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Original Article

Stochastic Model for Linear Consecutive 2-out-of-3: G System with Pre-emptive Priority Resume Repair

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Abstract *- The purpose of present exploration is to develop a stochastic model concerning consecutive 2-out-of-3: G system. The developed model involves three linearly ordered units A, B and C having persistent failure rates. The repair of recently failed unit is given priority and the system is repaired under pre-emptive priority resume repair discipline. The failure of units is modeled by exponential distribution, whereas repair of each unit follows general distribution. The differential equations of the proposed model are formulated by means of supplementary variable method. Laplace Transform tool is employed for solving the resultant equations. The system's performance metrics, including availability, reliability, mean time to failure (MTTF), as well as expected profit are appraised by considering certain specific cases. The effect of failure of each unit on system performance is also studied. The influence of failure of non-consecutive units A and C on system performance is found almost alike, however it is understood with the help of examples and graphs that the role of unit B is very crucial to system performance.*

Keywords *- Availability, Consecutive 2-out-of-3: G system, MTTF, Pre-emptive priority resume repair, Reliability.*

1. Introduction

The vigorous competition amid various industries to bring highly reliable, durable and safe products in market strongly influences customers choice. The long-lasting efficient goods are preferred by buyers and delivery of such products enhances customers satisfaction. The increased level of customer satisfaction aids in retaining customers and also rises product popularity. For profitability of industries and in order to meet customers expectation, it is indispensable to produce extremely reliable products. Today's automated technology driven systems are too sophisticated and advance and their failure free performance for definite period of time is of prime importance. The failure of safety critical systems cannot be afforded as it may lead to sever damage, injury, financial losses and security issues. The reliability of such systems like aircraft control system, communication system and power grid system need full consideration.

System reliability at any time t is expressed in terms of probability that the system will perform its necessary work satisfactorily for a desired time period t as per the fixed environmental situations. The branch of reliability engineering lends essential support to almost all engineering fields like software engineering, electrical engineering, automobile engineering, food process engineering, aircraft engineering etc. Reliability analysis is an indispensable tool for designing of the complex systems. Reliability studies are quintessential as they help industries in improving their systems performance. Such studies assist organizations in planning and developing reliability centered systems. Reliability metrics namely availability, reliability as well as MTTF are immensely important as they help in identifying the potential causes of system failure.

Over the past six decades, reliability engineering has made significant advancements. Researchers have developed many stochastic models by considering different types of failures (human error, common cause failure, critical error etc.) and repair policies. Numerous studies are available in literature on various types of systems ranging from series systems, parallel systems, standby systems to k-out-of-n: G/F structure systems. Many researchers have investigated the reliability of various systems while taking the common cause of failure into account by using the supplementary variable technique [1–9]. The standby redundancy is one of the most effective ways of improving system reliability. Several authors have appraised the reliability measures of standby systems [10-16].

The reliability models of k-out-of-n: G/F systems using supplementary variable technique have been investigated recently in [16-19]. The concept of copula has attracted many researchers working in the field of reliability analysis. It has applications in modeling two types of repairs between failed and normal states. A mathematical model was developed pertaining to

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complex system that can flop in *n*-mutually exclusive ways of total failure or because of common cause failure in [20]. They have utilized Gumbel-Hougaard family copula to model two types of repair facilities. Additionally, this copula technique was used in [13] to examine a stochastic model for standby system by incorporating waiting time to repair. Human error has been considered in this model and repair in such case is handled by copula. Recently, in [16] workers investigated the reliability of complicated repairable system comprised of two subsystems connected in series along with the controllers using Gumbel-Hougaard family copula. They have assumed that the system under consideration can fail on account of catastrophic failure too. Numerical example discussed in this study proves the efficiency of copula repair in improving system performance. In [21] researchers further used the same copula repair approach to investigate the reliability model of hybrid series-parallel system with two operators. They have considered an example and evaluated various reliability measures. The findings suggest that the failures because of manual operations are more crucial for efficient working of the system. Moreover, authors have mentioned that for improving system performance it is vital to have regular repair. Some other reliability studies involving supplementary variable technique and copula approach can be found in [22-25].

As per literature, for repairable systems the three most commonly used repair polices are 'pre-emptive priority resume repair', 'pre-emptive priority repeat repair' and 'head of line repair'. In [26], the reliability of a complex system with two subsystems operating under the pre-emptive resume repair discipline was investigated. The first subsystem consists of n units connected in series, while the second subsystem including one standby unit may fail during the course of its shelf life. The priority is given to the repair of first subsystem. Three models pertaining to standby redundant systems were further investigated by considering head-of-line and pre-emptive resume repair in [27]. Author has derived the formulae for the Laplace transform of several state probabilities, availability as well as the steady-state availability with the help of supplementary variable technique. The analytic behaviour of a complex system possessing two kinds of sub-components having imperfect switching and pre-emptive resume repair policy was also discussed in [28]. For complicated system having two components in sequence, time dependent probabilities were obtained in [29]. They have considered partial and catastrophic failures in their model and investigated the system under pre-emptive resume repair discipline. The complex system consisting of 1-out-of-2: G and 1-out-of-*n*: F arrangements was taken into consideration in [10]. The system is repaired under pre-emptive repeat repair discipline using supplementary variable technique and copula. In [30], researchers again used copula tool to develop a stochastic model for a system having three units namely super priority, priority and ordinary unit. The repair policy is pre-emptive priority resume and it is assumed that the super priority unit is never in standby mode. The similar type of system having three units along with an additional auto switch was further worked out in [31]. The both presented stochastic models in [31, 32] involve human error, copula approach and pre-emptive priority resume repair policy.

The consecutive structures are particular type of *k*-out-of-*n*: G/F systems that have significant applications in telecommunication systems and gas pipeline systems. The linear consecutive *k*-out-of-*n*: G system has a sequence of *n* units, and it performs when minimum *k* consecutive components are operational. Such systems have been considered in the studies of [33-36]. There are very few studies in literature on the stochastic models of linear consecutive *k*-out-of-*n*: G/F systems based on supplementary variable technique. Therefore, the present study is carried out to develop the stochastic model of linear consecutive 2-out-of-3: G system. The system is studied under pre-emptive priority resume repair discipline. In pre-emptive priority resume repair discipline, the repair of priority unit anticipates any other unit being repaired and once the anticipated unit is again considered for its repair, then the repair begins from the stage where it was left previously. The proposed model is studied using Laplace transform and supplementary variable technique.

