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Original Article

Mathematical Modelling of Bifurcation Analysis on the Effect of Random Perturbation Value on a Dynamical System: Alternative Numerical Approach

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Abstract *- Mathematical modelling of a Bifurcation analysis on the effect of random perturbation value of 0.64 on a dynamical system was investigated with the help of numerical approach of ordinary differential equation of order 45 (ODE45) and it was observed that the proposed dynamical system was purely unstable when the length of the growing season ranges from 19 days to 44 days. But when the length of the growing season increases to 49 days, Bifurcation was noticed when the length of the growing season is 54 days up to the harvesting season (99 days) and beyond. The randomization equally affects the steady-state since their values are fluctuating.*

Keywords *- Bifurcation, Dynamical system, Numerical approach, Steady-state, Random perturbation value and length of growing season.*

1. Introduction

The history of dynamical system is not complete without mentioning the effort of [1] in Newtonian mechanics. [2] now said dynamical systems theory, as it is more accurately if less spectacularly called, deals with the behaviour of mathematical objects and [3] defines a dynamical systems as a systems that are strongly associated with time with a defined rule. See also [4]. In the theory of mathematical modeling and numerical simulations, a dynamical systems can be characterized basically as ordinary differential equations, as partial differential equations, as delay differential equation and as delay-stochastic differential equation; [5], [6], [7], [8], [9], however, the linking of real life problems to ordinary differential equations, delay differential equation is not a new concept [10] and [11]. [12], defines stability as the return to equilibrium state as determines by eigenvalues of the Jacobian Matrix of the model. This definition is accepted by [13] but emphasized that the type of stability for specific steady state solutions should be tested for continuity and partial differentiability of the interacting functions that are imposed on the dynamical system. The mathematical modelling of stability analysis of a dynamical systems with continuous time delays with the help of Lambert W-function and obtain a stable system, was investigated by [14]. Though [15] investigated the stability analysis of a dynamical system using iterative algorithm techniques and found a region of stability in a delay system. Meanwhile, to investigate steady-state solution and its type of stability of the intrinsic growth rate of two interacting plant species and obtained a region of stability irrespective of the changes of the intrinsic growth rates. But [17] studied the survival of two competing species in a polluted environment with the aid of local stability analysis and the outcome revealed that the competition is affected in the presence of a toxicant. See also [18]. [19] have extended the work of [20] with the application of a differential equation system to investigate whether or not the concept of constructing a feedback control with which to stabilize an unstable steady-state is applicable to stabilize a market population system. [21], then used feedback control in constructing a controlled so that the two unstable steady-states of two interacting stock market population where stabilized which aided him to investigate the relationship between intraspecific and inter-specific competition. But [22] studied stability of a dynamical system perturbed by white noise and obtained a local stability with the help of stochastic differential equation. [23], then analyzed a dynamical model using numerical approach and obtained a local stability. [24], looked at comparism between analytical and numerical result of stability analysis of a dynamical system and obtained an unstable system with the help of ODE45 numerical techniques and see also [25]. Meanwhile, in this paper, we consider mathematical modelling of Bifurcational analysis of a dynamical system: Alternative numerical approach.

2. Mathematical Formulation

The following multi-parameter continuous first order nonlinear dynamical system was considered.

$$\frac{dx}{dt} = \propto_1 x - \beta_1 x^2 - r_1 xy - k_1 x^2 y$$

$$\frac{dy}{dt} = \propto_2 y - \beta_2 y^2 - y_2 xy - k_2 xy^2$$

Where the initial condition $x(o) = x_0 > 0$ and $y(0) = y_0 > 0$ and all parameters are assumed to be positive constants which can be any real constant.

- $x(t)$ specifies the biomass of cowpea at time t in the unit of days
- $y(t)$ specify the biomass of groundnut at time t in the unit of days
- $\propto_1$ and $\propto_2$ species the growth rate of cowpea and groundnut respectively
- β_1 and β_2 specifies the intra-competition coefficient of cowpea and groundnut respectively
- r_1 and r_2 specifies the inter-competition coefficient of groundnut and cowpea respectively in which r_1 is the contribution of the groundnut legume to inhibit the growth of the cowpea legume wheat as r_2 is the contribution of the cowpea legume to inhibit the growth of the groundnut legume
- k_1 and k_2 species the plant disease factors that inhibit the growth of the two competing legumes within an agricultural setting
- with the following precise model parameters $\propto_1 = 0.0226, \propto_2 = 0.0445,\ \beta_1 = 0.006902,\ \beta_2 = 0.133,\ r_1 = 0.0012,\ r_2 = 0.0012,\ k_1 = 0.01,\ k_2 = 0.01$, npv = random perturbation value.

