dominated by the same number of vertices from S, that is, $|N(u) \cap S| = |N(v) \cap S|$ for every two distinct vertices u and v from $V(G)\backslash S$ and a subset S of $V(G)$ is a k-fair dominating set in G if for every vertex $v \in V(G)\backslash S$, $|N(v) \cap S| = k$. The minimum cardinality of a fair dominating set of G, denoted by $\gamma_{fd}(G)$, is called the *fair domination number* of G. A fair dominating set of cardinality $\gamma_{fd}(G)$ is called γ_{fd}-set. Some studies on fair domination in graphs were found in the paper [29, 30, 31, 32].

In this paper, we introduced the study of fair doubly connected domination in graphs. A dominating set $S \subseteq V(G)$ is called doubly connected dominating set of G if both $\langle S\rangle$ and $\langle V(G)\backslash S\rangle$ are connected. A doubly connected dominating set is called a fair doubly connected dominating set of G if every distinct vertices u and v from $V(G)\backslash S$, $|N_G(u) \cap S| = |N_G(v) \cap S|$. Furthermore, the fair doubly connected domination number, denoted by $\gamma_{fcc}(G)$, is the minimum cardinality of a fair doubly connected dominating set of G. A fair doubly connected dominating set of cardinality $\gamma_{fcc}(G)$ is called γ_{fcc}-set. In this paper, we characterize the fair doubly connected domination in the corona and Cartesian product of two graphs and give some important results.

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For the general terminology in graph theory, readers may refer to [33]. A *graph* G is a pair $(V(G), E(G))$, where $V(G)$ is a finite nonempty set called the *vertex-set* of G and $E(G)$ is a set of unordered pairs $\{u, v\}$ (or simply uv) of distinct elements from $V(G)$ called the *edge-set* of G. The elements of $V(G)$ are called *vertices* and the cardinality $|V(G)|$ of $V(G)$ is the *order* of G. The elements of $E(G)$ are called *edges* and the cardinality $|E(G)|$ of $E(G)$ is the *size* of G. If $|V(G)| = 1$, then G is called a trivial graph. If $E(G) = \emptyset$, then G is called an empty graph. The *open neighborhood* of a vertex $v \in V(G)$ is the set $N_G(v) = \{u \in V(G): uv \in E(G)\}$. The elements of $N_G(v)$ are called *neighbors* of v. The *closed neighborhood* of $v \in V(G)$ is the set $N_G[v] = N_G(v) \cup \{v\}$. If $X \subseteq V(G)$, the *open neighborhood* of X in G is the set $N_G(X) = \bigcup_{v \in X} N_G(v)$. The *closed neighborhood* of X in G is the set $N_G[X] = \bigcup_{v \in X} N_G[v] = N_G(X) \cup X$. When no confusion arises, $N_G[x]$ [resp. $N_G(x)$] will be denote by $N[x]$ [resp. $N(x)$].

2. Results

In the paper of Cyman et al [25], they define that for each connected graph G the set of all vertices of G is a doubly connected dominating set of G. Moreover, Caro et al. [28] mentioned that a fair dominating set $S = V(G)$ is a k-fair dominating set since vacuously every vertex in $V(G)\setminus S = \emptyset$ satisfies the desired property. Thus, following the results of Cyman et al. and Caro et al., all connected nontrivial graphs has a fair doubly connected dominating set.

Definition 2.1 *A dominating set $S \subseteq V(G)$ is called a doubly connected dominating set of G if both $\langle S \rangle$ and $\langle V(G)\setminus S \rangle$ are connected. A doubly connected dominating set is called a fair doubly connected dominating set of G if every distinct vertices u and v from $V(G)\setminus S$, $|N_G(u) \cap S| = |N_G(v) \cap S|$.*

Definition 2.2 *The corona of two graphs G and H, denoted by $G \circ H$, is the graph obtained by taking one copy of G of order n and n copies of H, and then joining the $i-th$ copy of H. For every $v \in V(G)$, we denote by H^v the copy of H whose vertices are joined or attached to the vertex v.*

Remark 2.3 *If $S = V(G \circ H)$, then S is a fair doubly connected dominating set of $G \circ H$.*

The following is the result for the fair doubly connected dominating set in the corona of one trivial graph and a connected graph.

Proposition 2.4 *Let $G = K_1$ and H be a nontrivial connected graph. Then a nonempty $S \subseteq V(G \circ H)$ is a fair doubly connected dominating set of $G \circ H$ if and only if one of the following is satisfied.*

(i) $S = V(G \circ H)$.
(ii) $S = V(G)$
(iii) S is an $|S|$-fair dominating set of H and the $\langle S \rangle$ is connected

Proof: Suppose that a nonempty $S \subseteq V(G \circ H)$ is a fair doubly connected dominating set of $G \circ H$.