2. System Description and Assumptions

Three units, designated as unit A, unit B, and unit C, are included in the system in an orderly sequence. The system operates if at minimum two consecutive units are in good operating condition. The recently failed unit is repaired at priority. The symbols used in this study are described in Table 1. The system has seven states that have been explained in Table 2 and shown in transition diagram (Figure 1).

The assumptions associated with this model are as follows:

- The considered consecutive 2-out-of-3: G system possess three distinct units.
- Units A, B and C are arranged consecutively in the system.
- At time $t = 0$, all the three units are in good operational conditions.
- Failure of one unit among unit A and unit C leads the system to the degraded operational state.
- The absence of two consecutive working units in system brings it to the completely failed non-operational state.
- Failures of all the three units are statistically independent and follows exponential distribution.
- The failure rates λ_A, λ_B and λ_C are constant.
- All the three failed units are repairable and these are attended by the single repair man.
- System repair is carried out in both the degraded operational states (S_1 / S_2) and the completely failed states.

- The system is repaired as per pre-emptive priority resume repair discipline.
- The recently failed unit in the system is given the highest priority.
- The repair mechanism is perfect and hence the repaired restored system is as good as the brand new.

Table 1. Notations

t	Time scale
s	Laplace transform variable
S_i (i= 0, 1, 2, 3, 4, 5,6,7)	Transition states for i =0, 1, 2, 3, 4, 5,6,7
A / B / C	Unit A / B / C is in workable condition
$\bar{A}_r$ / $\bar{B}_r$ / $\bar{C}_r$	Unit A / B/ C is broken down and is under repair
$\bar{A}_{wi}$ / $\bar{B}_{wi}$ / $\bar{C}_{wi}$	Repair of Unit A / B/ C is anticipated and it is waiting for repair to resume
$\lambda_A, \lambda_B, \lambda_C$	Respective Failure rates for unit A, B, and C
$\mu_A(x), \mu_B(y), \mu_C(z)$	Rate of repair of unit A, unit B and unit C, respectively
$P_{ABC}(t)/P_{\bar{A}_rBC}(t)$ / $P_{AB\bar{C}_r}(t)/$ $P_{A\bar{B}_rC}(t)/$ $P_{\bar{A}_rB\bar{C}_{wi}}(t)/$ $P_{\bar{A}_{wi}B\bar{C}_r}(t)/$ $P_{A\bar{B}_r\bar{C}_{wi}}(t)$ / $P_{\bar{A}_{wi}\bar{B}_rC}(t)$	Respective probability of system to be in state S_i (i= 0, 1, 2, 3, 4, 5,6,7) at time t
$\overline{P_{ABC}}(s)/\overline{P_{\bar{A}_rBC}}(s)$ / $\overline{P_{AB\bar{C}_r}}(s)/$ $\overline{P_{A\bar{B}_rC}}(s)/$ $\overline{P_{\bar{A}_rB\bar{C}_{wi}}}(s)/$ $\overline{P_{\bar{A}_{wi}B\bar{C}_r}}(s)/$ $\overline{P_{A\bar{B}_r\bar{C}_{wi}}}(s)$ / $\overline{P_{\bar{A}_{wi}\bar{B}_rC}}(s)$	Respective Laplace transform of Probability of system to be in state S_i S_i (i= 0, 1, 2, 3, 4, 5, 6,7) at time t
$P_{\bar{A}_rBC}(x,t)$ / $P_{AB\bar{C}_r}(z,t)/$ $P_{A\bar{B}_rC}(y,t)$	Respective Probability that the system is in state S_1 / S_2 / S_3 at an epoch t if repair time x / z / y has passed
$P_{\bar{A}_rB\bar{C}_{wi}}(x,z,t)$	Probability of system to be in state S_4 at time t with an elapsed repair time x and z for repair of unit A and unit C, respectively
$P_{\bar{A}_{wi}B\bar{C}_r}(z,x,t)$	Probability of system to be in state S_5 *at* time t with an elapsed repair time z and x for repair of unit C and unit A, respectively
$P_{A\bar{B}_r\bar{C}_{wi}}(y,z,t)$	Probability of system to be in state S_6 at time t with an elapsed repair time y and z for repair of unit B and unit C, respectively
$P_{\bar{A}_{wi}\bar{B}_rC}(y,x,t)$	Probability of system to be in state S_7 at time t with an elapsed repair time y and x for repair of unit B and unit A, respectively
$\bar{Q}_A(s)$ / $\bar{Q}_B(s)$ / $\bar{Q}_C(s)$	Laplace transform of repair rates' probability density functions $\mu_A(x), \mu_B(y)$, and $\mu_C(z)$, respectively

Table 2. Description of States

S_0	Good state in which all the three consecutive linearly ordered units A, B and C are working properly
S_1	Degraded operational state in which Unit A is broken down and is being repaired
S_2	Degraded operational state in which Unit C is broken down and is being repaired
S_3	Completely non-functional condition in which Unit B is failed and is being repaired
S_4	Completely unproductive condition in which both units C and A are not working, repair of unit C is interrupted and ceased whereas unit A is put on repair
S_5	Completely failed state in which both units A and C are not working, repair of unit A is interrupted and ceased whereas unit C is put on repair
S_6	Completely failed state in which both units C and B are not working, repair of unit C is interrupted and ceased whereas unit B is put on repair
S_7	Completely failed state in which both units A and B are not working, repair of unit A is interrupted and ceased whereas unit B is put on repair