3. Method of Analysis

We have fully employ the ordinary differential equation of order 45 as a numerical approach to model and predict the effect random perturbation value on the proposed dynamical system.

4. Results

On the implementation of the above mention numerical approach the following results are obtained which are presented and displayed as shown in table 1 – 22.

Table 1. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 19 days on the type of stability using ODE 45 numerical method, scenario one.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	21.199	34.7141	0.0242	-0.0343	Unstable
2	0.64	22.0938	34.6399	0.0241	-0.0343	Unstable
3	0.64	20.5000	33.4257	0.0245	-0.0313	Unstable
4	0.64	22.4135	33.5909	0.0237	-0.0301	Unstable
5	0.64	21.1366	34.3638	0.0242	-0.0305	Unstable
6	0.64	21.457	33.9385	0.0233	-0.0327	Unstable
7	0.64	21.6989	33.7489	0.024	-0.035	Unstable
8	0.64	21.5426	33.0531	0.0231	-0.034	Unstable
9	0.64	22.1791	34.6319	0.0236	-0.0317	Unstable
10	0.64	21.2753	34.4251	0.0229	-0.0325	Unstable
11	0.64	21.9367	33.7636	0.0234	-0.0313	Unstable
12	0.64	21.2063	33.2744	0.023	-0.0321	Unstable
13	0.64	21.9615	33.9799	0.0242	-0.0343	Unstable
14	0.64	21.7756	32.9885	0.0235	-0.0315	Unstable
15	0.64	21.6100	34.1642	0.0231	-0.0319	Unstable

Table 2. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 24 days on the type of stability using ODE 45 numerical method, scenario two.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	21.9055	34.1084	0.0240	-0.0335	Unstable
2	0.64	21.6679	33.7221	0.0253	-0.0324	Unstable
3	0.64	21.6522	33.9229	0.0234	-0.0341	Unstable
4	0.64	21.3556	33.8603	0.0234	-0.0319	Unstable
5	0.64	23.2684	33.5351	0.0237	-0.0335	Unstable
6	0.64	21.8287	33.6365	0.0243	-0.0312	Unstable
7	0.64	21.7572	33.0479	0.0243	-0.0312	Unstable
8	0.64	21.0204	34.1534	0.0241	-0.0314	Unstable
9	0.64	22.1307	34.0080	0.0250	-0.0329	Unstable
10	0.64	22.0269	33.7175	0.0239	-0.0332	Unstable
11	0.64	21.3426	33.3448	0.0234	-0.0337	Unstable
12	0.64	22.0970	33.8953	0.0242	-0.0325	Unstable
13	0.64	22.4135	33.9045	0.0236	-0.0325	Unstable
14	0.64	22.6357	33.7554	0.0224	-0.0350	Unstable
15	0.64	20.8894	34.3290	0.0250	-0.0318	Unstable

Table 3. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 29 days on the type of stability using ODE 45 numerical method, scenario three.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	21.7709	34.0961	0.0244	-0.0340	Unstable
2	0.64	21.5092	34.3157	0.0244	-0.0318	Unstable
3	0.64	21.9254	33.9681	0.0238	-0.0325	Unstable
4	0.64	21.8298	33.9584	0.0242	-0.0353	Unstable
5	0.64	22.1829	34.1293	0.0242	-0.0329	Unstable
6	0.64	21.6942	33.5007	0.0238	-0.0330	Unstable
7	0.64	22.1862	34.1924	0.0228	-0.0315	Unstable
8	0.64	22.2668	34.0296	0.0237	-0.0306	Unstable
9	0.64	20.5667	33.1476	0.0237	-0.0311	Unstable
10	0.64	21.6840	33.3240	0.0239	-0.0315	Unstable
11	0.64	22.5271	33.8093	0.0228	-0.0339	Unstable
12	0.64	21.8027	33.6558	0.0247	-0.0339	Unstable
13	0.64	22.1287	34.0623	0.0238	-0.0327	Unstable
14	0.64	22.0737	33.9411	0.0224	-0.0347	Unstable
15	0.64	20.9474	33.6556	0.0227	-0.0336	Unstable

