International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT-V69I2P506

Volume 69 Issue 2, 45-52, February 2023

Original Article

On the Detour Eccentric D-Distance of some Graph Classes

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Received: 25 December 2022 Revised: 30 January 2023 Accepted: 11 February 2023 Published: 20 February 2023

Abstract *- In this article we introduce a new concept of detour eccentric D- distance by considering both the length of the path and eccentricities of the vertices present in the path of the graph. Then we derive some general theorems of detour eccentric D- distance in a connected simple graph. We also study the properties of detour eccentric D-distance on different graph classes and prove some results on detour eccentric D-radius, detour eccentric D-diameter and eccentric D-self centered graphs.*

Keywords *- Average distance, Average eccentricity, Bistar graph, Detour distance, Hamiltonian connected.*

1. Introduction

The importance of distance between any vertices in Graph Theory has been explained by the Mathematicians F.Harary[1], D.B West[22], Chatrand et al.[7], V. Venkateswara Rao[4], Doyale J.K[8], P.A. Ostrand[10], D. Reddy Babu et al.[22] Balci M A[17] and Babu DR[23] . In this paper we discuss the detour eccentric D-distance of a connected graph in terms of eccentricity and length of path. Here we restrictour study to those graphs which are simple, connected, undirected and with finite number of edges and vertices. The concept of distance theory is widely expanding area in graph theory and eccentric D-distance have useful applications in network theory and software development.

Let G(V, E) be a graph with finite non empty vertex set V and E is a binary relation on V . Take any arbitrary vertices u, v from the vertex set V of G, then

Definition 1.1. [6] Length of the shortest path between u and v, is called the geodesic distance and is denoted by $d(u, v)$.

Definition 1.2. [18] Length of the longest path between u and v is called the detour distance and is denoted by $D(u, v)$.

Definition 1.3. [19] The eccentricity of the vertex v is defined as $e(v) = max\{d(u, v), \forall\, u \in G\}$.

Definition 1.4. [20] The radius of a graph G is the minimum eccentricity of any vertex in the graph and is denoted by $rad(G) = min\{e(v), \forall\, v \in G\}$

Definition 1.5.[10]. The diameter of a graph is the maximum eccentricity of any vertex in the graph and is denoted by $diam(G) = max\{e(v), \forall\, v \in G\}$.

Definition 1.6. [21] The centre of a graph G are those vertices of G whose eccentricity equal to the radius of G.

Definition 1.7. [5] A peripheral vertex in a graph of diameter d is one that is at a distance d from some other vertex.

Definition 1.8. [3] A helm graph is a graph obtained from a wheel graph by adding a pendant edge at each node of the cycle.

Definition 1.9. [22] A star graph Sn, is a graph with one vertex of degree $n - 1$ and other vertices with degree one.

Definition 1.10. [2] Bistar is obtained by joining the Centre vertices of two copies of $K_{1,n}$ by an edge.

Definition 1.11.[11] A graph is said to be Hamiltonian-connected if there is a Hamiltonian path between every pair of vertices.

Definition1.12. [12] The distance of a vertex v, $\sigma_G(v)$ in graph G is the sum of minimum path lengths of v from the other vertices of G.

Definition1.13.[16] Average eccentricity *avec(G)* of a graph G is the mean of the eccentricities of the vertices of G.

Definition1.14..[14]Average distance of G is $\mu(G) = \frac{1}{n(n-1)} \sum_{v\epsilon\sigma_G(v)} \sigma_G(v)$.

2. New Results

Definition 2.1. If P is a path with end vertices u and v, then the eccentric D-length of u-v path P is defined as $l^e(P) = l(P) + e(u) + e(v) + \sum e(w)$ where $l(P)$ is the length of the path and $\sum$ runs over all the eccentricities of the vertices in the u-v path.

Definition 2.2. The eccentric D-distance $d^e(u,v)$ between any two vertices u and v is defined as $d^e(u,v) = min\{l^e(P)\}$, if u and v are distinct vertices and $d^e(u,v) = 0$, if u and v are same, where the minimum is taken over all the paths between u and v in G.

i.e.,

$$d^e(u,v) = \begin{cases} min\{l(P) + e(u) + e(v) + \sum e(w)\}, & if\ u \neq v \\ 0, & if\ u = v \end{cases}$$

where the minimum value is taken over all the u − v paths P in G.