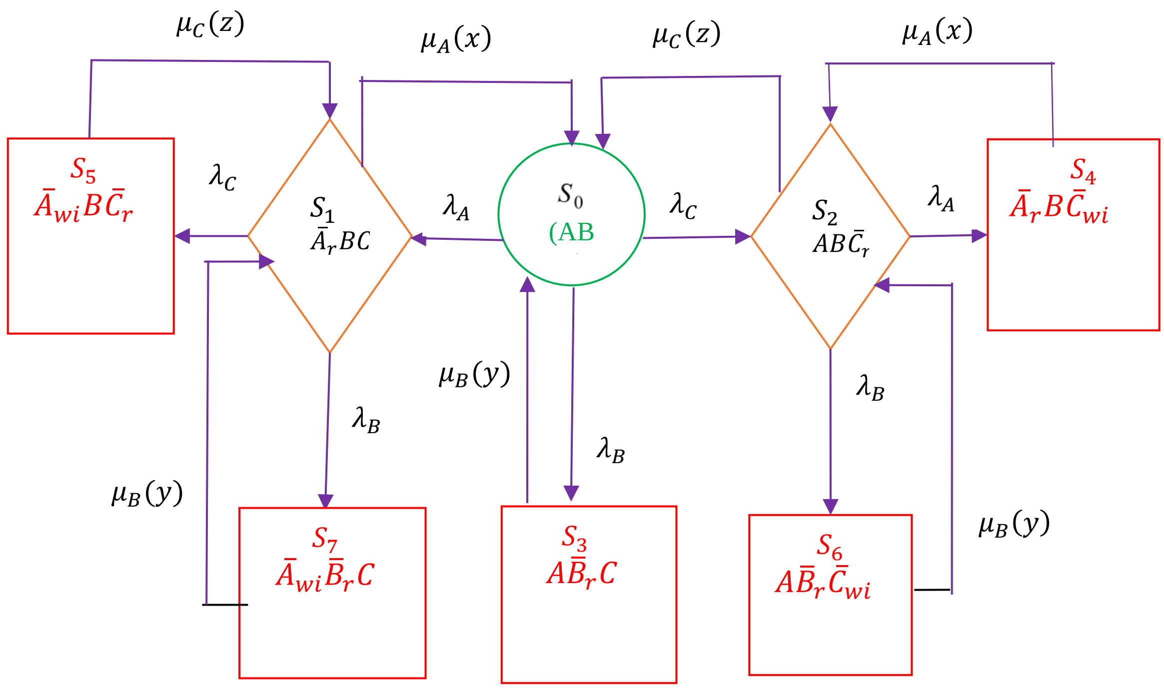

Fig. 1 Transition Diagram

3. Analysis

The supplementary variable technique has been employed to solve the current model. The model is administered by the under mentioned equations:

$$\left(\frac{\partial}{\partial t}+\lambda_A+\lambda_B+\lambda_C\right)P_{ABC}(t)=\int_0^\infty \mu_A(x)\,P_{\bar{A}_rBC}(x,t)\,dx+\int_0^\infty \mu_B(y)\,P_{A\bar{B}_rC}(y,t)\,dy\;+\int_0^\infty \mu_C(z)\,P_{AB\bar{C}_r}(z,t)dz \tag{1}$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\lambda_C+\lambda_B+\mu_A(x)\right)P_{\bar{A}_rBC}(x,t)=\int_0^\infty \mu_B(y)\,P_{\bar{A}_{wi}\bar{B}_rC}(y,x,t)dy+\;+\int_0^\infty \mu_C(z)\,P_{\bar{A}_{wi}B\bar{C}_r}(z,x,t)\,dz \tag{2}$$

$$\left(\frac{\partial}{\partial z}+\frac{\partial}{\partial t}++\lambda_A+\lambda_B+\mu_C(z)\right)P_{AB\bar{C}_r}(z,t)=\int_0^\infty \mu_A(x)\,P_{\bar{A}_rB\bar{C}_{wi}}(x,z,t)dx+\int_0^\infty \mu_B(y)\,P_{A\bar{B}_r\bar{C}_{wi}}(y,z,t)\,dy \tag{3}$$

$$\left(\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu_B(y)\right)\;P_{A\bar{B}_rC}(y,t)=0 \tag{4}$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_A(x)\right)P_{\bar{A}_rB\bar{C}_{wi}}(x,z,t)=0 \tag{5}$$

$$\left(\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\mu_C(z)\right)P_{\bar{A}_{wi}B\bar{C}_r}(z,x,t)=0 \tag{6}$$

$$\left(\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu_B(y)\right)P_{A\bar{B}_r\bar{C}_{wi}}(y,z,t)=0 \tag{7}$$

$$\left(\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu_B(y)\right)P_{\bar{A}_{wi}\bar{B}_rC}(y,x,t)=0. \tag{8}$$

The initial conditions associated with this model are as follows:

$$P_0(0)=1 \tag{9}$$

$$P_{\bar{A}_rBC}(0,t)=\lambda_A P_0(t) \tag{10}$$

$$P_{AB\bar{C}_r}(0,t)=\lambda_C P_0(t) \tag{11}$$

$$P_{A\bar{B}_rC}(0,t)=\lambda_B P_0(t) \tag{12}$$

$$P_{\bar{A}_rB\bar{C}_{wi}}(0,z,t)=\lambda_A P_{AB\bar{C}_r}(z,t) \tag{13}$$

$$P_{\bar{A}_{wi}B\bar{C}_r}(0,x,t)=\lambda_c P_{\bar{A}_rBC}(x,t) \tag{14}$$

$$P_{A\bar{B}_r\bar{C}_{wi}}(0,z,t)=\lambda_B\ P_{AB\bar{C}_r}(z,t) \tag{15}$$

$$P_{\bar{A}_{wi}\bar{B}_rC}(0,x,t)=\lambda_B P_{\bar{A}_rBC}(x,t). \tag{16}$$

Taking Laplace transform of Equation 2 and using Equation 10, we obtain

$$\overline{P_{\bar{A}_rBC}}(x,s)=\ \lambda_A\, e^{-W(s)x}\ e^{-\int_0^x \mu_A(x)dx}\,\overline{P_{ABC}}(s) \tag{17}$$

where,

$$W(s)=s+\lambda_B\left(1-\overline{Q_B}(s)\right)+\lambda_C\left(1-\overline{Q_C}(s)\right).$$

Integrating Equation 17 with respect to *x*, we get

$$\overline{P_{\bar{A}_rBC}}(s)=\lambda_A\left(\frac{1-\overline{Q_A}\left(W(s)\right)}{s}\right)\overline{P_{ABC}}(s).$$