Table 4: Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 34 days on the type of stability using ODE 45 numerical method, scenario four.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	22.4197	34.4920	0.0234	-0.0307	Unstable
2	0.64	21.7984	34.0143	0.0236	-0.0341	Unstable
3	0.64	22.4459	33.4174	0.0240	-0.0316	Unstable
4	0.64	22.1795	33.6141	0.0242	-0.0305	Unstable
5	0.64	22.5924	34.2370	0.0243	-0.0356	Unstable
6	0.64	21.1237	34.0796	0.0243	-0.0316	Unstable
7	0.64	22.1410	33.5590	0.0243	-0.0359	Unstable
8	0.64	22.0202	33.6090	0.0231	-0.0338	Unstable
9	0.64	22.1978	34.0535	0.0235	-0.0337	Unstable
10	0.64	21.8211	34.2838	0.0253	-0.0345	Unstable
11	0.64	21.4470	33.5558	0.0244	-0.0319	Unstable
12	0.64	20.7081	34.1251	0.0229	-0.0332	Unstable
13	0.64	21.5746	33.9748	0.0244	-0.0313	Unstable
14	0.64	22.4798	34.6869	0.0235	-0.0334	Unstable
15	0.64	22.0976	34.0169	0.0234	-0.0357	Unstable

Table 5. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 39 days on the type of stability using ODE 45 numerical method, scenario five.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	22.2108	34.2426	0.0237	-0.0336	Unstable
2	0.64	21.5518	33.5707	0.0240	-0.0349	Unstable
3	0.64	23.0092	33.9204	0.0257	-0.0341	Unstable
4	0.64	21.6037	33.3791	0.0231	-0.0341	Unstable
5	0.64	22.0925	33.8916	0.0234	-0.0345	Unstable
6	0.64	21.1610	33.9996	0.0255	-0.0353	Unstable
7	0.64	21.2779	34.5960	0.0225	-0.0350	Unstable
8	0.64	21.7355	33.7345	0.0250	-0.0324	Unstable
9	0.64	21.3215	34.8609	0.0242	-0.0314	Unstable
10	0.64	21.6739	33.5955	0.0237	-0.0331	Unstable
11	0.64	22.5316	33.9135	0.0241	-0.0301	Unstable
12	0.64	21.4931	33.6102	0.0256	-0.0320	Unstable
13	0.64	21.0718	34.2723	0.0227	-0.0317	Unstable
14	0.64	21.8067	34.1752	0.0238	-0.0319	Unstable
15	0.64	21.9873	33.3749	0.0231	-0.0330	Unstable

Table 6. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 44 days on the type of stability using ODE 45 numerical method, scenario six.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	21.8634	33.4195	0.0242	-0.033	Unstable
2	0.64	22.0711	33.4297	0.0245	-0.0328	Unstable
3	0.64	22.2466	34.5512	0.0244	-0.0310	Unstable
4	0.64	21.5972	33.4765	0.0236	-0.0345	Unstable
5	0.64	22.1346	33.3137	0.0232	-0.0344	Unstable
6	0.64	22.0864	34.7379	0.0235	-0.0318	Unstable
7	0.64	22.2232	33.5441	0.0230	-0.0341	Unstable
8	0.64	22.4999	34.3656	0.0231	-0.0325	Unstable
9	0.64	21.1818	34.0618	0.0249	-0.0306	Unstable
10	0.64	22.4039	32.5635	0.0241	-0.0364	Unstable
11	0.64	22.2463	34.3141	0.0258	-0.0306	Unstable
12	0.64	21.6138	34.4817	0.0240	-0.0343	Unstable
13	0.64	22.2403	33.7953	0.0243	-0.0355	Unstable
14	0.64	21.0279	33.4235	0.0238	-0.0341	Unstable
15	0.64	22.1971	34.0292	0.0244	-0.0310	Unstable