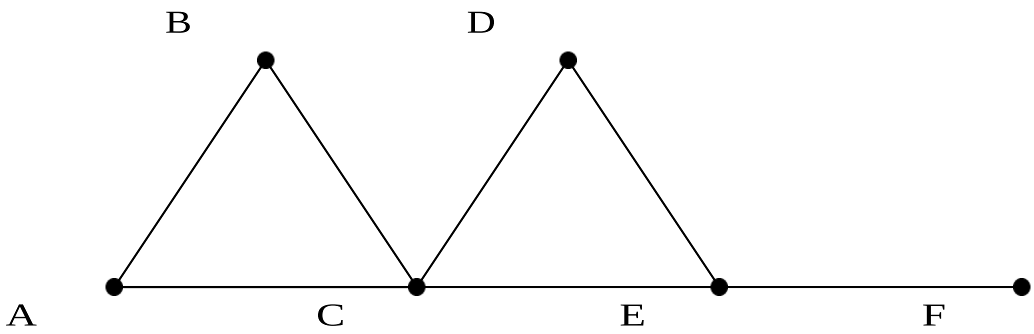
Definition 2.3. The detour eccentric *D*-distance between two vertices *u*, *v* of a connected graph *G* is defined as $D^e(u,v) = max\{l^e(P)\}$, if *u* and *v* are distinct vertices and $D^e(u,v) = 0$, if u and v are same, where the maximum is taken over all the paths between u and v in G

I.e.,

$$D^e(u,v) = \begin{cases} max\{l(P) + e(u) + e(v) + \sum e(w)\}, & if\ u \neq v \\ 0, & if\ u = v \end{cases}$$

where the maximum value is taken over all the u − v paths P in G.

Illustration 2.4. Consider a simple Graph *G* with 6 vertices and 7 edges



In this example we will compute the detour eccentric *D* distance between the vertices *A* and *E*. The eccentricities of the vertices are given by $e(A) = 3, e(B) = 3, e(C) = 2, e(D) = 2, e(E) = 2$ and $e(F) = 3$. Consider all paths between *A* and *E*. Various paths between *A* and *E* and their *D*−lengths are given below.

$P_1 : A - C - E$, the eccentric *D*−length of $P_1 = l^e(P_1) = 2 + (3 + 2 + 2) = 9$.
$P_2 : A - B - C - E$, the eccentric *D*−length of P_2 is
$l^e(P_2) = 3 + (3 + 2 + 3 + 2) = 13$.
P_3 : A−C−D−E, the eccentric D−length of $P_3 = l^e(P_2)=3+(3+2+2+2)=12$
$P_4 : A - B - C - D - E$, the eccentric D−length of P_3 D length of P_4
$= l^e(P_4) = 4 + (3 + 2 + 3 + 2 + 2) = 16$.

From this we reach the conclusion $D^e(A, E) = \max\{P_1, P_2, P_3, P_4\} = 16$.

The following table details the detour distance and detour eccentric $D - dis$tances between all other vertices in the given graph .

Table 1. detour distance between the vertices of *G*

D(u,v)	A	B	C	D	E	F
A	0	2	2	3	4	5
B	2	0	2	3	4	5
C	2	2	0	2	2	3
D	3	3	2	0	2	3
E	4	4	2	2	0	1
F	5	5	3	3	1	0

Table 2. Detour eccentric *D*−distance between the vertices of *G*

$D^e(u,v)$	A	B	C	D	E	F
A	0	10	10	13	16	20
B	10	0	10	13	16	20
C	10	10	0	8	8	12
D	13	13	8	0	8	12
E	16	16	8	8	0	6
F	20	20	12	12	6	0

From these tables, we reach the conclusion that $D^e(u,v) \geq D(u,v)$.

Theorem 2.5. The detour eccentric D- distance between the vertices of a connected graph is a metric.

Proof. Let G be a connected graph with vertex set V and edge set E. Let u and v be any two vertices of G. Then $D^e(u,v) > 0$ and $D^e(u,v) = 0$ only when $u = v$.

Also, $D^e(u,v) = D^e(v,u) > 0$, so that the D distance is symmetric.

Now to prove the triangle inequality, let u, v and w be any three arbitrary vertices of $V(G)$ such that the vertex w lies in the path from u to v with $D^e(u,v) > a$.

Now, let P_1 be the path from u to w and P_2 be the path from w to v with detour eccentric D− distances b and c respectively. Therefore $a = b + c$. Then

$$D^e(u,v) = a = b + c \leq D^e(u,w) + D^e(w,v).$$

Thus, triangle inequality is verified if w is in a path between u and v. The result is also true if w is not in a u-v path. Hence detour eccentric D distance is a metric on the set of vertices of a connected graph G.

Definition 2.6. If G is a connected graph and v is any vertex of G, then the detour maximal D–eccentricity of v is denoted by $e_D{}^e(v)$ and is defined as

$e_D{}^e(v) = max\{D^e(u,v): for\ every\ u\ in\ V(G)\}$. Thus, the maximal detour D-eccentricity of v denote the maximum detour eccentric D distance from other vertices of G.