Similarly, we have solved the Equation 3 with the help of Laplace Transform. Using the initial condition, Equation 11 we attain,

$$\overline{P_{AB\bar{C}_r}}(z,s)=\lambda_C\, e^{-V(s)z}\ e^{-\int_0^z \mu_C(z)dz}\overline{P_{ABC}}(s) \tag{18}$$

where,

$$V(s)=s+\lambda_B\left(1-\overline{Q_B}(s)\right)+\lambda_A\left(1-\overline{Q_A}(s)\right).$$

Integrating Equation 18 with respect to *z* we have,

$$\overline{P_{AB\bar{C}_r}}(s)=\lambda_C\left(\frac{1-\overline{Q_C}\left(V(s)\right)}{s}\right)\overline{P_{ABC}}(s).$$

Repeating the same process on the differential Equation 4 with initial condition given in Equation 12, we get

$$\overline{P_{A\bar{B}_rC}}(y,s) = \lambda_B\, e^{-sy}\; e^{-\int_0^y \mu_B(y)dy}\overline{P_{ABC}}(s). \tag{19}$$

Integrating above Equation 19 with respect to y we get

$$\overline{P_{A\bar{B}_rC}}(s) = \lambda_B\left(\frac{1-\overline{Q_B}(s)}{s}\right)\overline{P_{ABC}}(s).$$

The left-over differential equations are further solved by using the Laplace Transform tool. Using Equation 17, 18 and 19 corresponding to $\overline{P_{\bar{A}_rBC}}(x,s)$, $\overline{P_{AB\bar{C}_r}}(z,s)$ and $\overline{P_{A\bar{B}_rC}}(y,s)$ respectively and by assuming $\frac{1-\overline{Q_A}(s)}{s} = S_A(s)$, $\frac{1-\overline{Q_B}(s)}{s} = S_B(s)$ and $\frac{1-\overline{Q_C}(s)}{s} = S_C(s)$, we have obtained:

$$\overline{P_{ABC}}(s) = \frac{1}{H(s)}$$

$$\overline{P_{\bar{A}_rBC}}(s) = \lambda_A\, S_A\big(W(s)\big)\frac{1}{H(s)}$$

$$\overline{P_{AB\bar{C}_r}}(s) = \lambda_C\, S_C\big(V(s)\big)\frac{1}{H(s)}$$

$$\overline{P_{A\bar{B}_rC}}(s) = \lambda_B\, S_B(s)\frac{1}{H(s)}$$

$$\overline{P_{\bar{A}_rB\bar{C}_{wl}}}(s) = \lambda_A\,\lambda_C\, S_A(s)\, S_C\big(V(s)\big)\,\frac{1}{H(s)}$$

$$\overline{P_{\bar{A}_{wl}B\bar{C}_r}}(s) = \lambda_C\,\lambda_A\, S_C(s)\, S_A\big(W(s)\big)\,\frac{1}{H(s)}$$

$$\overline{P_{A\bar{B}_r\bar{C}_{wl}}}(s) = \lambda_B\,\lambda_C S_B(s)\, S_C\big(V(s)\big)\frac{1}{H(s)}$$

$$\overline{P_{\bar{A}_{wl}\bar{B}_rC}}(s) = \lambda_B\,\lambda_A S_B(s)\, S_A\big(W(s)\big)\,\frac{1}{H(s)}$$

where,

$$H(s) = s + \lambda_A(1-\overline{Q_A}(W)) + \; s\lambda_b S_B(s) + \;\lambda_C(1-\overline{Q_C}(V))\,.$$

Also, we have

$$\overline{P_{ABC}}(s) + \overline{P_{\bar{A}_rBC}}(s) + \overline{P_{AB\bar{C}_r}}(s) + \overline{P_{A\bar{B}_rC}}(s) + \overline{P_{\bar{A}_rB\bar{C}_{wl}}}(s) + \overline{P_{\bar{A}_{wl}B\bar{C}_r}}(s) + \overline{P_{A\bar{B}_r\bar{C}_{wl}}}(s) + \overline{P_{\bar{A}_{wl}\bar{B}_rC}}(s) = \frac{1}{s}.$$

The reliability measures namely availability, reliability and MTTF are appraised in the subsequent subsections by considering some particular cases. For numerical calculation, we have assumed that the repair of all the units follows exponential distribution with the constant parameters μ_A, μ_B and μ_c, respectively. Corresponding to these constant repair rates μ_A, μ_B and μ_c, we have $S_A(s) = \frac{1}{S+\mu_A}$, $S_B(s) = \frac{1}{(S+\mu_B)}$ and $S_C(s) = \frac{1}{(S+\mu_C)}$. Thus, the expressions for $W(s)$, $V(s)$ and $H(s)$ reduce to –

$$W_1(s) = s\left[1 + \frac{\lambda_B}{(s+\mu_B)} + \frac{\lambda_C}{(s+\mu_C)}\right]$$

$$V_1(s) = s\left[1 + \frac{\lambda_A}{(s+\mu_A)} + \frac{\lambda_B}{(s+\mu_B)}\right]$$

and

$$H_1(s) = s + \lambda_A \frac{W_1}{(W_1+\mu_A)} + \frac{S\,\lambda_B}{(S+\mu_B)} + \lambda_C \frac{V_1}{(V_1+\mu_C)}.$$

The Laplace transform of the probability of the system being in working state is assumed as $\overline{P_{up}}(s)$, and is given by following equation

$$\overline{P_{up}}(s) = \overline{P_{ABC}}(s) + \overline{P_{\bar{A}_r BC}}(s) + \overline{P_{AB\bar{C}_r}}(s)$$

$$= \frac{1}{H_1(s)}\left[1 + \frac{\lambda_A}{W_1+\mu_A} + \frac{\lambda_C}{V_1+\mu_C}\right]. \tag{20}$$

The Laplace transform of the probability of the system being in down state is considered as $\overline{P_{down}}(s)$ and it can be expressed as

$$\overline{P_{down}}(s) = \overline{P_{A\bar{B}_r C}}(s) + \overline{P_{\bar{A}_r B\bar{C}_{wi}}}(s) + \overline{P_{\bar{A}_{wi} B\bar{C}_r}}(s) + \overline{P_{A\bar{B}_r\bar{C}_{wi}}}(s) + \overline{P_{\bar{A}_{wi}\bar{B}_r C}}(s)$$

$$= \frac{1}{H_1(s)}\left[\frac{\lambda_B}{S+\mu_B} + \frac{\lambda_A\lambda_C}{(S+\mu_A)(V_1+\mu_C)} + \frac{\lambda_B\lambda_C}{(S+\mu_B)(V_1+\mu_C)} + \frac{\lambda_A\lambda_C}{(S+\mu_C)(W_1+\mu_A)} + \frac{\lambda_A\lambda_B}{(S+\mu_B)(W_1+\mu_A)}\right].$$