Table 7. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 49 days on the type of stability using ODE 45 numerical method, scenario seven.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	21.7991	33.1979	0.0246	-0.0323	Unstable
2	0.64	22.0988	33.5270	0.0244	-0.034	Unstable
3	0.64	22.3404	33.5561	0.0227	-0.0335	Unstable
4	0.64	21.4463	33.7906	0.0242	-0.0325	Unstable
5	0.64	21.5918	34.7066	0.0235	-0.0330	Unstable
6	0.64	22.0242	34.8582	0.0245	-0.0334	Unstable
7	0.64	22.5062	33.4026	0.0235	-0.0332	Unstable
8	0.64	21.7605	34.2611	0.0234	-0.0358	Unstable
9	0.64	20.8266	34.7454	0.0233	-0.0307	Unstable
10	0.64	22.1278	34.1332	0.0247	-0.0319	Unstable
11	0.64	21.4558	33.1893	0.0243	-0.0357	Unstable
12	0.64	21.5178	33.6680	0.0247	-0.0342	Unstable
13	0.64	21.5388	33.6506	0.0234	-0.0321	Unstable
14	0.64	21.5408	33.6255	0.0231	-0.0326	Unstable
15	0.64	21.7426	33.8735	0.0233	-0.0333	Unstable

Table 8. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 54 days on the type of stability using ODE 45 numerical method, scenario eight.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	21.8770	33.8163	0.0237	-0.0347	Unstable
2	0.64	20.6380	34.2772	0.0239	-0.0331	Unstable
3	0.64	22.1197	33.2685	0.0232	-0.0335	Unstable
4	0.64	22.2900	34.1257	0.0244	-0.0322	Unstable
5	0.64	22.6672	33.6703	0.0233	-0.0332	Unstable
6	0.64	21.3602	34.6379	0.0240	-0.0332	Unstable
7	0.64	22.6407	33.5346	0.0226	-0.0325	Unstable
8	0.64	21.5564	33.7559	0.0235	-0.0326	Unstable
9	0.64	21.7783	33.0913	0.0226	-0.0324	Unstable
10	0.64	21.3185	33.5544	0.0242	-0.0319	Unstable
11	0.64	22.2353	34.4778	0.0235	-0.0348	Unstable
12	0.64	22.0706	34.5156	0.0245	-0.0329	Unstable
13	0.64	21.9258	33.4531	0.0250	-0.0307	Unstable
14	0.64	21.8133	35.1858	0.0258	-0.0337	Unstable
15	0.64	21.3887	34.0930	0.0239	-0.0324	Unstable

Table 9. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 59 days on the type of stability using ODE 45 numerical method, scenario nine.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	20.9619	33.9871	0.0249	-0.0331	Unstable
2	0.64	22.2897	34.7967	0.0241	-0.0328	Unstable
3	0.64	23.1263	34.549	0.0243	-0.0328	Unstable
4	0.64	21.0556	32.8725	0.0244	-0.0317	Unstable
5	0.64	22.7557	33.9325	0.0245	-0.0332	Unstable
6	0.64	21.7114	33.9938	0.0247	-0.0322	Unstable
7	0.64	21.0055	34.3967	0.0246	-0.0342	Unstable
8	0.64	21.7755	33.6726	0.0234	-0.0321	Unstable
9	0.64	21.7210	34.5135	0.0229	-0.0327	Unstable
10	0.64	21.8297	32.8254	0.0246	-0.0348	Unstable
11	0.64	22.2850	34.4613	0.0230	-0.0315	Unstable
12	0.64	23.2305	34.3503	0.0240	-0.0330	Unstable
13	0.64	21.4010	33.7143	0.0229	-0.0345	Unstable
14	0.64	23.0219	34.5646	0.0241	-0.0330	Unstable
15	0.64	21.3820	33.5558	0.0236	-0.0343	Unstable

Table 10. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 64 days on the type of stability using ODE 45 numerical method, scenario ten.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	22.1727	34.5807	0.0244	-0.0324	Unstable
2	0.64	22.2161	33.8175	0.0241	-0.0332	Unstable
3	0.64	22.1758	33.5896	0.0238	-0.0329	Unstable
4	0.64	21.1307	33.0217	0.0237	-0.0347	Unstable
5	0.64	22.4067	33.6805	0.0251	-0.0330	Unstable
6	0.64	22.0163	34.0284	0.0231	-0.0329	Unstable
7	0.64	21.8553	33.9637	0.0221	-0.0343	Unstable
8	0.64	21.0508	33.8832	0.0228	-0.0332	Unstable
9	0.64	21.4986	33.1144	0.0253	-0.0350	Unstable
10	0.64	22.1015	32.6797	0.0241	-0.0330	Unstable
11	0.64	22.1573	34.1582	0.0238	-0.0336	Unstable
12	0.64	22.0915	33.6804	0.0239	-0.0343	Unstable
13	0.64	21.2484	34.3836	0.0241	-0.0311	Unstable
14	0.64	22.3305	33.0055	0.0232	-0.0328	Unstable
15	0.64	21.0162	34.0765	0.0241	-0.0326	Unstable