Definition 2.7. The minimum of detour maximal D-eccentricity among all the vertices of G is called the detour eccentric D-radius and is denoted by $r_D{}^e(G)$ ie,

$r_D{}^e(G) = min\{e_D{}^e(v); for\ every\ v\ in\ G\}$.

Definition 2.8. The maximum of detour maximal D-eccentricity among all the vertices of G is called the detour eccentric D-diameter and is denoted by $diam_D{}^e(G)$

I.e., $diam_D{}^e(G) = max\{e_D{}^e(v): for\ every\ v\ in\ V(G)\}$

Definition 2.9. If for a vertex the detour maximal D- eccentricity is equal to the detour eccentric D-radius, in a connected graph G, then v is called detour eccentric D-centre and is denoted by $C_D{}^e(G)$,

I.e., $C_D{}^e(G) = \{v \in G: r_D{}^e(G) = e_D{}^e(v)\}$

Definition 2.10.The detour eccentric D- peripheral vertex of a graph G of detour eccentric D-diameter d is one that is at a distance d from other vertex and is denoted by $P_D{}^e(G), ie\ P_D{}^e(G) = \{v \in G: e_D{}^e(v) = diam_D{}^e(G)\}$.

Definition 2.11.If for a graph the detour eccentric D- radius is same as the detour eccentric D-diameter the graph is said to be detour eccentric D-self centered.

From the above-described graph we can summaries that the detour maximal D- eccentricity, detour eccentric D-radius, detour eccentric D- diameter as follows.

Here, detour eccentric D- radius = 12
detour eccentric D- diameter = 20
detour eccentric D- centre = $\{C\}$
detour eccentric D- peripheral = $\{A, B, F\}$

Table 3. Detour maximal *D*−eccentricities

Vertex	**A**	**B**	**C**	**D**	**E**	**F**
Detour eccentricity	5	5	3	3	4	5
Detour maximal *D*−eccentricity	20	20	12	13	16	20

Theorem 2.12. For any connected simple graph G, $r_D{}^e(G) \leq diam_D{}^e(G) \leq 2r_D{}^e(G) \leq 2e_D{}^e(x)$ for any vertex x in G

Proof: . If G is a connected simple graph, we always have ,$r_D{}^e(G) \leq diam_D{}^e(G)$.

Now to prove the other inequalities, let u and v be any two vertices of G such that

$diam_D{}^e(G) = e_D{}^e(u) = D^e(u, v)$ and w is any vertex in the $u - v$ path with $r_D{}^e(G) = e_D{}^e(w)$.Then by triangle inequality we have

$$D^e(u, v) \leq D^e(u, w) + D^e(w, v)$$
$$\leq e_D{}^e(w) + e_D{}^e(w)$$
$$\leq r_D{}^e(G) + r_D{}^e(G)$$
$$\leq 2r_D{}^e(G)$$

Since, $r_D{}^e(G) \leq e_D{}^e(x)$ for any vertex x, we have $2r_D{}^e(G) \leq 2e_D{}^e(x)$.

Theorem 2.13. In a connected graph G, which is simple $e_D{}^e(x) + e_D{}^e(y) \geq diam_D{}^e(G)$

Proof: Let $diam_D{}^e(G) = D^e(x, z)$ for some vertices x,z of G. Then

$$diam_D{}^e(G) = D^e(x, z) \leq D^e(x, y) + D^e(y, z) \leq e_D{}^e(x) + \leq e_D{}^e(y)$$

Thus $e_D{}^e(x) + e_D{}^e(y) \geq diam_D{}^e(G)$.

Theorem 2.14. For a connected graph $e_D{}^e(x) - e_D{}^e(y) \leq r_D{}^e(x)$

Proof: Let $e_D{}^e(x) = D^e(x, z)$ where x,z in V(G).

Then $e_D{}^e(x) = D^e(x,z) \leq D^e(x,y) + D^e(y,z)$

$$\leq r_D{}^e(G) + e_D{}^e(y) \text{ and hence the result.}$$

Theorem 2.15. In a connected simple graph G, $e_D{}^e(u) = D^e(u,v)$ if and only if $d(u,v)$ is maximum for some vertex v in G.

Proof. Suppose that $d(u,v)$ is maximum for the $u-v$ path P in G and let $D(u,v) = C$. Hence there exist a greater number of vertices in $u-v$ path P than in other $u-v$ paths. Hence sum of the eccentricities in path P will be greater than in other paths. Therefore $D^e(u,v)$ is maximum.