3.1. Availability and Reliability

Availability and reliability are two significant measures to assess the performance of any system. Availability is the probability that the system is working adequately at any instant *t* whereas, reliability measures the probability that the system will work up to an instant *t* without any failure. The inverse Laplace transformation of Equation 20 gives the availability of the considered system at any time *t*. The expression for the availability, $A(t)$ at time t corresponding to the parameters $\lambda_A = 0.25, \lambda_B = 0.30$, $\lambda_C = 0.50$, $\mu_A = 1$, $\mu_B = 1$ and $\mu_C = 1$ is

$$\begin{aligned} A(t) = &-0.080484286065594\, e^{-2.632882514507982\,t} + 0.0027734450864471\, e^{-2.233569762016935\,t} \\ &+ 0.342904176733480\, e^{-1.392221955062082\,t} - 0.000000000025436\, e^{-0.999999999999900\,t} \\ &+ 0.000000000000106\, t\, e^{-0.999999999999900\,t} + 0.037977809502024\, e^{-0.702005101380235\,t} \\ &+ 0.003759547812852\, e^{-0.439320667032960\,t} \\ &+ 0.693069306931014. \end{aligned} \tag{21}$$

For reliability calculation, we have assumed all repair rates involved in Equation 20 to be zero. As a result, Equation 20 reduces to

$$\bar{R}(s) = \frac{1}{(s+\lambda_A+\lambda_B+\lambda_C)}\left[1 + \frac{\lambda_A}{s+\lambda_B+\lambda_C} + \frac{\lambda_C}{s+\lambda_B+\lambda_A}\right]. \tag{22}$$

The inverse Laplace transformation of Equation 22 gives the reliability of the system at any time *t*. The expression for the reliability, $R(t)$ at time t corresponding to the parameters $\lambda_A = 0.25, \lambda_B = 0.30$ and $\lambda_C = 0.50$ is

$$R(t) = -1.000000000000068\, e^{-1.049999999999993\,t} + 1.000000000000082\, e^{-0.800000000000007\,t} + 0.999999999999988\, e^{-0.549999999999998\,t}.$$

3.2. MTTF

The limiting behaviour of $\bar{R}(s)$ in Equation 22 as s tends to zero helps in determining the MTTF of the system. The expression for the MTTF of present system is

$$\text{MTTF} = \frac{1}{(\lambda_A + \lambda_B + \lambda_C)}\left[1 + \frac{\lambda_A}{\lambda_B + \lambda_C} + \frac{\lambda_C}{\lambda_B + \lambda_A}\right]. \tag{23}$$

3.3. Expected Profit

Cost benefit analysis is an important tool in decision making process. Consequently, it is employed by industries and organizations in system designing and planning. It considers involved total costs and then subtract it from the earned revenue. This systematic approach enables us in determining the expected profit and efficiency of any system. The expected profit of any system, $E_p(t)$ over the time interval $[0\ t)$ can be evaluated by using the basic formula:

$$E_p(t) = k_1 \int_0^t P_{up}(t)dt - t\,k_2$$

where, k_1 and k_2 represent revenue and service cost of system per unit time. With the use of Equation 21 and considering parameters as $\lambda_A = 0.25, \lambda_B = 0.30$, $\lambda_C = 0.50$, $\mu_A = 1$, $\mu_B = 1$ and $\mu_C = 1$, the obtained expression for the expected profit of present system is given below.

$$\begin{aligned} E_p(t) = k_1(&0.03056888623860020\ e^{-2.632882514507982\,t} - 0.00124170963164479\,e^{-2.233569762016935\,t} \\ &- 0.24629993478172800\ e^{-1.392221955062082\,t} + 0.00000000002543600\,e^{-0.999999999999900\,t} \\ &- 0.00000000000010600\ t\,e^{-0.999999999999900\,t} - 0.00000000000010600\ e^{-0.999999999999900\,t} \\ &- 0.05409905060141960\ e^{-0.702005101380235\,t} - 0.00855763931672702\,e^{-0.439320667032960\,t} \\ &+ 0.693069306931014\,t + 0.27962944806758900) - k_2 t. \end{aligned}$$

4. Result and Discussion

Table 3. Availability and Reliability of the system

$\boldsymbol{\lambda_A = 0.25, \lambda_B = 0.30, \lambda_C = 0.50}$		
Time	**Availability**	**Reliability**
0	1	1
1	0.794045757	0.676341025
2	0.724753594	0.412311173
3	0.703935374	0.239915735
4	0.697315164	0.136569786
5	0.694947657	0.076995982
6	0.693982162	0.043276610
7	0.693541858	0.024335008
8	0.693324386	0.013714030
9	0.693211147	0.007751305
10	0.693150031	0.004394698
11	0.693116156	0.002498959
12	0.693096965	0.001424725

Availability and reliability of the system is evaluated at different points of time and presented in Table 3. Graphs shown in Figure 2 and Figure 3 exhibit that both measures decrease with the passage of time. Table 3 and availability graph given in Figure 2 reveal that after some instant i.e., $t = 6$ system availability becomes approximately constant. The difference in availability and reliability graphs (Figure 2 and Figure 3) justifies the significance of corrective maintenance of three subunits of the system. Moreover, these graphs suggest that the performance of system could be further ameliorated by considering preventive maintenance.