Table 11. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 69 days on the type of stability using ODE 45 numerical method, scenario eleven.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	21.7323	33.8864	0.0239	-0.0343	Unstable
2	0.64	22.2426	33.8402	0.0239	-0.0344	Unstable
3	0.64	22.5207	34.5757	0.0238	-0.0311	Unstable
4	0.64	21.9955	34.1013	0.0247	-0.0338	Unstable
5	0.64	22.0539	33.7356	0.0240	-0.0333	Unstable
6	0.64	21.9436	33.7401	0.0215	-0.0329	Unstable
7	0.64	22.1574	33.9910	0.0233	-0.0360	Unstable
8	0.64	22.4879	34.3069	0.0252	-0.0322	Unstable
9	0.64	21.1493	33.9290	0.0241	-0.0327	Unstable
10	0.64	21.6052	33.9187	0.0234	-0.0316	Unstable
11	0.64	22.5371	34.4136	0.0224	-0.0322	Unstable
12	0.64	21.8460	33.3638	0.0234	-0.0329	Unstable
13	0.64	20.7091	34.4962	0.0233	-0.0337	Unstable
14	0.64	21.6285	34.7501	0.0238	-0.0311	Unstable
15	0.64	22.2955	35.2605	0.0233	-0.0319	Unstable

Table 12. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 74 days on the type of stability using ODE 45 numerical method, scenario twelve.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	21.7036	33.4170	0.0249	-0.0327	Unstable
2	0.64	21.3531	33.3064	0.0235	-0.0322	Unstable
3	0.64	21.8726	33.9705	0.0256	-0.0355	Unstable
4	0.64	21.4514	33.9846	0.0237	-0.0319	Unstable
5	0.64	21.9712	33.8079	0.0244	-0.0324	Unstable
6	0.64	21.6672	33.9361	0.0246	-0.0314	Unstable
7	0.64	21.5066	33.7101	0.0239	-0.0335	Unstable
8	0.64	22.2427	32.7241	0.0246	-0.0326	Unstable
9	0.64	23.5331	34.1584	0.0243	-0.0316	Unstable
10	0.64	22.227	33.8635	0.0250	-0.0324	Unstable
11	0.64	22.0698	33.4508	0.0239	-0.0332	Unstable
12	0.64	21.987	33.5736	0.0240	-0.0331	Unstable
13	0.64	22.418	34.1339	0.0229	-0.0340	Unstable
14	0.64	21.6586	34.2211	0.0237	-0.0327	Unstable
15	0.64	22.4715	34.2047	0.0238	-0.0326	Unstable

Table 13. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 79 days on the type of stability using ODE 45 numerical method, scenario thirteen.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	21.1990	34.7141	0.0242	-0.0343	Unstable
2	0.64	22.0938	34.6399	0.0241	-0.0343	Unstable
3	0.64	20.5000	33.4257	0.0245	-0.0313	Unstable
4	0.64	22.4135	33.5909	0.0237	-0.0301	Unstable
5	0.64	21.1366	34.3638	0.0242	-0.0305	Unstable
6	0.64	21.4570	33.9385	0.0233	-0.0327	Unstable
7	0.64	21.6989	33.7489	0.0240	-0.0350	Unstable
8	0.64	21.5426	33.0531	0.0231	-0.0340	Unstable
9	0.64	22.1791	34.6319	0.0236	-0.0317	Unstable
10	0.64	21.2753	34.4251	0.0229	-0.0325	Unstable
11	0.64	21.9367	33.7636	0.0234	-0.0313	Unstable
12	0.64	21.2063	33.2744	0.0230	-0.0321	Unstable
13	0.64	21.9615	33.9799	0.0242	-0.0343	Unstable
14	0.64	21.7756	32.9885	0.0235	-0.0315	Unstable
15	0.64	21.6100	34.1642	0.0231	-0.0319	Unstable

