*Original Article*

# Confidentiality in Communication by using Sumudu transform and Cryptography

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*Abstract - In this Paper our aim is to present how to keep confidentiality in between two or more persons by using Sumudu transform and Cryptography. In the first part of the paper we will consider one message in plain-text and convert it in confidential form by applying Sumudu transform to trigonometric sine series. In second part we will convert this confidential message to our original message by applying inverse Sumudu transform.*

*Keywords - Sumudu transform, Inverse sumudu transform, Plain-text, Cipher text, Encryption, Decryption.*

## 1. Introduction

Confidentiality plays an important role in some fields like military services, C.I.D. Department, examination systems, etc. To keep confidentiality in communication between two or more than two persons some secured  password is necessary. To create such secured password cryptography branch is useful which deals with the study of encryption and decryption. The word cryptography originates from the Greek word crypto which means hidden and graph y means to write. The process to convert plaintext i.e. our original message to a new message in hidden form which cannot be able to open without some key or password is known as encryption. The reverse process that is to convert hidden message to original message is known as decryption. In University examination or any competitive examinations this cryptographic method will be more useful to keep security in examination work like question paper setting, results, etc. Now a days people p refers cashless transactions like mobile banking, phone pay, etc and to operate these facilities security is necessary. Original message which is to be converted in hidden form is known as cipher text and this process for conversion is called encryption. The reverse process to convert cipher text to plain text is known as decryption. There are various methods of cryptography i by applying number theory. Some integral transforms like Laplace transform, Sumudu transform, etc contributes in the process of cryptography i.e. encryption and decryption in literature [2]. Some authors have developed a cryptographic scheme by applying Laplace transform and Sumudu transform to hyperbolic sine and cosine function [4]. In 1990 Gamage K. Watugala has introduced a new integral transform namely Sumudu transform which has similarity with Laplace transform. Sumudu means smooth which is a Sinhala word [7]. In this paper we have applied Sumudu transform and inverse Sumudu transform to trigonometric sine function for the process of encryption and decryption.

### 1.1. Some Important Definitions and Theorems [7]

**Def.1.1.1** Sumudu Transform : The Sumudu transform of f(t) is defined by

$$G(u) = \int_0^\infty \frac{e^{-\frac{t}{u}}}{u} f(t)\, dt \text{ over the set C of functions defined by}$$

$$C = \left\{ f(t) \text{ such that there exists N}, t_1, t_2 > 0, |f(t)| < N e^{\frac{|t|}{|t_j|}}, t\epsilon(-1)^j X\, [0,\infty] \right\}$$

**Def.1.1.2:** If $G(u)$ is the Sumudu transform of $f(t)$ then the inverse Sumudu transform of  $G(u)$ is f(t) and we write $S^{-1}(G(u)) =$  f(t)

**Def.1.1.2: [3]** [The relation of congruent modulo n]

Let n be a positive integer. Then an integer $\alpha$ is congruent to an integer $\beta$ modulo n if n divides $\alpha - \beta$ .If $\alpha$ is congruent to $\beta$ modulo n then symbolically we write $\alpha \equiv \beta(\mod n)$. If $\alpha$ is not congruent to $\beta$ modulo n then we denote it as $\alpha \not\equiv 2$.

*Methodology of encryption & decryption by applying Sumudu transform to trigonometric sine function*

Suppose that we are provided A,B,C,D,E,………,Z as a plain text which is to be converted to cipher text. In the first step let us give the allotment to letters in the given plain text as given below

A→ 0, B → 1, C → 2, D → 3, E → 4, F → 5, G → 6, H → 7, I → 8, J → 9, K → 10, L → 11, M → 12, N → 13, O → 14, P → 15, Q → 16, R → 1 → 18, T → 19, U → 20, V → 21, W → 22, X → 23, Y → 24, Z → 25

In this method we consider trigonometric sine series given as

$$\sin nx = nx - \frac{n^3 x^3}{3!} + \frac{n^5 x^5}{5!} - \frac{n^7 x^7}{7!} + \frac{n^9 x^9}{9!} + \cdots \cdots$$

Therefore we have

$$x^m \sin nx = nx^{m+1} - \frac{n^3 x^{m+3}}{3!} + \frac{n^5 x^{m+5}}{5!} - \frac{n^7 x^{m+7}}{7!} + \ldots \quad (1)$$

Let $H_0$, H, $H_2$, ... be the coefficients of the eq$^n$ (1)
∴ We write

$$Hx^m \sin nx = H_0 \, nx^{m+1} - H_1 \frac{n^3 x^{m+3}}{3!} + H_2 \frac{n^5 x^{m+5}}{5!} \ldots \ldots \ldots \ldots (2)$$

By operating Sumudu transform to eq$^n$ (2) we will obtain one equation having some new variable in denominator and some values in the numerator (we call them as resulting values say $r_i$) adjusting these resulting values such that $r_i \equiv H_i{}' \bmod 26$ for i = 0,1, ⋯⋯, j we obtain $H_i{}'$ which is our required cipher text.
As decryption is the reverse process of encryption we can obtain plain text by applying Inverse Sumudu transform of $S\{Hx^m \sin nx\}$.