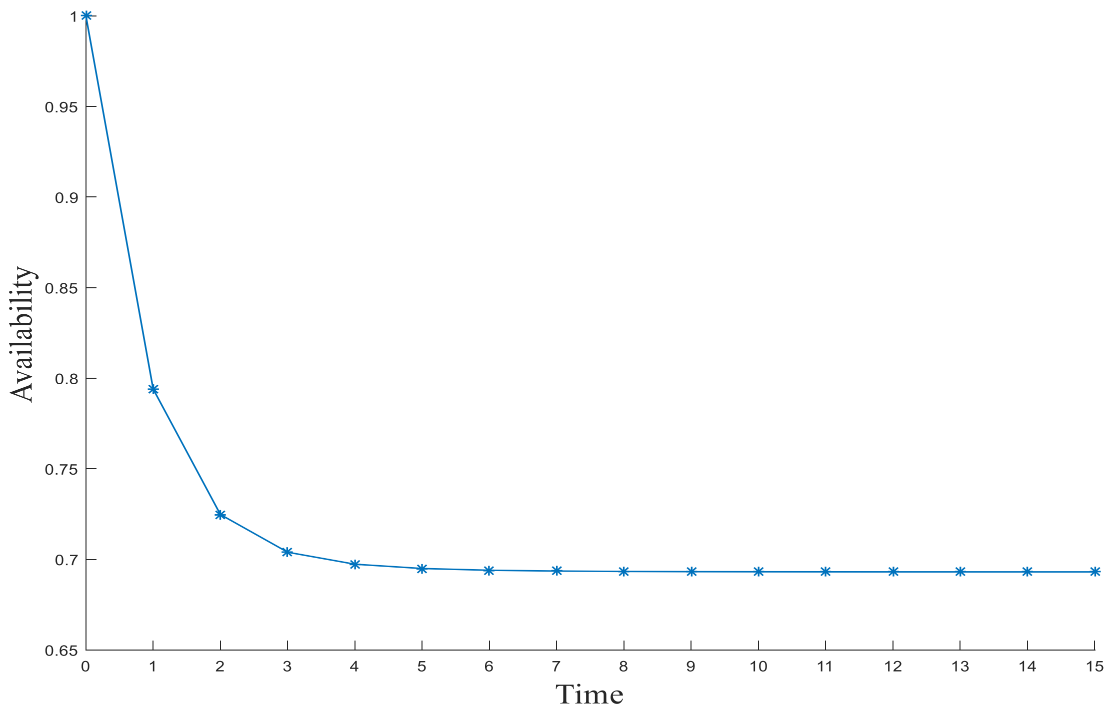

Fig. 2 Time vs Availability

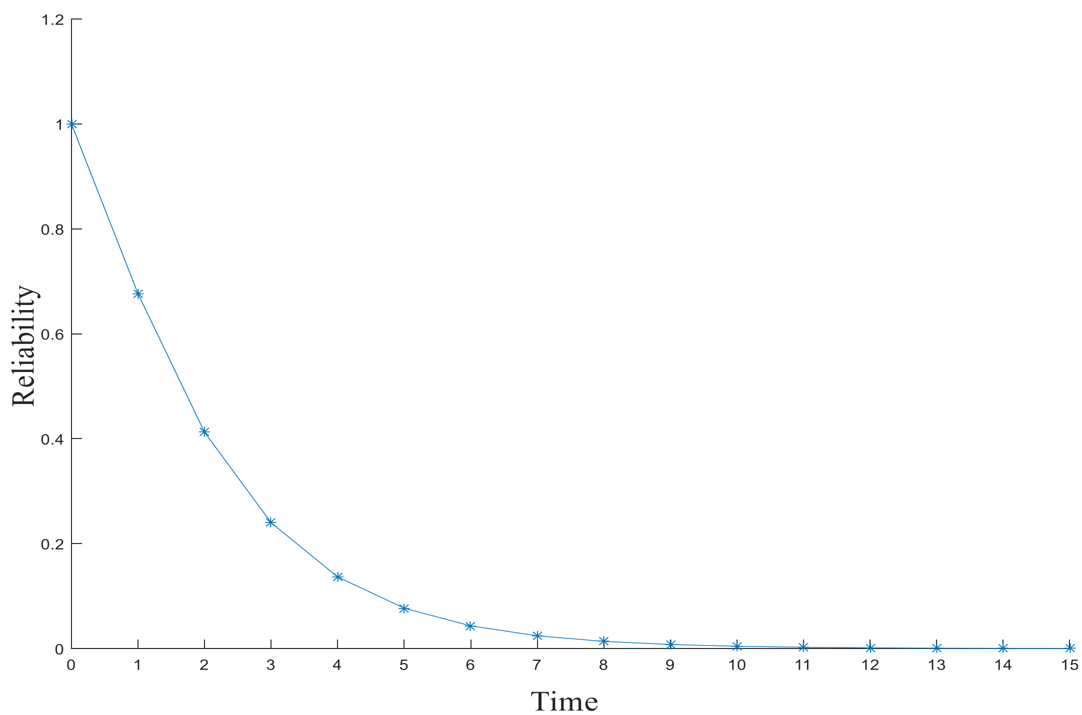

Fig. 3 Time vs Reliability

The effect of failure of each of the three units on system reliability is also examined by considering some special cases. For this we have assumed parameters as $\lambda_A = 0.25, \lambda_B = 0.25$ and $\lambda_C = 0.30$. System reliability is evaluated at certain points of time by altering hazard rate individually of each unit in series of 0.10, 0.20, 0.40, 0.60 and 0.80. The impact of failure of each unit on system performance is observed by plotting reliability graphs. The curves in Figure 4 and Figure 6 are closely located, as compared to those in Figure 5. The similarity in reliability curves of Figure 4 and Figure 6 indicates that at any specific time *t* the decrease in system reliability by increment in hazard rates of non-consecutive units A and C is almost identical. However, Figure 5 indicates that the rise in failure rate of unit B results into strong decrease in system reliability at any time *t*.