Table 14. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 84 days on the type of stability using ODE 45 numerical method, scenario fourteen.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	21.1764	32.9229	0.0234	-0.0347	Unstable
2	0.64	22.3321	33.2965	0.0229	-0.0326	Unstable
3	0.64	21.8937	33.0059	0.0221	-0.0334	Unstable
4	0.64	21.5086	33.5768	0.0237	-0.0319	Unstable
5	0.64	21.9337	33.3827	0.0229	-0.0324	Unstable
6	0.64	22.0062	34.0630	0.0221	-0.0327	Unstable
7	0.64	22.7819	33.4283	0.0252	-0.0334	Unstable
8	0.64	21.9988	33.9698	0.0238	-0.0335	Unstable
9	0.64	21.0893	33.4712	0.0238	-0.0337	Unstable
10	0.64	21.8265	33.9385	0.0230	-0.0345	Unstable
11	0.64	22.4622	33.8052	0.0245	-0.0336	Unstable
12	0.64	22.1655	32.8609	0.0252	-0.0353	Unstable
13	0.64	21.8347	33.6289	0.0230	-0.0296	Unstable
14	0.64	22.2558	33.1016	0.0230	-0.0309	Unstable
15	0.64	21.4766	34.2560	0.0236	-0.0325	Unstable

Table 15. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 89 days on the type of stability using ODE 45 numerical method, scenario fifteen.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	21.9780	33.0476	0.0241	-0.0346	Unstable
2	0.64	21.8383	33.7319	0.0231	-0.0337	Unstable
3	0.64	21.2527	34.0206	0.0241	-0.0311	Unstable
4	0.64	22.1053	33.9157	0.0236	-0.0365	Unstable
5	0.64	21.8723	33.5910	0.0236	-0.0338	Unstable
6	0.64	21.8846	32.6106	0.0236	-0.0360	Unstable
7	0.64	22.3998	33.9713	0.0236	-0.0323	Unstable
8	0.64	21.3133	33.7276	0.0240	-0.0324	Unstable
9	0.64	22.9111	33.9446	0.0247	-0.0349	Unstable
10	0.64	21.8054	33.9813	0.0239	-0.0324	Unstable
11	0.64	22.2509	34.2613	0.0254	-0.0337	Unstable
12	0.64	22.0025	33.3419	0.0236	-0.0317	Unstable
13	0.64	22.0736	34.1884	0.0240	-0.0302	Unstable
14	0.64	21.5947	34.5104	0.0234	-0.0317	Unstable
15	0.64	22.6182	33.9385	0.0241	-0.0331	Unstable

Table 16. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 94 days on the type of stability using ODE 45 numerical method, scenario sixteen.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	27.4612	38.5443	-0.0563	0.0113	Unstable
2	0.64	29.7143	43.6802	-0.0723	0.0026	Unstable
3	0.64	33.0440	45.8607	-0.0851	-0.0039	Stable
4	0.64	32.9833	48.6676	-0.0906	-0.0072	Stable
5	0.64	33.2363	51.2287	-0.0963	-0.0106	Stable
6	0.64	33.6516	51.2753	-0.0975	-0.0111	Stable
7	0.64	33.3450	53.0248	-0.1002	-0.0128	Stable
8	0.64	31.7192	55.0555	-0.0133	-0.1002	Stable
9	0.64	32.5843	54.4793	-0.0137	-0.1012	Stable
10	0.64	32.0283	55.9658	-0.0148	-0.1028	Stable
11	0.64	32.3428	55.1698	-0.0142	-0.102	Stable
12	0.64	32.3329	54.9597	-0.0139	-0.1016	Stable
13	0.64	31.2674	55.2756	-0.013	-0.0996	Stable
14	0.64	30.9642	56.2788	-0.0138	-0.1008	Stable
15	0.64	31.7937	55.4369	-0.0138	-0.1012	Stable
16	0.64	30.8299	56.0713	-0.0134	-0.1001	Stable
17	0.64	30.8641	56.7544	-0.0143	-0.1015	Stable
18	0.64	30.8667	57.0429	-0.0146	-0.1021	Stable
19	0.64	31.4529	56.6603	-0.0149	-0.1028	Stable