Case 1. If $S = V(G \circ H)$, then we are done with statement (i).

Case 2. If $S \neq V(G \circ H)$, then $S \subset V(G \circ H)$. Consider that $S = V(G)$. Then we are done with statement (ii). Next, consider that $S \neq V(G)$. Then $S \subseteq V(H)$. If $S = V(H)$, then $\langle S \rangle$ is connected since H is connected. Let $V(G) = \{x\}$. Then $V(G \circ H)\setminus S = V(G \circ H)\setminus V(H) = V(G)$. Thus, $|N_{G \circ H}(x) \cap S| = |V(H)|$, that is, $S = V(H)$, is an $|S|$-fair dominating set of H. This shows statement (iii). If $S \neq V(H)$, then $S \subset V(H)$. Since S is a fair doubly connected dominating set of $G \circ H$, $\langle S \rangle$ is connected and $|N_{G \circ H}(x) \cap S| = |S| = |N_{G \circ H}(y) \cap S|$ for all $x, y \in V(G \circ H)\setminus S$. Since $S \subset V(H)$, S is an $|S|$-fair dominating set of H. This shows statement (iii).

For the converse, suppose that statement (i) is satisfied. Then $S = V(G \circ H)$. By Remark 2.3, S is a fair doubly connected dominating set of $G \circ H$.

Suppose that statement (ii) is satisfied. Then $S = V(G)$. Since $G = K_1$, $S = V(K_1)$, that is, $\langle S \rangle$ is trivially connected. Since

$$V(G \circ H)\setminus S = V(G \circ H)\setminus V(G) = V(H),$$

given that H is connected, it follows that $\langle V(G \circ H)\setminus S \rangle$ is connected. Thus, S is a doubly connected dominating set by definition 2.1. Further, $S \neq V(G \circ H)$, that is, $S \subset V(G \circ H)$. This implies that $V(G \circ H)\setminus S \neq \emptyset$. Since H is nontrivial, $|V(H)| \neq 1$. Let

$u, u' \in V(G \circ H)\backslash S$. Then $|N_{G\circ H}(u) \cap S| = 1 = |N_{G\circ H}(u') \cap S|$ for all $u, u' \in V(G \circ H)\backslash S$. Hence, S is a fair dominating set of $G \circ H$, that is, S is a fair doubly connected dominating set of $G \circ H$.

Suppose that statement (iii) is satisfied. Then S is an $|S|$-fair dominating set of H and the $\langle S\rangle$ is connected, that is $S \subseteq V(H)$.

$Case$ 1. If $S = V(H)$, then $\langle V(G \circ H)\backslash S\rangle = G$ is trivially connected. Since $\langle S\rangle$ is connected, it follows that S is a doubly connected dominating set of $G \circ H$. Now, S is an $|S|$-fair dominating set of H, implies that S is an $|S|$-fair dominating set of $G \circ H$. That is, S is a fair doubly connected dominating set of $G \circ H$.

$Case$ 2. If $S \neq V(H)$, then $S \subset V(H)$. Let $u \in V(H)\backslash S$ and $V(G) = \{v\}$. Then $u, v \in V(G \circ H)\backslash S$. This implies that $uv \in E(G \circ H)\backslash S$ for all $u \in V(H)\backslash S \subset V(G \circ H)\backslash S$. Thus, $\langle V(G \circ H)\backslash S\rangle$ is connected. Since $\langle S\rangle$ is connected, it follows that S is a doubly connected dominating set of $G \circ H$.

Now, S is an $|S|$-fair dominating set of H, implies that

$$|N_{G\circ H}(u) \cap S| = |N_H(u) \cap S| = |S| = |N_H(u') \cap S| = |N_{G\circ H}(u') \cap S|$$

for all $u, u' \in V(H)\backslash S \subset V(G \circ H)\backslash S$. Note that $|N_{G\circ H}(v) \cap S| = |S|$ with $v \in V(G) \subset V(G \circ H)\backslash S$. Thus, $|N_{G\circ H}(x) \cap S| = |N_{G\circ H}(y) \cap S|$ for all $x, y \in V(G \circ H)\backslash S$, that is, S is a fair dominating set of $G \circ H$. Hence, S is a fair doubly connected dominating set of $G \circ H$. ■

The next result is an immediate consequence of Proposition 2.4.