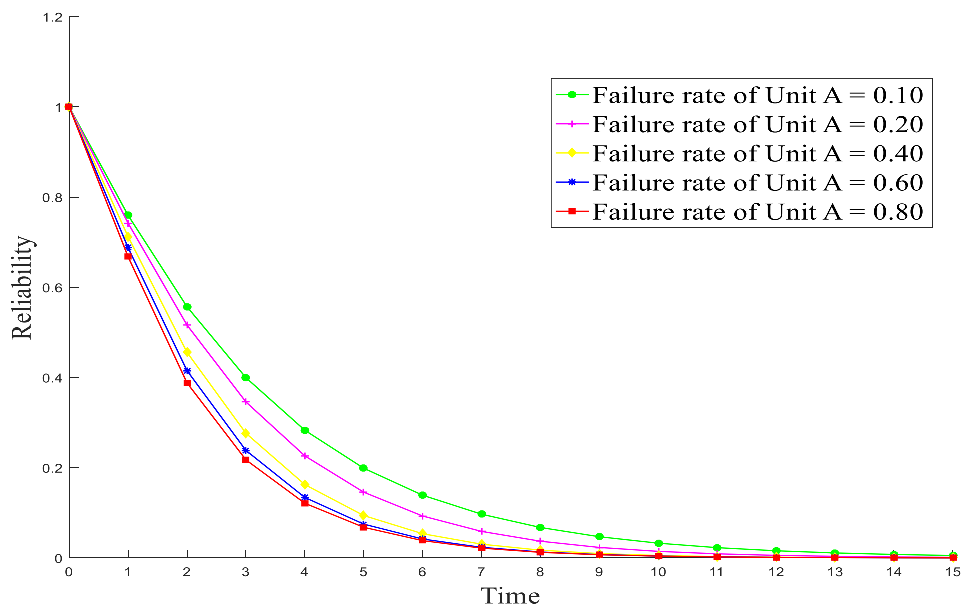

Fig. 4 Effect of increase of failure rate of unit A on reliability

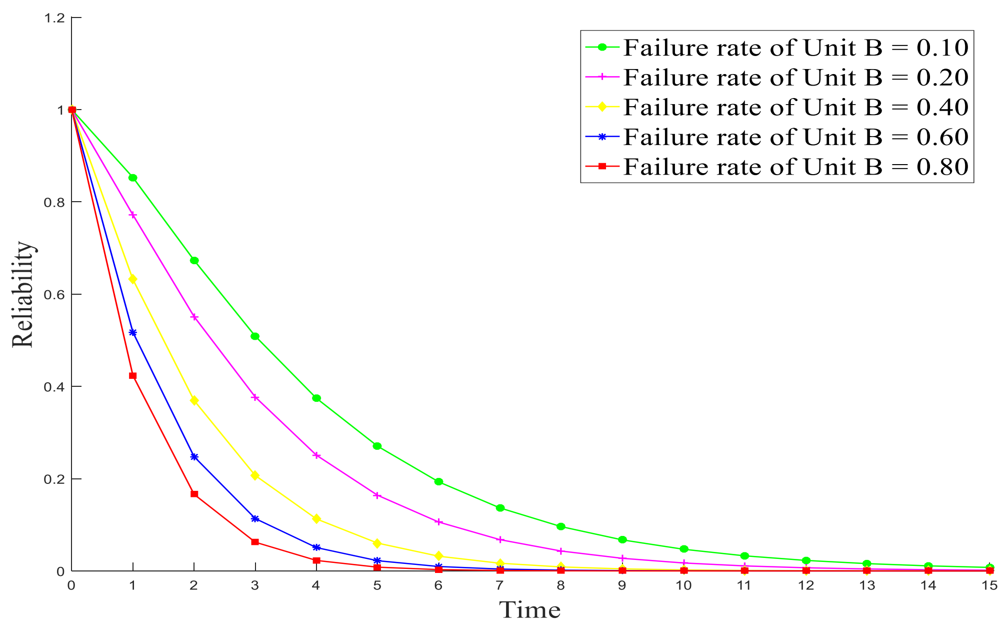

Fig. 5 Effect of increase of failure rate of unit B on reliability

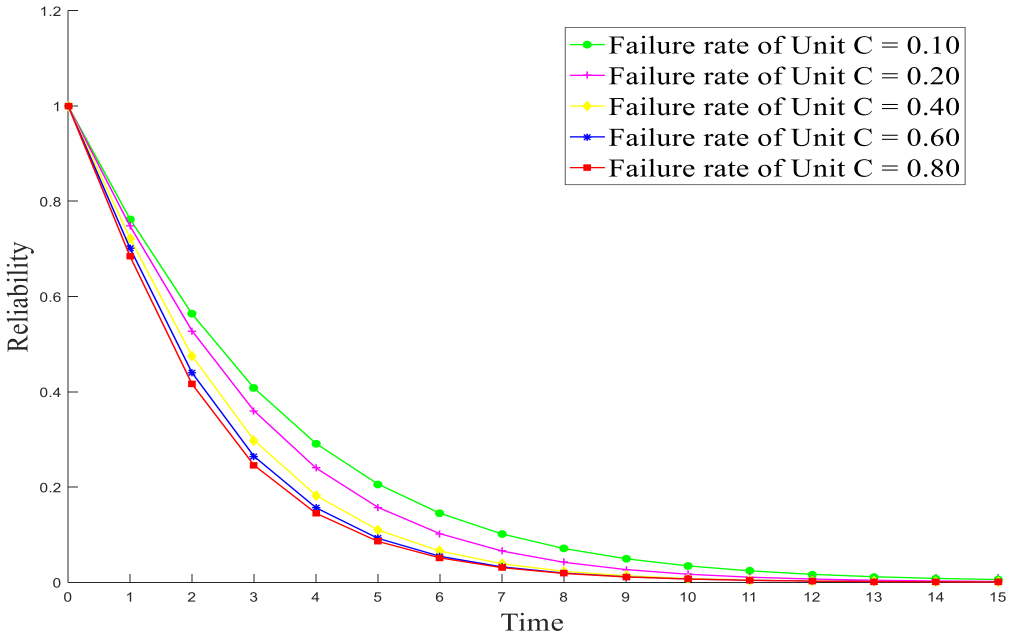

Fig. 6 Effect of increase of failure rate of unit C on reliability

Data given in Table 4 shows that at fixed instant $t = 2$, raising the failure rates of units A, B and C from 0.10 to 0.60 results in respective decrease of 0.14162883, 0.425658906 and 0.123510726 in system reliability. The observed drop in system reliability, $R(4)$ with rise in hazard rate of units A, B and C from 0.10 to 0.60 is 0.161327648, 0.395316347 and 0.143440362, respectively. Among three units, the highest decrease in $R(t)$ at any instant *t* is corresponding to the rise in failure rate of unit B.