Table 17. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 99 days on the type of stability using ODE 45 numerical method, scenario seventeen.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	30.0962	56.711	-0.0145	-0.1031	Stable
2	0.64	33.1582	57.1768	-0.0145	-0.1038	Stable
3	0.64	31.0555	57.7692	-0.0154	-0.102	Stable
4	0.64	31.1032	56.5793	-0.0148	-0.1029	Stable
5	0.64	30.9607	57.2308	-0.0152	-0.1016	Stable
6	0.64	31.0999	56.8068	-0.0156	-0.1011	Stable
7	0.64	31.5843	57.5226	-0.0140	-0.1018	Stable
8	0.64	30.0304	57.0894	-0.0153	-0.101	Stable
9	0.64	30.5044	56.5610	-0.0145	-0.1023	Stable
10	0.64	30.3894	56.2522	-0.0135	-0.1033	Stable
11	0.64	30.0992	55.5901	-0.0155	-0.1031	Stable
12	0.64	31.1321	56.7314	-0.0151	-0.1023	Stable
13	0.64	32.4102	57.6212	-0.0149	-0.1025	Stable
14	0.64	30.7683	56.1719	-0.0148	-0.1041	Stable
15	0.64	31.8479	58.0608	-0.0138	-0.1006	Stable

Table 18. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 104 days on the type of stability using ODE 45 numerical method, scenario eighteen.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	29.9465	57.9745	-0.0146	-0.1017	Stable
2	0.64	30.5322	58.2218	-0.0156	-0.1037	Stable
3	0.64	32.1170	55.1486	-0.0139	-0.1014	Stable
4	0.64	31.8178	56.0858	-0.0146	-0.1026	Stable
5	0.64	29.7171	58.1147	-0.0145	-0.1014	Stable
6	0.64	30.8098	57.3047	-0.0149	-0.1025	Stable
7	0.64	30.9348	56.2613	-0.0138	-0.1007	Stable
8	0.64	31.1641	56.8174	-0.0147	-0.1024	Stable
9	0.64	30.3934	56.7280	-0.0137	-0.1003	Stable
10	0.64	30.8942	57.0150	-0.0146	-0.1021	Stable
11	0.64	30.9440	56.7569	-0.0144	-0.1017	Stable
12	0.64	29.9880	58.4663	-0.0152	-0.1028	Stable
13	0.64	30.5920	57.4632	-0.0148	-0.1023	Stable
14	0.64	31.6116	56.7114	-0.0151	-0.1033	Stable
15	0.64	30.6316	56.8458	-0.0141	-0.1011	Stable

Table 19. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 109 days on the type of stability using ODE 45 numerical method, scenario nineteen.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	30.2864	57.6517	-0.0146	-0.1019	Stable
2	0.64	30.4020	57.6520	-0.0148	-0.1022	Stable
3	0.64	31.0986	57.1642	-0.0151	-0.1029	Stable
4	0.64	30.6135	57.3930	-0.0147	-0.1022	Stable
5	0.64	29.7683	57.9892	-0.0144	-0.1013	Stable
6	0.64	30.8916	57.1240	-0.0148	-0.1023	Stable
7	0.64	31.2030	56.9067	-0.0149	-0.1027	Stable
8	0.64	30.3465	57.5139	-0.0145	-0.1018	Stable
9	0.64	31.6161	56.1080	-0.0144	-0.1021	Stable
10	0.64	31.7441	56.2184	-0.0147	-0.1026	Stable
11	0.64	31.7153	57.2855	-0.0159	-0.1047	Stable
12	0.64	31.1826	57.1360	-0.0151	-0.1031	Stable
13	0.64	31.5492	55.9999	-0.0142	-0.1017	Stable
14	0.64	31.7071	57.0868	-0.0157	-0.1043	Stable
15	0.64	30.0436	57.5006	-0.0142	-0.1010	Stable

Table 20. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 114 days on the type of stability using ODE 45 numerical method, scenario twenty.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	31.0307	57.4088	-0.0153	-0.1033	Stable
2	0.64	31.5143	56.2193	-0.0144	-0.1021	Stable
3	0.64	29.6281	58.4973	-0.0148	-0.1020	Stable
4	0.64	30.4756	56.8279	-0.0139	-0.1007	Stable
5	0.64	30.6180	56.7915	-0.0140	-0.1010	Stable
6	0.64	31.4111	55.7937	-0.0138	-0.1010	Stable
7	0.64	32.0663	56.1391	-0.0150	-0.1033	Stable
8	0.64	30.2367	58.0529	-0.0151	-0.1026	Stable
9	0.64	31.0358	57.0192	-0.0148	-0.1025	Stable
10	0.64	30.6820	57.0553	-0.0144	-0.1017	Stable
11	0.64	30.9066	57.0734	-0.0147	-0.1023	Stable
12	0.64	31.6173	57.4091	-0.0160	-0.1047	Stable
13	0.64	31.3465	56.0379	-0.0140	-0.1013	Stable
14	0.64	31.8649	56.3491	-0.0150	-0.1032	Stable
15	0.64	31.9755	56.3076	-0.0151	-0.1034	Stable