Corollary 2.5 *Let $G = K_1$ and H be a nontrivial connected graph. Then,* $\gamma_{fcc}(G \circ H) = 1$.

Proof: Let $S = V(G) = \{x\}$. Then $\langle S\rangle$ is trivially connected. Since H is connected,

$$\langle V(G \circ H)\backslash S\rangle = \langle V(G \circ H)\backslash V(G)\rangle = H$$

is connected. Thus, S is a doubly connected dominating set of $G \circ H$. Let $u, u' \in V(G \circ H)\backslash S$. Then $u, u' \in V(H)$. Thus,

$$|N_{G\circ H}(u) \cap S| = |S| = |N_{G\circ H}(u') \cap S|$$

for all $u, u' \in V(G \circ H)\backslash S$, that is, S is a fair dominating set of $G \circ H$. Hence, $1 \leq \gamma_{fcc}(G \circ H) \leq |S| = 1$, that is, $\gamma_{fcc}(G \circ H) = 1$. ■

Definition 2.6 *The Cartesian product of two graphs G and H, denoted by $G\square H$, is the graph with vertex set $V(G\square H) = V(G) \times V(H)$ and edge-set $E(G\square H)$ if and only if either $u_1, u_2 \in E(G)$ and $v_1 = v_2$ or $u_1 = u_2$ and $v_1, v_2 \in E(H)$.*

Remark 2.7 *If $S = V(G\square H)$, then S is a fair doubly connected dominating set of $G\square H$.*

The following results are needed for the characterization of fair doubly connected dominating set in the Cartesian product of two graphs.

Theorem 2.8 *Let G and H be nontrivial connected graphs. Then a nonempty $S \subseteq V(GH)$ is a fair doubly connected dominating set of $G\square H$ if S' is a fair dominating set, $\langle S'\rangle$ is connected subgraph and one of the following is satisfied.*

(i) $S = S' \times V(H)$ *where* $S' \subseteq V(G)$ *and* $\langle V(G)\backslash S'\rangle$ *is connected.*
(ii) $S = V(G) \times S'$ *where* $S' \subseteq V(H)$ *and* $\langle V(H)\backslash S'\rangle$ *is connected.*

Proof: Suppose statement (i) is satisfied. Then $S = S' \times V(H)$ where $S' \subseteq V(G)$ and $\langle V(G)\backslash S'\rangle$ is connected. Further, S' is a fair dominating set of G, and $\langle S'\rangle$ is connected subgraph of G. Then

$$V(G\square H)\backslash S = \big(V(G) \times V(H)\big)\backslash(S' \times V(H))$$
$$= (V(G)\backslash S') \times V(H), \text{ since } S' \subseteq V(G).$$

If $S' = V(G)$, then $S = V(G) \times V(H) = V(G\square H)$. By Remark 2.7, S is a fair doubly connected dominating set of $G\square H$.

If $S' \neq V(G)$, then $S' \subset V(G)$ and $V(G)\backslash S' \neq \emptyset$. Since $\langle V(G)\backslash S' \rangle$ is connected and H is connected implies that $(V(G)\backslash S') \times V(H)$ is connected, it follows that $\langle V(G\Box H)\backslash S \rangle$ is connected. Similarly, since $S = S' \times V(H)$ where $\langle S' \rangle$ is connected and H is connected, it follows that $\langle S \rangle$ is connected. Further, S' is a dominating set of G implies that $S = S' \times V(H)$ is a dominating set of $G\Box H$. Hence, S is a doubly connected dominating set of $G\Box H$.

Let $(u,v),(u',v') \in V(G\Box H)\backslash S$. Then $u,u' \in V(G)\backslash S'$ and $v,v' \in V(H)$. Since S' is an $|S'|$-fair dominating set of G, it follows that $|N_G(u) \cap S'| = |N_G(u') \cap S'|$ for all $u,u' \in V(G)\backslash S'$. Thus, for all $v,v' \in V(H)$,

$$|N_{G\Box H}(u,v) \cap S| = |N_{G\Box H}(u',v') \cap S| \text{ for all } (u,v),(u'v') \in V(G\Box H)\backslash S,$$

that is, S is a fair dominating set of $G\Box H$. Accordingly, S is a fair doubly connected dominating set of $G\Box H$.