To determine cipher text by applying Sumudu transform trigonometric cosine function we may use the above method by considering some series of the form $Hx^m \cos nx$

3. Example: Let us consider one plain text given below
J  O  U  R  N  A  L  and we will create one cipher text for this plain text

By our allotment given plain text is equivalent to
9    14    20    17    13    0    11

Suppose that Let $H_0 = 9$, $H_1 = 14$, $H_2 = 20$, $H_3 = 17$, $H_0 H_4 = 13 H_4 = 13$, $H_5 = 0$, $H_6 = 11$
Case No.1: Let m=n=1 then equation (2) becomes

$$Hx \sin x = 9x^2 - 14\frac{x^4}{!3} + 20\frac{x^6}{!5} - 17\frac{x^8}{!7} + 13\frac{x^{10}}{!9} - 0\frac{x^{12}}{!11} + 11\frac{x^{14}}{!13} \quad (2)$$

Applying Sumudu transform to both sides we have
$$S\{Hx \sin x\} = 9\, S\{x^2\} - \frac{14}{!3} S\{x^4\} + \frac{20}{!5} S\{x^6\} - \frac{17}{!7} S\{x^8\} + \frac{13}{!9} S\{x^{10}\} - \frac{0}{!11} S\{x^{12}\} + \frac{11}{!13} S\{x^{14}\}$$

$$= 9! \, 2 \, u^2 - \frac{14}{!3} \, !4 \, u^4 + \frac{20}{!5} \, !6 \, u^6 - \frac{17}{!7} \, !8 \, u^8 + \frac{13}{!9} \, !10 \, u^{10} - \frac{0}{!11} !12 \, u^{12} + \frac{11}{!13} !14 \, u^{14} \quad (3)$$

$$\{Hx \sin x\} = 18 \, u^2 - 56 \, u^4 + 120 \, u^6 - 136 \, u^8 + 130 \, u^{10} - 0 \, u^{12} + 154 \, u^{14}$$

Let $r_0 = 18$, $r_1 = -56$, $r_2 = 120$, $r_3 = -136$, $r_4 = 130$, $r_5 = 0$, $r_6 = 154$

Now we will find $H_i'$ such that $r_i$ is congruent to $H_i'$ modulo 26.Then we have obtained $H_0' = -8$, $H_1' = -4$, $H_2'=16$, $H_3'=-6$, $H_4' = 0$, $H_5' = 0$, $H_6' = -2$

Thus we have obtained cipher (I,e, hidden text for our original message) for given plain text given below.

8   4   16   6   0   0   2

I.e. I   Q   G   H   A   A   C

If we want to keep confidentiality for our message JOURNAL we may provide this message in hidden form i.e. IQGHAAC .But to open this message one key is required which we can create by applying inverse Sumudu transform to equation (3)

Therefore applying inverse Sumudu transform equation (3) we have

$S^{-1}(S\{Hx\sin x\}) = 18\,S^{-1}(u^2) - 56\,S^{-1}(u^4)$

$+120\,S^{-1}(u^6) - 136 S^{-1}(u^8) + 130 S^{-1}(u^{10}) - 0\,S^{-1}(u^{12}) + 154\,S^{-1}(u^{14})$

$Hx\sin x = 18\dfrac{x^2}{2!} - 56\dfrac{x^4}{4!} + 120\dfrac{x^6}{6!} - 136\dfrac{x^8}{8!} + 130\dfrac{x^{10}}{10!} - 0\dfrac{x^{12}}{12!} + 154\dfrac{x^{14}}{14!}$

$Hx\sin x = 9\,x^2 - \dfrac{14}{3!}x^4 + \dfrac{20}{5!}\,x^6 - \dfrac{17}{7!}\,x^8 + \dfrac{13}{9!}\,x^{10} - \dfrac{0}{11!}\,x^{12} + \dfrac{11}{3!}\,x^{14}$

This is equation (2) whose coefficients are letters of plain text i.e.JOURNAL

## 2. Conclusion

From this paper we can encrypt and decrypt the given message by applying Sumudu transform & inverse Sumudu transform respectively to trigonometric sine function with the help of cryptography. In this way we can keep confidentiality to given message by using Sumudu transform & Cryptography.

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