Thus $e_D{}^e(u) = D^e(u,v)$

Conversely, let $e_D{}^e(u) = D^e(u,v)$ for some vertex v in G. Then obviously $d(u,v)$ is maximum.

Theorem 2.16. Petersen graph is a self centred graph w.r.t detour eccentric D-distance.

Proof. Eccentricity of each vertex of Petersen graph, with 10 vertices and 15 edges, is 2 and the detour distance between the vertices of the graph is 9. Then, detour maximal D−eccentricity of any vertex v is $e_D{}^e(v) = 9 + (10 * 2) = 29$.

Thus, in Petersen graph $r_D{}^e(v) = diam_D{}^e(v) = 29$. Hence Petersen graph is a detour eccentric *D*- self centered graph.

Theorem 2.17. In a Hamiltonian-connected graph,

$$\mu(G) + n - 1 \leq \text{diam}_D{}^e(G)$$

where $\mu(G)$ is the distance of a vertex, n is the total number of vertices and $diam_D{}^e(G)$ is the detour eccentric D-diameter of G

Proof: Let G be a Hamiltonian-connected graph. Then

$$\text{diam}_D{}^e(G) = e_D{}^e(u), \text{for some vertex in G}$$

$$= D(u,v) + eccentricites\ of\ the\ vertces\ in\ the\ u-v\ path$$

Since the graph is Hamiltonian -connected with n vertices, $D(u,v) = n-1$

Therefore, $\text{diam}_D{}^e(G) = n - 1 + \sum_{v\in V} e(v)$

$$\geq n - 1 + \frac{\sum_{v\in V} e(v)}{n}$$

$$\geq n - 1 + avec(G)$$

$$\geq n - 1 + \mu(G), \text{ since } avec(G) \geq \mu(G)$$

Thus, $\mu(G) + n - 1 \leq \text{diam}_D{}^e(G)$.

Theorem 2.18. In any graph G the detour maximal D-eccentricity of a vertex v has the upper bound

$$e_D{}^e(v) \leq n + \frac{1}{n}\sigma(G) + rad(G)$$

Proof: Let G be a connected simple graph. Then,

$$e_D{}^e(v) = D(u,v) + eccentricites\ of\ the\ vertces\ in\ the\ u-v\ path$$

$$\leq n - 1 + \sum_{v\in V} e(v)$$

$$\leq n + avec(G)$$

$\leq n + \frac{1}{n}\sigma(G) + rad(G)$.

Thus, the proof.

3. Results of detour eccentric D−distance on some graph classes

Theorem 3.1. For the Complete graph K_n, the detour maximal D-eccentricity, $e_D{}^e$ of any vertex v is $2n - 1$
Proof. Since K_n is a complete graph on n vertices, the eccentricity of each vertex v is one.

Then by definition, for any vertices u, v in K_n, $D(u, v) = n - 1$ and therefore,

$$D^e(u, v) = (n - 1) + (n.1) = 2n - 1.$$

Thus, the detour maximal D− eccentricity of each vertex in K_n is $2n - 1$

Corollary 3.2. K_n is detour eccentric D−self centered graph.

Theorem 3.3. For a cycle graph C_n, the detour maximal D−eccentricity of any vertex v is

$$e_D{}^e(v) = \begin{cases} \frac{n^2+2n-2}{2}, if\ n\ is\ even \\ \frac{n^2+n-2}{2}, if\ n\ is\ odd \end{cases}$$

Proof: In a cycle graph $C_{n,}$ with n vertices, take two vertices u and v such that the detour distance between them is $n - 1$. Eccentricity of each of vertices is $n/2$ if n is even and $(n - 1)/2$ if n is odd. Then

$D^e(u, v) = (n - 1) + (n.n/2) = \frac{n^2+2n-2}{2}$, if n is even and

$D^e(u, v) = (n - 1) + \frac{n(n-1)}{2} = \frac{n^2+n-2}{2}$, if n is odd.

Hence the maximal D-eccentricity of each vertex is

$$e\ _D{}^e(v) = \begin{cases} \frac{n^2 + 2n - 2}{2}, if\ n\ is\ even \\ \frac{n^2 + n - 2}{2}, if\ n\ is\ odd \end{cases}$$

Thus, the result.

Corollary 3.4. Cycle graph C_n is a detour eccentric D-self centred graph.

Theorem 3.5. Given a Wheel graph W_n with n vertices. Then the detour maximal D−eccentricity of any vertex of v, $e_D{}^e(v)$ is $3n - 2$.