Suppose that statement (ii) is satisfied. Then $S = V(G) \times S'$ where $S' \subseteq V(H)$ and $\langle V(H)\backslash S' \rangle$ is connected. Further, S' is a fair dominating set of H, and $\langle S' \rangle$ is connected subgraph of H. Then

$$V(G\Box H)\backslash S = \big(V(G) \times V(H)\big)\backslash(V(G) \times S')$$

$$= \big(V(G) \times (V(H)\backslash S')\big), \text{ since } S' \subseteq V(H).$$

If $S' = V(H)$, then $S = V(G) \times V(H) = V(G\Box H)$. By Remark 2.7, S is a fair doubly connected dominating set of $G\Box H$.

If $S' \neq V(H)$, then $S' \subset V(H)$ and $V(H)\backslash S' \neq \emptyset$. Since G is connected and $\langle V(H)\backslash S' \rangle$ is connected implies that $V(G) \times (V(H)\backslash S')$ is connected, it follows that $\langle V(G\Box H)\backslash S \rangle$ is connected. Similarly, since G is connected and $S = V(G) \times S'$ where $\langle S' \rangle$ is connected, it follows that $\langle S \rangle$ is connected. Further, S' is a dominating set of H implies that $S = V(G) \times S'$ is a dominating set of $G\Box H$. Hence, S is a doubly connected dominating set of $G\Box H$.

Let $(u,v),(u',v') \in V(G\Box H)\backslash S$. Then $u,u' \in V(G)$ and $v,v' \in V(H)\backslash S'$. Since S' is an $|S'|$-fair dominating set of H, it follows that $|N_H(v) \cap S'| = |N_H(v') \cap S'|$ for all $v,v' \in V(H)\backslash S'$. Thus, for all $u,u' \in V(G)$,

$$|N_{G\Box H}(u,v) \cap S| = |N_{G\Box H}(u',v') \cap S| \text{ for all } (u,v),(u',v') \in V(G\Box H)\backslash S,$$

that is, S is a fair dominating set of $G\Box H$. Accordingly, S is a fair doubly connected dominating set of $G\Box H$. ■

The following result, is an immediate consequence of Theorem 2.8.

Corollary 2.9 *Let G and H be a connected graphs of order $m \geq 2$ and $n \geq 2$, respectively. If S' is a minimum fair doubly connected dominating set of G or H. Then*

$$\gamma_{fcc}(G\Box H) = \min\{|S'|n, m|S'|\}$$

Proof: Suppose that S' is a minimum fair doubly connected dominating set of G or H. If S' is a minimum fair doubly connected dominating set of G, then $S' \subseteq V(G)$ is a fair dominating set, $\langle S' \rangle$ is connected, $\langle V(G)\backslash S' \rangle$ is connected, and $S = S' \times V(H)$. By Theorem 2.8 (i), S is a fair doubly connected dominating set of $G\Box H$. This implies that $\gamma_{fcc}(G\Box H) \leq |S|$.

If $S = S' \times V(H)$ is a minimum fair doubly connected dominating set of $G\Box H$, then $\gamma_{fcc}(G\Box H) = |S| = |S' \times V(H)| = |S'| \cdot |V(H)| = |S'|n$.

If $S = S' \times V(H)$ is not a minimum fair doubly connected dominating set of $G\Box H$, then consider that S' is a minimum fair doubly connected dominating set of H, that is, $S' \subseteq V(H)$ is a fair dominating set, $\langle S' \rangle$ is connected, $\langle V(H)\backslash S' \rangle$ is connected, and $S = V(G) \times S'$. By Theorem 2.8 (ii), S is a fair doubly connected dominating set of $G\Box H$. This implies that $\gamma_{fcc}(G\Box H) \leq |S|$.

Since $S = S' \times V(H)$ is not a minimum fair doubly connected dominating set of $G\Box H$, it is clear that $S = V(G) \times S'$ is a minimum fair doubly connected dominating set of $G\Box H$. Thus, $\gamma_{fcc}(G\Box H) = |S| = |V(G) \times S'| = |V(G)| \cdot |S'| = m|S'|$. ■

The next result is a direct consequence of Corollary 2.9.

Corollary 2.10 *Let G and H be a complete graph of order $m \geq 2$ and $n \geq 2$, respectively. Then* $\gamma_{fcc}(G\square H)$ =min $\{m, n\}$.

3. Conclusion and Recommendations

In this paper, we introduced a new parameter of domination of graphs- the fair doubly connected domination in graphs. The fair doubly connected domination in the corona and Cartesian product of two graphs were characterized. The exact fair doubly connected domination number resulting from the corona and Cartesian product of two graphs were computed. This study will pave a way to new researches such bounds and other binary operations of two connected graphs. Other parameters involving fair doubly connected domination in graphs may also be explored. Finally, the characterization of a fair doubly connected domination in graphs of the lexicographic product, and its bounds are promising extension of this study.

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