Table 4. System reliability at *t* =2 and *t* = 3

Varying failure rate	$\lambda_A = 0.25, \lambda_B = 0.25, \lambda_C = 0.30$					
	R(2)			R(3)		
	Varying λ_A	Varying λ_B	Varying λ_C	Varying λ_A	Varying λ_B	Varying λ_C
0.10	0.55692460	0.67338248	0.56327053	0.399713586	0.508857889	0.40776902
0.20	0.51631059	0.55131894	0.52785213	0.345890945	0.376971196	0.35991399
0.40	0.45583426	0.36956014	0.47511234	0.276479659	0.206886179	0.29819871
0.60	0.41529576	0.24772357	0.43975981	0.238385938	0.113541542	0.26432866
0.80	0.38812200	0.16605408	0.41606229	0.217479661	0.06231292	0.24574037

Table 5. MTTF of the System

λ_a	λ_b	λ_c
2.638888889	3.347338936	2.77972028
2.25	2.598162072	2.484848485
2.007575758	2.115800866	2.308377897
1.845238095	1.780007432	2.194121668
1.730769231	1.533333333	2.115800866
1.646825397	1.344820757	2.059727712
1.583333333	1.19630974	2.018181818
1.534090909	1.076450431	1.986531987
1.495098039	0.977790326	1.961859979

The variation in MTTF is studied by varying failure rate of one unit at a time in Equation 23. The fixed values assigned to failure rates λ_A, λ_B and λ_C are 0.25, 0.30 and 0.50, respectively. Failure rates are varied from 0.1 to 0.9 by giving an increment of 0.1. Table 5 indicates that MTTF of the system decreases with increase in failure rate of each of the three units. The curves corresponding to unit A and Unit C in Figure 7 have similar pattern. This shows that the enhancement in the failure rates of both non-consecutive units A and C effects the system MTTF in the same manner. However, Figure 7 exhibits that an initial increase in the failure rate of unit B results in the sharp decrease in system MTTF. The comparison among three curves in Figure 7 indicates sharp decrease in MTTF with the elevating failure rate of unit B. Hence, the proper working of Unit B is very crucial for the performance of system.

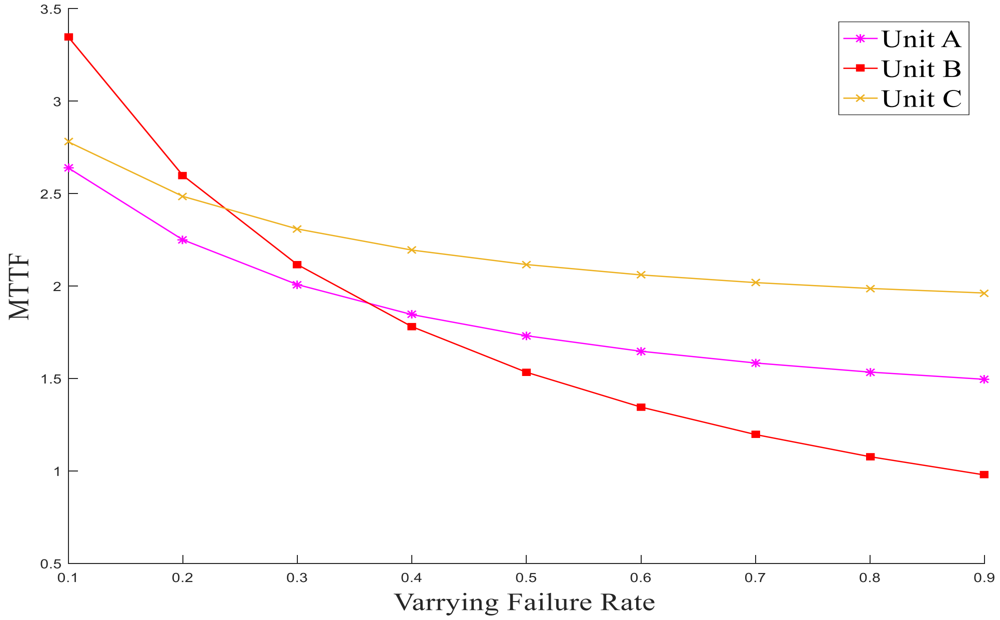

Fig. 7 Failure rate of each unit vs MTTF

Table 6. Expected Profit of the system

Time	$k_2 = 0.10$	$k_2 = 0.30$	$k_2 = 0.50$	$k_2 = 0.70$
0	0	0	0	0
1	0.7812254985	0.5812254985	0.4812254985	0.1812254985
2	1.4338577225	1.0338577225	0.8338577225	0.2338577225
3	2.0461908272	1.4461908272	1.1461908272	0.2461908272
4	2.6462279713	1.8462279713	1.4462279713	0.2462279713
5	3.2421737275	2.2421737275	1.7421737275	0.2421737275
6	3.8365725378	2.6365725378	2.0365725378	0.2365725378
7	4.4303077590	3.0303077590	2.3303077590	0.2303077590
8	5.0237287702	3.4237287702	2.6237287702	0.2237287702
9	5.6169906183	3.8169906183	2.9169906183	0.2169906183
10	6.2101681621	4.2101681621	3.2101681621	0.2101681621

The expected profit is evaluated at different points of time by taking revenue per unit time as 1. The influence of service cost on expected profit is studied by varying it from 0.10 to 0.70. Table 6 confirms that the expected profit lowers with raising the service cost.

5. Conclusion

In present study, a stochastic model is developed and three reliability metrics namely availability, reliability and MTTF of the consecutive 2-out-of-3: G system are explored. The graphical presentation pertaining to reliability confirms that it shrinks with increasing time while availability of the system initially declines and later on after certain period of time it attains the constant value. This is due to pre-emptive priority resume repair corrective maintenance policy adopted in this model. It is also observed that at any instant reliability decreases strongly with rise in hazard rate of unit B. Reliability curves corresponding to different failure rates of unit B along with MTTF graphs indicate that the improved behaviour of unit B can enhance the system efficiency. Thus, the role of unit B is very central in the system performance. The investigated model aids in improving the productivity of linear consecutive 2-out-of-3: G system. Such type of linear consecutive *k*-out-of-*n*: G/F systems are in very much demand and hence there is strong need to develop and study more reliability models. The present stochastic model can be extended by considering preventive maintenance and copula repair.

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