Table 21. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 119 days on the type of stability using ODE 45 numerical method, scenario twenty-one.

Example	rpv	x_e	y_e	λ_1	λ_2	TOS
1	0.64	31.6215	56.2199	-0.0146	-0.1023	Stable
2	0.64	31.1256	56.9456	-0.0148	-0.1026	Stable
3	0.64	30.2660	56.6093	-0.0134	-0.0998	Stable
4	0.64	31.7125	55.4453	-0.0138	-0.1010	Stable
5	0.64	31.6894	56.1662	-0.0146	-0.1024	Stable
6	0.64	30.6495	56.9953	-0.0143	-0.1015	Stable
7	0.64	31.2750	55.8598	-0.0137	-0.1008	Stable
8	0.64	32.2730	55.5906	-0.0146	-0.1027	Stable
9	0.64	30.6385	56.0587	-0.0132	-0.0996	Stable
10	0.64	31.2170	56.6684	-0.0146	-0.1022	Stable
11	0.64	30.1600	57.6052	-0.0144	-0.1015	Stable
12	0.64	31.5667	56.6811	-0.0150	-0.1031	Stable
13	0.64	29.8946	57.9990	-0.0146	-0.1016	Stable
14	0.64	31.3782	56.0747	-0.0141	-0.1014	Stable
15	0.64	32.1588	56.3642	-0.0154	-0.1040	Stable

Table 22. Quantifying the effect of a random perturbation value (rpv) of 0.64 when the length of the growing season that define the growth of the two legumes cowpea and groundnut is specified by 124 days on the type of stability using ODE 45 numerical method, scenario twenty-two.

Example	**rpv**	x_e	y_e	λ_1	λ_2	**TOS**
1	0.64	29.8017	57.9240	-0.0144	-0.1013	Stable
2	0.64	30.2982	57.1140	-0.0140	-0.1009	Stable
3	0.64	30.2001	57.6619	-0.0145	-0.1017	Stable
4	0.64	30.7218	57.2232	-0.0147	-0.1021	Stable
5	0.64	32.2285	55.9920	-0.0150	-0.1034	Stable
6	0.64	30.0517	57.7898	-0.0145	-0.1016	Stable
7	0.64	30.3567	57.4346	-0.0145	-0.1016	Stable
8	0.64	30.9446	56.2413	-0.0138	-0.1007	Stable
9	0.64	32.0553	55.9221	-0.0147	-0.1028	Stable
10	0.64	30.9809	56.8996	-0.0146	-0.1021	Stable
11	0.64	30.6913	58.3850	-0.0160	-0.1044	Stable
12	0.64	31.4309	56.7358	-0.0149	-0.1029	Stable
13	0.64	31.6377	56.5126	-0.0149	-0.1030	Stable
14	0.64	30.9655	57.2211	-0.0150	-0.1027	Stable
15	0.64	31.0164	56.1545	-0.0138	-0.1007	Stable

5. Discussion of Results

The results show that on the implementation of the random perturbation value of (0.64) the proposed dynamical system was purely unstable when the length of the growing season ranges from 19 days to 44 days but when the length of the growing season increases to 49 days, we noticed a bifurcation (changing from instability to stability) and the system maintain stability from when the length of the growing season is 54 days up to the harvesting season (99 days) and beyond. The result also shows that the randomization affected the proposed dynamical system greatly as each time we run the analysis, a new result turns out whether the length of growing season is the same or not. It was equally found that the co-existence the steady-states fluctuate.

6. Conclusion

We have applied numerical approach of order 45 to ascertain the effect of randomization on the dynamical system and observed that when the length of growing season increases up to 49 days that there is bifurcation and the system maintain stability up to the harvesting time and beyond.

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