Proof. In a wheel graph W_n, the eccentricity of the centre vertex is one and the others are 2. Let u be a vertex of W_n. Choose an arbitrary vertex v such that their detour distance is $n - 1$. Then

$D^e(u, v) = (n - 1)+((n - 1)\times 2)+1 = 3n - 2$.

Hence the detour maximal D- eccentricity of any vertex is $3n - 2$. Hence the proof.

Corollary 3.6. W_n is a detour eccentric D−self centred graph.

Remark 1: Hamiltonian-connected graphs Petersen graph, Complete graph *Kn,* wheel graph *Wn*, Cycle graph *Cn* are all detour eccentric self centred.

Relation between distance of a vertex and the detour eccentric D-diameter in a Hamiltonian-connected graph

Remark 2: Hamiltonian-connected graphs satisfies the relation $\sigma_G(v) + n - 1 \leq \text{diam}\ {}_D{}^e(G)$ where $\boldsymbol{\sigma_G(v)}$ is the distance of a vertex, n is the total number of vertices and diam ${}_D{}^e(G)$ is the detour eccentric D-diameter of G

Theorem 3.7. For any complete bipartite graph $K_{m,n}$ with mn edges and $m + n$ vertices, $m \leq n$, the detour maximal D−eccentricity of any vertex v is $4m - 1$.

Proof. Let the graph $K_{m,n}$ is bi-partitioned into X and Y with m and n vertices respectively. We have, eccentricity of each vertex v of $K_{m,n}$ is 1 and the detour distance between any two vertices is $2m - 1$.

If the vertex u is selected from X and v is from Y then,

$D^e(u, v) = 2m - 1 + 2m - 1 + 1 = 4m - 1.$

Hence the result follows.

Corollary 3.8. The complete bipartite graph $K_{m,n}$ is self centred with respect to detour eccentric D−distance.

Theorem 3.10. For a star graph, $S_n(n \geq 5)$ with n vertices, the detour maximal

D- eccentricity of any pendent vertex is 7 and that of the center vertex is 4.

Proof. Eccentricity of pendant. vertex of star graph is 2 and that of the centre vertex is 1. For a pendent vertex v, ${e_D}^e(v) = 2 + (2 + 2 + 1) = 7$ and for the centre vertex u, ${e_D}^e(u) = 1+(1+2) = 4$. Thus ${r_D}^e(S_n) = 4$, ${diam_D}^e(S_n) = 7$.

Remark 2: ${diam_D}^e(S_n) - {r_D}^e(S_n) = 4$

Theorem 3.11. For helm graph H_n, $n \geq 4$ *with* $2n + 1$ vertices, the detour maximal D- eccentricity of the pendant vertices are $2n + 10$ and that of the other vertices are $2n + 5$.

Proof. In helm graph H_n, eccentricity of the pendant vertices, centre vertex and the other vertices are 4, 2, 3 respectively. For a pendant vertex v,

$${e_D}^e(v) = D^e(u, v) = (\tfrac{(n-1)}{2} + 2) + (2 + (2 \times 4) + (\tfrac{n-1}{2})3)$$

$$= 2(n - 1) + 12 = 2n + 10.$$

The detour maximal $D-$ eccentricity of the other vertices are

$${e_D}^e(u) = D^e(u, v) = (\tfrac{(n-1)}{2} + 1) + (2 + (1 \times 4) + (\tfrac{(n-1)}{2}3)$$

$$= 2(n - 1) + 7 = 2n + 5.$$

Corollary 3.12: For Helm graph H_n, ${r_D}^e(H_n)$=2n+5 and ${diam_D}^e(H_n) = 2n + 10, n \geq 4$.

Theorem 3.13. In a bistar graph $B_{n,n}$, the detour D-eccentricity of the pendant vertices are 13 and that of others are 9.

Proof. Eccentricity of the pendant vertices in $B_{n,n}$ is 3 and that of othersare 2. The detour maximal D-eccentricity of a pendent vertex v is ${e_D}^e(v))$ =3 + (3 + 3 + 2 + 2) = 13. The detour maximal D-eccentricity of other two

vertices are ${e_D}^e(u))$ = 2 + (2 + 3 + 2) = 9.

Corollary 3.14. For Star graph $B_{n,n}$, ${e_D}^e(B_{n,n}) = 9$ and $diam\ {}_D{}^e(B_{n,n}) = 13$.

4. Conclusion

In this paper we found the detour eccentric $D-$distance using eccentricity and path length of some standard graphs like K_n, wheel graph, cycle graph, helm graph etc. Also, we have derived some theorems on detour eccentric $D-$distance for simple graphs. Further we can study the detour eccentric $D-$distance of other graph classes and the relation between different distance concepts of graph theory.

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